KWAME NKRUMAH UNIVERSITY OF SCIENCE AND

TECHNOLOGY, KUMASI

INTEGER LINEAR PROGRAMMING MODEL OF COMMODITY TRADING: A CASE STUDY OF OBUASI AND TECHIMAN CENTRAL MARKETS.

BY

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A thesis submitted to the College of Science in the partial fulfillment of the requirement for the degree of MSc. in Industrial Mathematics at Kwame Nkrumah University of Science and Technology

JUNE, 2014

Certification

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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Dedication

This thesis is dedicated to my two children, Paapa Anyemadu Ankomah and Adwoa Nuako Ankomah

ABSTRACT

The goal of every commodity trader is to maximize his/ her profit, but what should traders do to maximize their profit? In this thesis the wholesale spot prices of six agricultural commodities in Obuasi and Techiman Central markets were examined. The trader purchases commodities from one market place and sells in the other market taking into consideration the spot price of the commodities in questions. In this thesis we are interested in finding the volume of the six commodities to be purchased from Techiman and sold in Obuasi central market to make maximum profit knowing the demand of those commodities. Linear programming Solver (software) was used to solve real trader's problem of a trader in the presence of some constraints

ACKNOWLEDGEMENT

The fear of the Lord is undeniably the beginning of Wisdom. At the outset I thank and praise the Almighty whose blessings and grace had bestowed in me the power and confidence to successfully complete this research work.

My gratitude is highly expressed to the lecturers in the Department of Mathematics for the guidance given to me to accomplish this work. I particularly owe an appreciable gratitude to my supervisor, Mr. K.F. Darkwah for his guidance and constructive suggestions leading to the realization of this work. I thank Mrs. Georgette Adams for editing the grammar in this thesis. I am also deeply indebted to my father Mr. John Carr who graciously gave me perfect education and always encourages me that the sky should always be my limit. I would like to extend my heartfelt thanks and gratitude to him.

Motivation will almost always beat mere talents. Hence I would be failing in my duty if I do not thank my well-wishers: Elder Charles Nti (presiding elder, C.O.P Maranatha Assembly-Akaporiso District) ,Mr. Peter Amponsah (former headmaster, Akrofuom Senior High School) for motivating me to do the work with great confidence.

My sincere thanks to my adorable wife Mrs. Charlotte Ankomah who rendered her support both mentally and physically throughout the writing of this thesis.

Finally, thanks to all the people who were directly and or indirectly involved in bringing out this research work. May God bless you all.

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CHAPTER ONE

1.0 Introduction

These days, agricultural commodities have become the building blocks of a large part of our economy. Commodities are relatively easily traded and can be delivered physically anywhere in the world. More than 91 million 60kg coffee bags were traded in 2006, and more than 84 million barrels of crude oil are consumed every day (Source Reuters: commodity price facts). Now it is observed that, the volume of agricultural commodity trading in the world is very huge. Many less-developed countries, like Ghana, depend heavily on the exports of a small number of primary commodities for income. In some countries there are big firms with huge departments dedicated exclusively to commodity trading.

Commodities are traded in very active markets. In Africa there are two main commodity markets – Africa Mercantile Exchange (AfMX) based in Nairobi, Kenya which deals with agricultural, equity and energy products, and Ethiopia Commodity Exchange (ECX) based in Addis Ababa which deals only in agricultural products. Other examples of commodity markets are the Chicago Board of Trade (CBOT), the New York Mercantile Exchange (NYMEX), and the London Metal Exchange (LME). Prices are determined by the market, rather than by the large suppliers or the large buyers, with the exception perhaps of the Organization of the Petroleum Exporting Countries (OPEC) which has the sole right to determine prices. Spot prices at commodity markets exhibit several salient features: they are highly auto-correlated and extremely volatile with rare but violent explosions in price. Figure 1.1 is an example of the time series plot of a commodity (Cocoa) with high volatility and spikes in price (Source Reuters: commodity price facts). Deaton and Laroque (1996) explained the volatility by inventory holding dynamics: where speculators carry inventory, the price remains stable, but when they are out-of-stock, the price fluctuates wildly.

1.1 Commodity Market in Ghana

Over the years, commodity prices have displayed a more volatile behavior than non-agricultural commodity prices. The graph in figure 1.1 shows the spot prices of Cocoa over ten years. In the chart, it could be observed that in October 2009 and April 2011 the sport prices was highly volatile.

The high degree of volatility could be attributed to two main factors:
Production uncertainty and stock shifting. Output uncertainty for agricultural biological factors such as diseases and pest. On the other hand stocks are important because production is normally seasonal and anticipated or demand shocks are instantly reflected in the current price. For example, if during the growing season a major production area of Maize is flooded, the expected Maize production will be lower. Traders will have a reason to hold on to their current stock since they know that future price on that commodity is likely to increase, all things being equal. Couple with these, supply effects with a general inelastic demand for agricultural commodities can lead to a large price swings (Dhingra, 1986).

The figure 1.1 is the price chart for Cocoa in the world market between October 2006 and October 2012.

Fig. 1.1 Cocoa price chart.

Figure 1.1 shows that, spot price of Cocoa in a commodity markets is extremely volatile with rare but violent explosions in price (Source Reuters: commodity price facts).

It is commonly assumed that the price of a commodity is the same everywhere in the world at any point in time, as occurs for stocks or bonds. This is not so for agricultural commodities, as the price in each location is adjusted for logistics costs and local market conditions among others (Hull, 2003). For example, a maize seller who buys from one market and sells in another markets located at different places will pay different prices for deliveries in each one of the markets. One would assume that these differences are very small or at least stable over time. This is true in the long-run but in some cases price spreads, i.e., price difference between two locations can be significant in the short term (Hull, 2003).

1.2 Background of the study

The word commodity came into use in English in the 15th century and is derived from the French word commodite, similar in meaning to "convenience" in terms of quality of services. The Latin root is commoditas, referring variously to the appropriate measure of something; a fitting state, time or condition; a good quality; efficaciousness or propriety; and advantage, or benefit. The German equivalent is die Ware, i.e., wares or goods offered for sale. The French equivalent is produit de base like energy, goods, or industrial raw materials. The Spanish translation is mercancia and mercaderiain Catalan. In the original and simplified sense, commodities were things of value, of uniform quality, that were produced in large quantities by many different producers. The items from each different producer were considered equivalent. In a broader sense one can think of a commodity to be anything for which there a demand is, but which is supplied without qualitative differentiation across a given market. Examples include not only minerals, metals and agricultural products such as iron ore, aluminum, silver, gold, sugar, Cocoa, rice, wheat, but also energy sources such as coal, oil, and natural or liquefied gas, and even intermediary or manufactured products such as chemicals or generic drugs (Seidel and Ginsberg, 1983).

In economics, a commodity is the generic term for any marketable item produced to satisfy wants or needs. In this thesis, commodity will be applied to goods only for which there is a demand. Hard commodities are those that are extracted through mining such as Gold, Manganese, etc., whereas soft commodities are goods that are grown or obtained through agricultural activities. There is another important class of energy commodities which includes electricity, gas, coal and oil (Seidel and Ginsberg, 1983).

Market power is a term used to explain demand and supply, representing the aggregate influence of self-interested buyers and sellers on price and quantity of the goods and services offered in a market. In general, excess demand causes prices and quantity of supply to rise and excess supply causes them to fall (Seidel and Ginsberg, 1983).

Trading is the activity of buying and selling between people at different locations with the aim of making a profit. In Ghana, agricultural commodity trading moves huge volumes of goods and money throughout the country (Seidel and Ginsberg, 1983).This thesis considers the bi-directional problem of a commodity trader and how they purchase in one market and resell in the other (or vice versa) depending on the spot price at the two markets.

1.2.1 Spot market price

A spot market is a place where individuals and groups of people exchange commodities and money. There are two parties involved in trading, one party sells and one party buys, that is pays for the commodity at an agreed price. The agreed price is the spot price. In a spot market, delivery and payment takes place immediately, which is termed settlement. This is mostly the case in our traditional market today. This thesis is much interested in the spot prices of some agricultural commodities in two different market places. If the time between the purchase or payment for a commodity is two or more days then the trade is termed forward or future transaction. In the last chapter of this thesis, the pros and cons of the future contracts shall be analyzed. Spot transactions can take place in organized markets such as weekend markets or market days (Tomek and Robinson, 2003).

1.2.2 Market fundamentals of demand and supply

Apart from the different forms of risk, the role of prices and price formation is very crucial. For agricultural commodity trading, Traders make decisions to buy or sell based on the spot price. For instance, a buyer may decide to buy more if price is relatively low. It is therefore important to analyze the market fundamentals of demand and supply (Merton, 1973).

1.2.3 Normal and inferior goods

It is necessary to understand the difference between normal and inferior goods. This is important in agricultural commodity demand and supply because foodstuff, for instance, satisfies basic needs. The classification is useful when it comes to elasticity (Tomek and Robinson, 2003).

Normal Goods are items which are consumed more when an individual's income goes higher. Consider, for example, filet mignon. If you have more income and you aren't a vegetarian, you will end up consuming more filet mignons that you would with a lower income. In other words, quantity demanded rises as income rise.

Inferior Good: These goods are not of poorer quality than goods are not of poorer quality than other goods, as is suggested by the adjective "inferior" . instead, they are goods that you consume less of if your income rises. Consider for example, ramen noodles. People with lower income will probably consume more ramen noodles than they would if their incomes start to rise to a level there they could afford more expensive and more wholesome food. In other words, quantity demanded of ramen noodles falls as incomes rise.

1.3 Problem Statement

The decision that traders would have to make on what commodities to trade in and its volume is always a difficult one. Generally, the primary aim of every commodity trader is to make profit and then maximize it. Today many commodity traders fail to make profit, sometimes they break even or run at a loss. Because of the fear of losing one's capital, lots of traders do not wish to enter into commodity trading. It is believed, the problem is s a result of lack of knowledge in the market dynamics, and this is what this thesis seeks to provide. If this problem is ignored;

i) People involved in the commodity trading businesses will be trailing profit.

ii) Traders with bank loans will suffer to repay and they may end up paying more or in jails.

iii) Unemployment will be on the rise; a consequential effect on the growth of the economy.

This thesis brings to light what traders should fully understand and consider before venturing into a particular commodity trader.

1.4 Objective

The objectives of this thesis are as follows:

i) To model quantities of commodities traded at Obuasi as a Linear Programming problem (LPP)

ii) To determine optimal quantities of commodities using Integer Linear Programming (ILP).

1.5 Methodology

In this thesis a survey was conducted in Obuasi and Techiman central markets to know the demands of six agricultural commodities and their market dynamics.

Data was collected through face-to-face interview. The interview was conducted on a sample of 20 market women on the wholesale prices and demands of the six commodities under consideration. The duration of the data collection was between January –June 2013

The trader's problem was modeled as an Integer programming problem because integer variables represent quantities that can only be integer. For tractability, the thesis considers no fractional volume of commodities. The problem was solved using Lip solver. A Linear Optimization Solver was downloaded free-of – charge from the internet to aid in the computation of the optimality which may take several iterations and computation time.

1.6 Justification

The study has many policy and useful implications to current and prospective commodity traders (stakeholders). It is believed that the outcome of this thesis will benefit stakeholders in various ways:

1. The thesis will pre-inform the traders trading between Obuasi and Techiman central markets which commodity to trade in for maximum profit.
2. Given limited financial resources of traders, what volume of commodities should be traded between the two markets to maximize profit?

3. The work will also inform the famers in the two market areas as to which commodity should be produced more knowing the demand of each of the commodities under consideration.

4. This work when completed will be made available to the Obuasi Municipal Assembly. It is believed that the work will make the trading activity attractive for current traders and prospective traders as well.

1.7 Thesis organization

The Thesis is organized as follows. Chapter 1, a short introduction to commodity trading and the major commodity markets in the world. The chapter also captured spot market price dynamics and the benefits of the thesis are stated.

In Chapter 2, previous literature were reviewed as related to this thesis. It considered spot market pricing; characteristic of Ghanaian markets, forward and futures contracts, risk minimization in trading and strategies involved in commodity trading.

Chapter 3 focused on the method employed in solving the problem that was formulated. The components involved in formulating a linear programming problem were discussed.

Chapter 4 dealt with the methods used in solving LPP and the assumptions of the model. It also described in detail the model and its notations. The model is applied to six agricultural commodity prices in Ghana, which gave a favorable response.

Finally in Chapter 5, conclusions and recommendations were drawn with the discussion of future research directions discussed.

CHARPTER TWO

LITERATURE REVIEW

2.1 Introduction

It is useful to begin with a summary of the essential mathematical and financial concepts involved in the modeling of commodity pricing and market dynamics. There is a lot of research work on commodity pricing and analysis of commodity trading in two different literature streams: the first one, from economics and finance, and the second one, from operations Research points of view.

2.2 Spot Price Modeling

In Deaton and Laroque (1992), a first attempt to confront the model with actual commodity prices using annual data on 12 commodities from 1900 to 1987 were made. Under the assumption of independently and identically distributed (i.i.d.) harvest shocks, an assumption that has dominated previous discussion in the literature, they simulated the model in an attempt to reproduce some of the stylized facts of commodity price behavior, and they tested some of its implications, for example, the relationship between current and expected future prices. Their results were encouraging, at least in some respects. Simulated data reproduced a pattern of "doldrums" interrupted by upward "spikes" that is characteristic of many actual commodity prices. Furthermore, their limited econometric tests could not reject the implication of the model that, below a fixed cutoff, one-period-ahead price expectations are current prices multiplied by a factor greater than unity, whereas above the cutoff, expectations are constant. However, without starting from auto-correlated shocks, and thus building autocorrelation into the prices by construction, they did not find parameter values whose associated simulations reproduced the high levels of positive autocorrelation that are displayed by the actual series. Whether or not such parameters exist is a question that they could not answer because the application of generalized method of moments (GMM) estimation to the commodity price model leaves crucial parameters unidentified. Without estimates of all the parameters, it is impossible to calculate the autocorrelations implied by the model and to compare them with the data. Their model fails to explain or answer the central question of whether the highly auto-correlated price data are consistent with profit-maximizing and risk-neutral speculators' acting on an (i.i.d). weather-driven process. The Deaton-Laroque (1992) had the following limitations.

First, they utilize low frequency (annual) data for a wide variety of very heterogeneous commodities. Since in reality economic agents make decisions regarding storage daily, if not intraday, the frequency of their data is poorly aligned with the frequency of the economic decisions they are trying to assess empirically. Moreover, Deaton-Laroque impose a single model on very different commodities. Their commodities include those that are produced continuously and have non-seasonal demand (e.g., industrial metals such as tin and copper), those that are planted and produced seasonally (e.g., corn and wheat), and others that are produced seasonally from perennial plants (e.g., coffee and cocoa).

Finally, their use of annual data forces them to estimate their model with decades of data encompassing periods of major changes in income, technology, policy regimes, and trade patterns (not to mention wars), but they do not allow for structural shifts.

This work was later extended in Deaton and Laroque (1996), and proved beyond doubt that speculation is not the only cause of the high positive out correlation in commodity prices.

They explain that other factors such as demand and supply also contribute to the positive auto correlation in price of commodities.

Schwartz and Smith (2000) present a two-factor spot model where the stochastic processes are for short- and long-term variations, with no explicit inclusion of convenience yield. The model is found to be equivalent to that of Gibson and Schwartz (1990) and so does not offer anything particularly innovative, though has practical advantages. In the same vein, Cortazar and Schwartz (2003) develop other two- and three-factor models based on Gibson and Schwartz (1990) and Schwartz (1997), but with alternative notation where the benefit arises from a simplification in fitting to data.

Mikosch (2000) is known for stochastic calculus, Baxter and Rennie (1996) for a very accessible text on pricing products using the martingale approach, particularly when interest rates are involved in the modelling of commodity price.Chambers and Bailey (1996). They developed a competitive rational expectations model to explain the properties of commodity prices in markets. They all consider the existence of convenience yields, which are defined in Hull (2003) as a measure of the benefits from owning a commodity asset versus holding a long futures contract on the asset. A futures contract is a contract that obligates the holder to buy or sell an asset at a predetermined delivery price during a specified future time period. A convenience yield is generally positive since carrying inventory allows the owner to make profits from trading opportunities that may arise, for instance from temporary local shortages; on the other hand, holding a futures contract does not allow it. As an example, an oil refiner is unlikely to regard a futures contract on crude oil as equivalent to crude oil held in inventory: the crude oil in inventory can be an input to the refining process whereas a futures contract cannot be used for this purpose. The convenience yield typically reflects the market's expectations concerning the future availability of the commodity. As noted in Hull (2003), the greater the chances that shortages will occur, the higher the convenience yield. One can describe spot prices evolution by modeling convenience yields and spot prices as separate stochastic processes, possibly correlated. Schwartz develops three variations of a mean-reverting stochastic model driven by one, two or three factors taking into account mean reversion of commodity prices, stochastic spot price model determined by the combination of short-term and long-term factors that allow volatility in both terms. This two-factor model is equivalent to the stochastic convenience yield model developed in Gibson and Schwartz (1990). These models are empirically validated for copper, gold and crude oil. The work does not directly model mean-reverting spot price processes or convenience yields, as in these papers, but focuses instead on modeling price spreads.

Nielson and Schwartz (2004) develop a two-factor spot model that extends the Gibson and Schwartz (1990) model so that the volatility of both factors is a function of convenience yield, in an attempt to capture the effect of inventories on volatility. They find little difference in pricing forwards and futures, but notable differences when pricing options. Similarly, Schwartz and Smith (2000) present a two-factor model where both volatilities are proportional to the square root of convenience yield (i.e. convenience yield follows a square root process, x1.4), however they do not price options. Schwartz (1997) present a latent three-factor model of commodity price, convenience yield, and interest rate that nests some of the previously mentioned models and tests against a comprehensive data set. The following two model descriptions have formed the basis for much of the literature in the field. It is suspect that this is because the models offer analytic solutions to futures and forward prices, and subsequently option prices. Gibson & Schwartz have done for commodity derivatives what Black, Scholes (1976) & Merton (1973) did for stock options.

Hull (2003) considered the correlation between prices of different commodities within broad families (e.g., natural gas and electricity in energy). They modeled the substitutability of these commodities and find existence of equilibrium price processes, where rational agents convert one commodity into the other. In particular, they focused on natural gas and electricity, and analyze the spread (price difference) between these two commodities. This is known in the industry as the spark spread, and is used extensively by commodity traders. The industry also uses the dark spread, i.e., the spread between coal and electricity prices. Reuters (the world's largest international multimedia news agency) provides to its users calculators for these two types of spreads. In this work, price spreads between different geographical locations is the focus, although the work could be extended to spreads between different commodities, as discussed later. A similar approach is employed, and specifically considers the individual operational actions of rational traders, which allows us to describe in closed-form the agents' actions.

Carlson *et al*. (2007) analyze an exhaustible resource and find, amongst other results, that there is a U-shaped relationship between spot price volatility and the slope of the term structure of forward prices. Hull (2003) obtain a similar result for futures price volatility in a model that features irreversible investment and a capacity constraint, a result that they claim cannot be captured by standard storage-based models of commodity prices. Spatt*et al*. (2000) build an equilibrium model involving a commodity that is used as an input into a production process. There is no storage in their model, but a friction in the commodity extraction process is sufficient to induce an endogenous convenience yield.

2.3 Convenience Yields

The second stream of literature on commodities exists in operations management. The papers in this group typically focus on the management of inventory of commodities, in the presence of price uncertainty, with buy/sell decisions in a single market. There is extensive literature on inventory management models, see for example Wright and Williams (1989) offer another explanation for positive inventory in the presence of low expected returns to storage that is based on mis-measurement. They argue that commodities that are aggregated for reporting purposes are often economically distinct. They show that if the cost of transforming one commodity into another is higher when carried out in a later period, then one commodity may be stored in positive quantities even though (apparent) excess returns are available from storing the other commodity. Such a situation will appear in the data as positive industry-wide storage with a negative expected return to storage.

Zipkin (2000), Goel and Gutierrez (2004, 2006 and 2007) apply these type of models specifically into commodities. These papers try to incorporate the information given by the convenience yield in the inventory and buy/sell decisions, e.g. Caldentey*et al.*(2007) for mining operations in Chile, where they use the stochastic process in Schwartz (1997) to model copper spot prices. In this sense, they try to combine finance and operations models. In this group, Golabi (1985) models the prices of the commodity in future periods as random variables with known distribution functions. Assuming constant demand, he proves that a sequence of critical price levels at a given period determines the optimal ordering strategy. Wang (2001), proves that a myopic inventory policy is optimal for a multi-period model with stochastic demand and decreasing prices. Secomandi (2004), considers optimal commodity trading and provides a much more detailed view of the operations involved in trading. He focuses on storage assets, i.e. storage facilities or contracts that ensure that one will have the inventory at a pre-determined time. His model is based on inventory and well-behaved flow constraints. He shows that the optimal policy is, depending on the region, to buy and withdraw, to do nothing or to sell and inject. This type of policy is used for contract valuation in Secomandi (2004) and Wang *et al* (2001).

This work combines the ideas of price equilibrium, from economics and finance, together with a more detailed view of operations. Here, the objective is to specifically optimize the inventory management policy, given that the trading activity may influence the price spread process.

Gustafson (1958). Implemented a structural model of the optimal storage of a commodity. He recognized the fundamentally dynamic nature of the problem, and utilized dynamic programming techniques, and numerical solutions to these programs. He used a piecewise function to approximate price as a function of storage. Subsequently, Newbery and Stiglitz (1982) and the Chicago Board of Trade (1989) employed this approximation to derive storage rules and prices.

The structural modeling of commodity prices, and the dynamic programming approach, was elegantly formalized by Scheinkman and Schectman (1983) under the assumption of a single i.i.d. demand shock.

Kogan *et al* (2000) presents a one factor model of commodity storage, and calibrate this model to certain moments of oil futures prices. Specifically, they choose the parameters of the storage model (the autocorrelation and variance of the demand shock, and the parameters of the net demand curve) to minimize the mean squared errors in the means and variances of oil futures prices with maturities between one and ten months. They found that the basic one factor model does a job at explaining the variances of longer-tenor futures prices. They propose a model with an additional, and permanent, demand shock that does not affect optimal storage decisions and which is not priced in equilibrium. They calibrate the variance of this parameter so as to match the variance of the 10 month oil futures price, and then choose the remaining parameters to minimize mean squared errors in the means and variances of the remaining futures prices. These scholars do not examine the behavior of correlations between futures prices of different commodities

2.4 Forward Contracts and Futures Contracts

The futures and the forward contracts are very similar because they both involve trading a commodity that is buying and selling at a future date for a fixed predetermined amount which is agreed by both parties involved in the trading. The economic difference between the two contracts is that future contracts are settled daily, whereas forward contracts are settled only at maturity. Secondly, forward contracts are less standardized, have poor secondary market, and are not guaranteed by the exchange.

Forward contracts could be explained as an agreement to sell or buy a commodity at a specific time (s) at a price known as forward price, determined at time (t). The volume of commodity at time (t) is zero and there is no payoffs from the contract until the maturity dates (s). While a future contract is also an agreement between traders that a buyer and a seller, such that a trader may want to buy a commodity at time s, at a future price of f(t) agreed at time t (Dhingra, 1986).

Black (1976) was one of the first researchers to extensively study the relationship between forward and futures contracts. He explicitly took into account the daily resettlement feature of futures contracts. Apart from Blacks' research, other researchers have also decided to concentrate on this distinction and derived some important analytical results and benefits. Jarrow and Oldfield (1981) and Margrabe (1978) both came out with a common result that if interest rates are non-stochastic, then forward prices and futures prices should be equal. The intuition behind this result is quite simple, as explained below. As a result of daily resettlement of future contracts, the investor benefiting from the futures price movement on any given day receives the cash proceeds from the investor holding the opposite position and has the opportunity to invest those proceeds at the current interest rate. The investor holding the opposite position must come up with the requisite cash, presumably by borrowing at the prevailing interest rate. For both investors the future interest rate is an important variable in determining the net benefit due to daily resettlement. Therefore, if there is no uncertainty regarding interest rate that will prevail at each point of time until maturity (assuming interest rates are non-stochastic), forward prices must be equal to futures prices. (See Jarrow and Oldfield (1981) for a clear proof of this proposition. Cox, Ingersoll and Ross (CIR) (1981) further investigate the effect of stochastic interest rates on the magnitude of the difference between forward prices and futures prices of commodities. CIR derive an arbitrage proof to show that the difference between the two prices depends upon the relationship between futures prices and short-term interest rates. However, If the two variables have positive covariance, then forward prices must be lower than futures prices. The opposite is true if futures prices and short-term interest rates have negative covariance. The magnitude of the difference depends on the magnitude of the covariance between futures price and short-term interest rates and the time to maturity of the contracts. The intuition behind the CIR result is explained well by Klemkosky and Lasser (1985) as follows. When the futures price falls, if there is a negative correlation between the futures price and short-term interest rates, the buyer of the contract must borrow for payment to the seller at a higher interest rate than existed when the contract was issued. When the futures price rises, the buyer will be able invest the resettlement, but at a lower rate. The seller, on the other hand, will be able to invest when rates rise and must borrow when rates fall. CIR (1981) show that if forward prices and futures prices do not behave in this fashion, an arbitrage profit can be obtained by undertaking the following strategy: Buy a forward contract, sell B(j) futures contracts in each period j, liquidate them in the next period and invest the (possibly negative) proceeds into risk free bonds. This arbitrage process prescribes the following relationship between futures prices and forward prices (CIR, Proposition 6).

$$f(tj - g(t)) = PVS[f(j+1) - f(j)]\left[\frac{B(j)}{\frac{B(j+1)-1}{B(t)}}\right]$$

$j = k$

where PV is the present value operator.

In a continuous-time framework this equation reduces to

the following equation,

$$g(t) - f(t) = PV$$

t [If (u)cov(f ' (u) ,B' (u))du]/B(t)………………………….. (2)

wherecov f' (u) ,B' (u)) is defined as the local covariance of the percentage change in the futures price, f'(u), and the percentage change in bond price. This result implies that if the local covariance between futures prices and bond prices is positive for every time from *t* to *s*, forward prices will be greater than futures prices. Conversely, for negative covariance futures prices will be greater than forward prices. Note that this equation does allow for the possibility that forward prices and futures prices may be equal even when interest rates are stochastic. This is possible if the local covariance between bond prices and futures prices is zero for each period until maturity. Cornell and Reinganum (1981) found that there is no significant difference between forward prices and futures prices on foreign currencies. Since they find that the covariance between short-term interest rates and currency futures prices is negligible, their findings are consistent with the CIR model. Cornell and Reinganum (1981) also find that T-bills show greater difference between forward prices and futures prices even though the covariance between short-term interest rates and T-bill futures prices is negligible. This difference is apparently inconsistent with the CIR model. Cornell and Reinganum suggest that the inconsistency may be caused by factors other than marking-to-market. They offer tax treatment of T-bills and problems associated with shorting T-bills as primary candidates for explaining the discrepancy.

French (1983) compares forward prices and futures prices on two commodities: silver and copper and finds significant differences between them. He finds some support for the CIR model in explaining the differences between the two prices. Park and Chen (1985) find that there are no significant differences between forward and futures prices on foreign currencies, but such differences are significant for contracts based on physical commodities. They find strong support for the CIR model.

The empirical studies of the above literature review that one or both of the following drawbacks prevailed. It was observed that data on forward contracts were difficult to obtain, and also of poor quality too. This problem is evident in the studies by French (1983) and, Park and Chen (1985). French compares forward and futures prices which are observed in different countries, and at different times, and are denominated in different currencies. Park and Chen have problems in getting a large number of observations because forward contracts and futures contracts trade under different conventions. Forward contracts are issued with standard maturity periods ,i.e., on every day, a one-month, a three-month, a six-month, and other such contracts are available. On the other hand, futures contracts are traded on the basis of standard maturity dates. Therefore, a three-month futures contract is available only on the day it is initiated or when a longer maturity contract has exactly three months left to maturity. For this reason it is difficult to obtain enough observations for which forward contracts and futures contracts have the same time of maturity. These studies use forward prices observed in the market, compare them to futures prices and attribute the difference to marking-to-market. Since forward contracts differ from futures contracts, along other qualitative dimensions too, it is not clear how the differences observed can be attributed solely to marking-to-market. In order to isolate the marking-to-market effect forward prices, that are free from these extraneous factors, are needed. It is quite obvious that one cannot hope to observe such "perfect" forward prices in the market. However, they can be determined quite accurately by a simple, yet powerful, arbitrage model. This well-known arbitrage model of forward prices, sometimes known as the cost-of -carry model, simply says that the forward price of an asset must equal its spot price plus the net costs associated with buying the asset today and holding it until maturity of the contract. If such is not the case then arbitrage will take place. For a financial asset which provides no intermediate cash flows, the cost associated with holding the asset is simply the interest cost (the opportunity cost of money). Thus, the forward price for such an asset is given by the following model.

$$g(T) = \frac{S(T)}{B(t)}$$

where g is the forward price, S is the spot price of the commodity and B is the price of a discount bond which pays \$1 at maturity.

If this price does not prevail, an arbitrage profit is available. For example, if

$$g(T) > \frac{S(T)}{B(t)}$$

an investor can buy the asset in the spot market by borrowing the money at the risk free rate, and short a forward contract on the same asset. This strategy costs nothing and gives a positive payoff of [g(T)-S(T)/B(T) J at maturity. If

$$g(T) > \frac{S(T)}{B(t)}$$

then the strategy is reversed to make a riskless profit. This model can be adjusted for assets that provide intermediate cash flows (e.g., dividends on common stock).

2.5 Optimal Hedge Ratios

Hedgers are firms and individuals with positions in the cash market, including producers, merchandisers, and end-users, who use commodity futures markets to transfer part of their risk of loss to speculators. Having a cash position separates hedgers from speculators who do not have a position in the underlying physical commodity.

According to Anderson and Danthine (1981), speculators take on the price level risk that hedgers are not willing to carry. In turn, hedgers are still left with basis risk, or price difference (spread) risk. The source of risk is most often attributed to market price fluctuations of the underlying commodities but may include the risk of supply shortages (Seidel and Ginsberg, 1983). The traditional view of hedging assumed that the optimal strategy is to hold a position in the futures market which is equal and opposite of the position in the underlying commodity (Rolfo, 1980). While this approach has intuitive appeal and ease of execution, there has been considerable research in the literature focused on improving the estimation of hedge ratios. Two main categories of hedge ratio estimation include the risk-minimizing and utility-maximizing approaches. In the literature review, both of these categories are discussed and summarized; then, the points of departure for this thesis are outlined. Most of the approaches used to find optimal hedge ratios follow the same general procedure, whereby the revenue or profit function is maximized in terms of the choice variables.

As the number of choice variables and possible risk-management instruments increases and correlations are also considered, finding analytical formulas for the variance of the profit functions becomes more complex. Hedging with futures contracts reduces the total amount of risk by substituting the risk of absolute price movements with the risk of movements in the basis. The basis tends to be less volatile than actual prices; therefore, the amount of risk assumed by the hedger can be substantially reduced (Seidel and Ginsberg, 1983).

2.6 Risk Minimization

In TradingInLence and Hayes (1994), Johnson (1960), Blank et al. (1991), and Rolfo (1980) derivation of risk-minimizing formula was documented. The risk-minimizing approach to finding hedge ratios uses the coefficient of regression, also given by the formula

$$H = -(\frac{\sigma_{sf}}{\sigma_f^2})$$

Where, H is the risk-minimizing hedge ratio, which is the ratio of the covariance between cash and futures markets to the variance of the futures. Since, traditionally, the reason for hedging is risk reduction, this hedge ratio is calculated with the objective of minimizing the variance of income once the position in the cash market has been determined.

Collins notes that the risk-minimizing hedge ratio is inappropriate for processors as it "does not match the behavior of processors and traders who frequently hedge only part or none of their commitments" (Collins, 1997). He concludes that the risk-minimizing model tends to be best at predicting the behavior of arbitrage traders who take close to equal and opposite positions in futures markets.

Lence and Hayes (1994) used the risk-minimizing approach in calculating hedge ratios but also allowed the estimation of uncertainty for the random variables in the model. The minimum-variance hedge ratio (MVH) is the optimal hedge position if futures prices are unbiased or if the agent is infinitely risk-averse. According to Lence and Hayes (1994), different authors estimated different MVHs for the same commodity. The estimation results imply that "MVH estimation risk is important" (Lence and Hayes, 1994). The MVH is sensitive to the estimated parameters because the statistical characteristics of cash and futures markets (such as volatilities and correlations) change over time.

Lence (1996) re-examines the performance of minimum-variance hedge ratios and finds that it is only consistent with maximizing utility under certain conditions. When the assumptions of MVHs are relaxed, such as allowing for production uncertainty, alternative investment opportunities, and brokerage fees, the optimal hedge ratios are substantially lower than those suggested by MVHs. His analysis suggests that MVHs are only optimal under very specific situations with a limited number of stochastic variables (Lence, 1996).

Vukina *et al*. (1996) developed a risk-minimizing model where both price and yield risk are hedged in futures markets. Their optimal hedge ratio calculation is similar to that in Blank *et al*. (1991) but includes two sources of uncertainty. An analytical model is also developed for measuring hedging effectiveness in terms of the reduction in the variance of outcomes (Vukina *et al*., 1996).

Hedging strategies of firms with both production and price risk are further complicated by including foreign exchange risk, as in the case of companies involved in international transactions. Nayak and Turvey (2000) developed a risk-minimizing hedge ratio model to address all of the above questions in a simultaneous decision-making problem. They solved a system of three equations simultaneously for price, currency, and yield futures hedge ratios. The effectiveness of minimum-variance hedges is measured by the amount of risk reduction gained from a particular strategy. One of the approaches to measure risk reduction is comparing the risk of unhedged positions to a variety of hedging strategies. As expected, the authors conclude: "The magnitude of risk reduction depends upon the correlation and covariance between the random outcomes" as stated in Nayak and Turvey, (2000).

Utility-maximizing models are used by Haigh and Holt (1995), Sakong*et al*. (1993), Lapan*et al*. (1991), Collins (1997), Garcia *et al*. (1994), and Rolfo (1980), among others. The models make explicit assumptions about the utility function of the decision-maker. These models typically include a risk-aversion parameter in the hedge ratio formula as well as the agent's expectations of futures price movements. Derivation of a representative model is described in Blank *et al*. (1991) and is also found in numerous articles in the literature such as Lapan*et al*. (1991), Sakong*et al*. (1993), and Collins (1997). The mean-variance model with a single source of uncertainty yields the following solution:

$$H^{*} = \frac{E(f_1) - f_0}{2\lambda\sigma_f^2} - \frac{\sigma_{sf}}{\sigma^2 f}$$

where the second component is the same as the risk-minimizing hedge ratio. The first component includes the risk aversion parameter, λ, and the bias term in the numerator. The first term is often referred to as the speculative component of the hedge ratio (Blank *et al*., 1991; Vukina*et al*., 1996). The bias term, $E(f_1)$-f_O refers to the hedger's point of view regarding futures prices. When current futures prices (f_O) are believed to be the best estimate of "future" futures prices (f_1), the bias is zero. However, if the hedger believes he or she possesses some unique knowledge which allows him or her to anticipate price changes, the utility-maximizing hedge ratio can include these expectations. Of course, the level of risk aversion will determine the degree to which the hedge position reflects anticipation of price movement.

2.7 Trading Strategies

This literature will review the analysis of advanced commodity trading strategies. Analysis of commodity trading involves the sole use of price and related summary statistics, such as volume, to inform trading decisions. Given its longstanding use in financial markets, Analysis of commodity trading has naturally become a focus of academic study. In part, this is because profits accruing from a strategy constructed entirely around the analysis of past prices runs counter to the least restrictive form of market efficiency. The above quotation from Malkiel (1999) expresses this opinion, based on a belief in efficient markets. This thesis examines several important areas of commodity trading analysis and finds that there is strong empirical evidence that opposes this point of view. Forecasting future price changes of commodities with the aid of charts of past prices has a long history of use by investors and traders. For example, Nison (1994) describes the development of candlestick charts, which provide a visual representation of the opening, closing, high and low prices for a discrete period. It is shown that such charts may have been used as early as the 1700s by traders in what was, in effect, the first rice futures market in Japan. Furthermore, it is certain that traders plotted candlestick charts and used them to inform trading decisions by the late 1800s. The so-called "book method", which was an early version of point and figure charting, was also in active use by 1900. Indeed, Charles Dow published a Wall Street Journal editorial on the subject in 1901 Murphy(1999). Thousands of books on commodity trading analysis aimed at traders have since been published, with many different forms of commodity trading strategies proposed, to be employed across the whole commodity trading markets, including equities, foreign exchange and futures. Indeed, all professional trading platforms, such as Reuters and Bloomberg, can perform commodity trading analysis. The common thread is the sole use of past price data for making buy and sell decisions. Importantly, it is clear that the continuing non-academic interest in commodity trading analysis translates into active use in the markets. For example, Taylor and Allen (1992) conducted a survey of foreign exchange traders in London. The results showed that where respondents employed in-house commodity trading analysts, there was a greater tendency for them to initiate trades as opposed to in-house economists.

Other surveys also provide convincing evidence that traders make significant use of commodity trading analysis, either in isolation or in conjunction with fundamental analysis for example, Lui and Mole, (1998); 2006); Cheung*et al*. (2004). If commodity trading strategies do not provide economically valuable information, then their continuing use proves somewhat perplexing, and provides strong motivation for increased academic study.

The analysis of commodity trading covers a multitude of different techniques and strategies to utilize price data. For example, moving averages, relative strength, trend indicator sand price patterns. There are also innumerable chart styles, such as bar charts, candle stick charts, and point and figure charts. However, previous academic research in this area has largely concentrated on what can be termed 'basic' technical analysis, such as moving averages. This is partly because it is relatively easy to construct algorithms to evaluate the profitability of basic commodity trading strategies. However, for the purposes of this thesis, 'advanced' commodity trading strategies will be discussed. Advanced commodity trading strategies are generally concerned with detecting and evaluating visual patterns displayed on charts of past price data. Whilst formations approximating a particular specification are usually clear to the human eye, it is a considerable problem to develop algorithms to allow the evaluation of advanced commodity analysis by computer.

Trading strategies such as the moving average, where buy and sell signals can be easily derived from a vector of past prices. It is only comparatively recently that appropriate econometric methods and sufficient computational power has existed to allow a full investigation of advanced trading analysis. A related point concerns high-frequency data. Many commodity trading methodologies are agnostic of the time frame over which they can be applied—for example, being equally valid using weekly charts, daily charts and intraday charts. Thus, a 50-period moving average could be employed over 50 weeks, 50 days or 50 minutes.

In addition, many other strategies are specifically proposed as being useful over short time horizons. Reading Lui and Mole, (1998); commodity trading strategies employed using high frequency data, as it is done here, is particularly important given the increasing numbers of day traders. Professional traders, and hedge funds in particular, also employ program trading strategies that utilise trading analysis. Yet the profitability of such strategies is still not know. High-frequency data has been available for some time from sources such as the New York Stock Exchange. However, it is only relatively recently that such data has been readily available to the academic community and, again, that computational power has allowed researchers to take full advantage of this. It is now possible and increasingly pressing to investigate the profitability of commodity trading strategies with high-frequency data.

In addition, most existing research such as Lui and Mole, (1998), has not succeeded in evaluating and applying commodity trading strategies as they are actually employed by traders, when making buy and sell decisions. For example, there is often a clear disparity between the head and shoulders pattern that is consistently seen in the literature aimed at practitioners and that which is evaluated in academic research. This is partly because of the aforementioned problem of computational power and suitable methodology. This study, however, makes considerable progress in addressing this issue.

Given its long history, commodity trading analysis has seen the development of innumerable indicators, patterns, chart types and trading strategies. Partly due to the depth and breadth of the subject, academic investigation has been severely limited or even nonexistent into a great many aspects of commodity trading. The sparsity of empirical evaluation into areas of commodity analysis, the lack of investigation into many trading strategies at time horizons employed by traders, and the scant knowledge about the profitability of advanced commodity trading strategies makes commodity trading analysis a compelling and timely area for study. This thesis seeks to examine the nature and profitability of a number of important technical trading strategies, and make a significant contribution in several important areas. There is no way of making an expected profit by extrapolating past changes in the futures price, by chart or any other esoteric devices of or magic. The market quotation already contains in itself all that can be known about the future and in that sense has discounted future contingencies as much as is humanly possible." (Samuelson, 1965) Samuelson succinctly expresses the opinion that in an efficient market we would not expect to be able to make profits through technical analysis. This review of the literature shows the increasing interest in technical analysis by researchers, often demonstrating that profits can be shown, in contradiction of weak-form efficiency. Existing research is classified accordingly into two broad groups: First, basic studies of trading analysis, which are recognisable by the evaluation of simple rules and trading strategies such as filter rules and moving average crossovers. Second, 'new' studies of trading analysis. This more recent work tends to possess more robust econometric methodology. More advanced technical analysis strategies including pattern recognition, its strength is in pointing the reader to the papers and research that has shaped academic understanding of technical trading analysis. Before this, however, it is important to establish a firm grip on what constitutes commodity trading analysis, and this is addressed in the next section.

A large body of work investigates simple technical strategies; however, as noted, it is the complex and predominantly visual patterns that are of specific interest here.

Park and Irwin (2007) provide a useful general overview of the literature in the area of technical analysis of commodity trading.

Lapan *et al*. (1991) extend the scope of the hedging decision by allowing the use of options. Their model includes production uncertainty, which is also referred to as yield risk for producers. Speculative positions can be taken as part of the hedging strategy when current prices are considered biased. Utility is maximized by the decision-maker through his or her choice of optimal production levels and hedge ratios in both futures and options contracts. The mean-variance framework is relaxed since options result in non-linear payoff functions. The results show that, in the absence of bias, only futures would be used for hedging and the futures position equals the minimum-variance hedge ratio (Lapan*et al*., (1991). Sakong *et al*. (1993) found that options may be optimal to use for risk management, combined with futures hedges when both price risk and production uncertainty are present. Their model is an expansion of the model developed by Lapan*et al*. (1991) except for allowing production uncertainty. The utility-maximizing solution is given by a combination of a futures position equal to the size of the minimum expected yield, and the additional production volume was hedged by put options. Hedging strategies using options and futures are often considered separately, but in this case, both instruments were part of the same risk management program (Sakong *et al*., 1993).

Rolfo (1980) incorporates price and production uncertainty into a risk-minimizing hedge ratio model, and applies it to four producer countries. The results indicate that, in the presence of yield risk, producers are made better off by selling less than the equal and opposite volume in the futures markets as opposed to entering a full short hedge. Two models, one of risk minimization under a mean-variance framework and a utility-maximizing model under a logarithmic utility function, are considered (Rolfo, 1980).

Martinez and Zering (1992) also consider price and yield uncertainty in a dynamic hedging model for grain producers. Regression analysis is used to estimate parameters of forecasting models, which are used to estimate the key parameters and their statistical relationships. The model was expressed as a linear exponential Gaussian optimal control problem. The mean-variance framework is used to examine the effect of hedging strategies on average returns and risk. The results indicate that, while dynamic hedging strategies may increase expected profits, the complexity and effort needed in their implementation often outweigh the benefits.

A similar model was developed by Lapan and Moschini (1994) for a risk-averse producer facing price, basis, and production risk. The producer's utility function was assumed to be of the Constant Absolute Risk Aversion (CARA) form, where utility is expressed as

$$U = -e^{(-\lambda\pi)}$$

with λ representing the risk aversion factor and Π the profit function. The suggested hedge ratio is very similar to that in Blank *et al*. (1991) with a pure hedge component and a speculative component which is affected by the bias and the risk aversion parameter. However, the inclusion of yield uncertainty results in continually updated futures positions as production was being realized. Assuming no production uncertainty and no bias, the hedge ratio is reduced to the familiar regression coefficient. One of the key findings is the negative relationship between production risk and the hedge ratio.

Haigh and Holt (1995) extended traditional hedge ratio models to include risks other than volatility in commodity prices. They moved beyond a domestic company and modeled the hedging decision of an international trading entity facing multiple sources of risk. They recognized the need to include both foreign currency exchange rates and transportation costs as a significant source of uncertainty which have to be taken into account. Results show the importance of correlation among closely related markets, especially when each market is the source of significant uncertainty. Due to large volumes of grain being transferred in international transactions, even small increases in transportation costs can potentially diminish profits.

Three common approaches to estimate hedge ratios are reviewed by Schroeder and Mintert (1988) and Blank (1989). The three methods include price-level models, where cash price is regressed on the nearby futures price; price-change models, where the change in cash price over the hedging period is regressed on the change in the respective futures price; and percentage-change models, where the percentage change in cash price is regressed on the percentage change in respective futures price. “Price difference models of hedge ratios vary depending upon the decision maker’s goal” (Blank, 1989). In addition, logarithmic returns are also often used, which represent the continuously compounded rate of return. Nevertheless, there is no consensus in the literature as to which model produces superior results. Many of the models, whether the objective is risk minimization or utility maximization, fail to explain the observed behavior of a wide range of agents. Collins (1997) constructs his positive model of hedging behavior as a financial decision where the firm’s objective is to maximize terminal equity by making choices about current operations. The model explains hedging as avoidance of financial failure, as opposed to an approach to reduce price uncertainty. It explains why some agents, such as most farmers, choose not to hedge at all while others, such as arbitragers, typically hedge most of their positions (Collins, 1997).

CHAPTER THREE

METHODOLOGY

3.1 Introduction to Linear Programming

In Mathematics Linear programming (LP) is a subset of operations research, branch of mathematics. LP is one of the best known method or procedure for optimization linear objective function which is subjected to some constraint. The LP approach assist us to calculate the best outcome expected from an activity such as maximizing profit and or minimizing cost in a given mathematical model given some limitations which should be satisfied by the equation , these limitations are best known as constraints. It is commonly accepted fact that every organization has constraints imposed on its decision variables limited by one or more of the following resources: Capital, human resources, facilities to mention a few. These constrains must be taken in to consideration in my calculations otherwise the LP techniques that will be applied to the problem may yield a solution that is unacceptable from a practical standpoint.

Lately, LP has undergone a lot of evolution, but it is interesting to note that, the idea of maximization of profit or minimization of cost still remains the objective of all advanced LP techniques. Over the years mathematicians have striven to formulate models that can assist manufacturing and production companies in maximizing their profit. Dantzig (1963) and Fourer and Mehrotra (1992) they proved that linear programming among other models in operations research is the best technique to solve constraints optimization problems.

The problem has always been to optimize the value of some objective function, subject to some constraints, such as behavioral, output, input restrictions etc.

In this thesis, the methodology would be narrowed to Integer Linear Programming. An integer programming is a mathematical optimization technique in which some or all of the variables are restricted to be integers, integer programming is popularly called Integer Linear Programming (ILP). This method is used for the modeling in this thesis because the integer variables in this model represent quantities of commodities that a trader can buy/sell in a market, the commodities can only be integers, for example a trader cannot buy 10.5 tubers of Yam.

3.2 Model Components

The following are the three main components in LP formulations

1. Decision Variables
2. Objective Function and
3. Constraints.

3.2.1 Decision Variables

Decision variables capture the level of activities that the model studies. Decision makers have some freedom (subject to Constraints, see below) to assign numerical values to decision variables. For example, number of bolts (screws) produced in a week, denoted by B (S), is a common decision variable at machining plants. Letting, say, B = 5000 and S = 7200, this means 5000 bolts and 7200 screws are produced in a week. These activity levels of 5000 and 7200 specify a (production) plan over a week. The plan is not as detailed as specifying what to do every day. It can be said that daily activity levels are abstracted out as they are aggregated into weekly levels to facilitate computability. Solving a mathematical model means finding these numerical values for decision variables to minimize or maximize an objective function in the presence of constraints.

3.2.2 Objective Function

With mathematical models, the desire is to maximize or minimize a quantity such as cost, profit, risk, net present value, number of employees, customer satisfaction, etc. The quantity one wish to maximize or minimize is known as objective (function). The objective function is said to highlight the fact that objective is a function of decision variables. Deciding on the correct objective in practical situations is not trivial. At one extreme there may be no clear objectives, at the other there may be multiple objectives. Multiple objectives, although possible in the case of a single decision maker, often arise with multiple decision makers. Reconciliation, weighing, or demotion of all but one of these objectives to constraints are among the methods to end up a single objective. This process of honing down to a single objective involves discussions between the developers of the formulation and users of the formulation and it then takes place before formulation starts. The users must check and approve the final objective; a wrong objective can be worse than no objective at all.

3.2.3 Constraints

Constraints represent the limitations such as available capacity, daily working hours, raw material availability, etc. Sometimes constraints are also used to represent relationships between decision variables.

3.2.4 Slack variable

The standard form requires that all constraints be in the form of equations (equalities).

A slack variable is added to a ≤constraint (weak inequality) to convert it to an equation (=). A slack variable typically represents an unused resource and contributes nothing to the objective function value.

3.2.5 Surplus variable

A surplus variable is subtracted from a ≥ constraint to convert it to an equation (=). It represents an excess above a constraint requirement level. A surplus variable also contributes nothing to the calculated value of the objective function.

3.3 Linear programming formulation

Assuming a Ship has three compartments for storing Ship: front, centre and rear, and these compartments have the following limits on both weight and space:

Compartment Weight capacity (tonnes) Space capacity (cubic metres)

The table below give information on weight and space capacity of a Ship

Table 3.1 Weight and space capacity of a Ship

Compartment	Maximum Weight capacity (tonnes)	Maximum space capacity (cubic metres)
Front	10	6800
Centre	16	8700
Rear	8	5300

Furthermore, assuming the weight of the Ship in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the Ship.

The following four Ships are available for shipment:

Table 3.2 Capacities and profit table of four Ships

Ship	Weight (tonnes)	Volume (cubic metres/tonne)	Profit (£/tonne)
S1	18	480	310
S2	15	650	380
S3	23	580	350
S4	12	390	285

Any proportion of these Ships can be accepted. The objective is to determine *how much* (if any) of each Ship S1, S2, S3 and S4 should be accepted and *how to distribute* each among the compartments so that the total profit for the journey ismaximized.

Now the above problem is formulated as shown below:

Variables

We need to decide how much of each of the four Ships to put in each of the three compartments. Hence let:

x_{ij} be the number of tonnes of Shipi (i=1,2,3,4 for S1, S2, S3 and S4 respectively) that is put into compartment j (j=1 for Front, j=2 for Centre and j=3 for Rear) where $x_{ij}>=0$ i=1,2,3,4; j=1,2,3

Note here that we are explicitly told we can split the Ships into any proportions (fractions) that we like.

Constraints

- cannot pack more of each of the four Ships than we have available

$$x_{11} + x_{12} + x_{13} \leq 18$$

$$x_{21} + x_{22} + x_{23} \leq 15$$

$$x_{31} + x_{32} + x_{33} \leq 23$$

$$x_{41} + x_{42} + x_{43} \leq 12$$

- the weight capacity of each compartment must be respected

$$x_{11} + x_{21} + x_{31} + x_{41} \leq 10$$

$$x_{12} + x_{23} + x_{32} + x_{42} \leq 16$$

$$x_{13} + x_{23} + x_{33} + x_{43} \leq 8$$

- the volume (space) capacity of each compartment must be respected

$$480x_{11} + 650x_{21} + 580x_{31} + 390x_{41} \leq 6800$$

$$480x_{12} + 650x_{23} + 580x_{32} + 390x_{42} \leq 8700$$

$$480x_{13} + 650x_{23} + 580x_{33} + 390x_{43} \leq 5300$$

- the weight of the Ship in the respective compartments must be the same proportion of that compartment's weight capacity to maintain the balance of the Ship

$$\frac{x_{11} + x_{21} + x_{31} + x_{41}}{10} = \frac{x_{12} + x_{23} + x_{32} + x_{42}}{16} = \frac{x_{13} + x_{23} + x_{33} + x_{43}}{8}$$

Objective function

The objective is to maximise total profit, i.e.

$$310[x_{11} + x_{12} + x_{13}] + 380[x_{21} + x_{22} + x_{23}] + 350[x_{31} + x_{32} + x_{33}] + 285[x_{41} + x_{42} + x_{43}]$$

3.4 Integer Linear programming

The Integer Linear programming (ILP) problem is a mathematical optimization program in which all the variable are restricted to be integers. The linear programming models that have been discussed earlier all have been continuous, in the sense that decision variables are allowed to be fractional. Often this is realistic assumption, for example it is possible to produce 10.5 gallons of wine.

If a model requires finding, example the number of people require to do a work within a set time, since the decision variable cannot be fractional, the problem could best be model as integer programming problem as fractional solution is not realistic. The general form is:

Maximize $\sum_{j=1}^{n} \mathbf{cj\,Xj}$

Subject to

$\sum_{j=1}^{n} \mathbf{cj\,Xj} = bi \quad (i=1,2,......................., m)$

$x_j \geq 0 \quad (j=1,2,......................., n)$

x_j integer (for some or all j= 1,2,............, n)

This problem is called the (Linear) integer programming problem.

3.5 Mixed Integer Programming

A mixed-integer program is also a model for minimization or maximization of a linear function subject to linear constraints. It is called a mixed integer program because some, but not all, variables are restricted to be integer and is called pure integer program when all decision variables must be integers

Mixed integer programs can be used to formulate just about any discrete optimization problem. They are heavily used in practice for solving problems in transportation and manufacturing: airline crew scheduling, vehicle routing, production planning, etc.

3.6 Integer programming models

Integer programming models arise in practically every area of application of mathematical programming to develop a preliminary application for the importance of these models. In these section two areas where integer programming has played an important role in supporting managerial decisions are discussed.

3.6.1 Capital Budgeting

In a typical Capital budgeting problems decision involve the selection of a number of the potential investment, the investment decision might be chosen among possible plant locations to select a configuration of capital equipment or to settle upon a set of research and development project often it make no sense to consider partial investment in these activities. There the decision variable are taken to be x_j=0 or 1 indicating that the j^{th} investment is rejected or accepted assuming that c_j is the contribution resulting from thej^{th}investment and that aij is the amount of resource *i* such as cash or manpower used on the j^{th}investment. We can state the problem finally as

Maximize $\sum_{j=1}^{n} \mathbf{cj\,Xj}$

Subject to

$\sum_{j=1}^{n} \mathbf{aij\,Xj} \leq b_i$ (i=1,2, ..., m),

x_j = 0 or 1 (j=1,2, ..., n).

The objective is to maximize total contribution from all investments with exceeding the limited availability bi for any resource.

3.6.2 Warehouse Location

In modeling distribution systems, decisions must be made about tradeoffs between transportation cost and cost of operating distribution centers. Examples if management should decide which of n warehouses to use for meeting the demands of m customers for a good. The decision to be made are which warehouses to operate and how much to ship from any warehouse to any customer. Let

$$yi= \begin{cases} 1, & \boldsymbol{ifwarehouseiisopened}, \\ \mathbf{0}, & \boldsymbol{ifwarehouseiisclosed} \end{cases}$$

x_{ij} = Amount to be sent from warehouse *i* to customer *j*the relevant cost are :

f_i= Fixed operating cost for warehouse i , if opened (for example, a cost to lease the warehouse)

C_{ij} = per unit operating cost at warehouse i plus the transportation cost for shipping from warehouse i to customer j

There are two types of constraints for the model:

1. The demand d_j of each customer must be filled from the warehouses.

2. Goods can be shipped from a warehouse only if it is opened

The model is

$$\text{Minimize}= \sum_{\boldsymbol{i=1}}^{\boldsymbol{m}} \sum_{\boldsymbol{j=1}}^{\boldsymbol{n}} \mathbf{cijXij} + \sum_{\boldsymbol{i=1}}^{\boldsymbol{m}} \mathbf{fiyi},$$

Subject to

$\sum_{i=1}^{m} \mathbf{xij}$ = dj (j= 1,2,....., n)

3.7Methods of Solution to LP

There are several approaches for solving the LP problems. Among these techniques are:

i) Graphical approach

ii) Simplex Algorithm

iii) Branch and Bond approach

iv) Interior Point algorithm

3.7.1Graphical Solution

In a case where there are exactly two decision variables ie x and y, the graphical method of solution is most suitable. The draw of inequalities (for example$\mathrm{x} + \mathrm{y} \leq 1$) can be drawn when the graph of the equation$x + y = 1$is drawn. The equation $x + y = 1$ is called the boundary equation of the inequality $\mathrm{x} + \mathrm{y} \leq 1$

Test Point of an inequality: Any point chosen at random on the x-axis or y-axis or at the origin is called a test point. The Test Points can be substituted in the Objective function to determine maximization or minimization of the Objective function.

Considering a company producing Bowls and Mugs. The problem is to know how many bowls and mugs should be produced to maximize profits given labor and materials constraints?

The table below gives information on the resource available to produce Bowls and Mugs

Table 3.3 Resource Requirement

Product	Labor(Hr./Unit)	Clay(Lb./Unit)	Profit($/Unit)
Bowl	1	4	40
Mug	2	3	50

Resource 40 hrs of labor per day

Availability: 120 lbs of clay

Decision x_1 = number of bowls to produce per day

Variables: x_2 = number of mugs to produce per day

Objective Maximize Z = 40x_1$ + 50x_2$

Function: Where Z = profit per day

Resource: $x_1 + x_2 \leq 40$ hours of labor

Constraints: $4x_1 + 3x_2 \leq 120$pounds of clay

Non-Negativity Constraints:

$x_1 \geq 0$; $x_2 \geq 0$

Maximize $Z = \$40x_1 + \$50x_2$

subject to: $x_1+2x_2 \leq 40$

$4x_1+3x_2 \leq 20$

$x_1, x_2 \geq 0$

A **feasible solution** does not violate **any** of the constraints:

Example: $x_1 = 5$ bowls

$x_2 = 10$ mugs

$Z = \$40x_1 + \$50x_2 = \$700$

Labor constraint check: $1(5) + 2(10) = 25 < 40$ hours

Clay constraint check: $4(5) + 3(10) = 70 < 120$ pounds

An **infeasible solution** violates **at least one** of the constraints

Example: $x_1 = 10$ bowls

$x_2 = 20$ mugs

$Z = \$40x_1 + \$50x_2 = \$1400$

Labor constraint check: $1(10) + 2(20) = 50 > 40$ hours

Fig. 3.1 Optimal Solution Coordinates

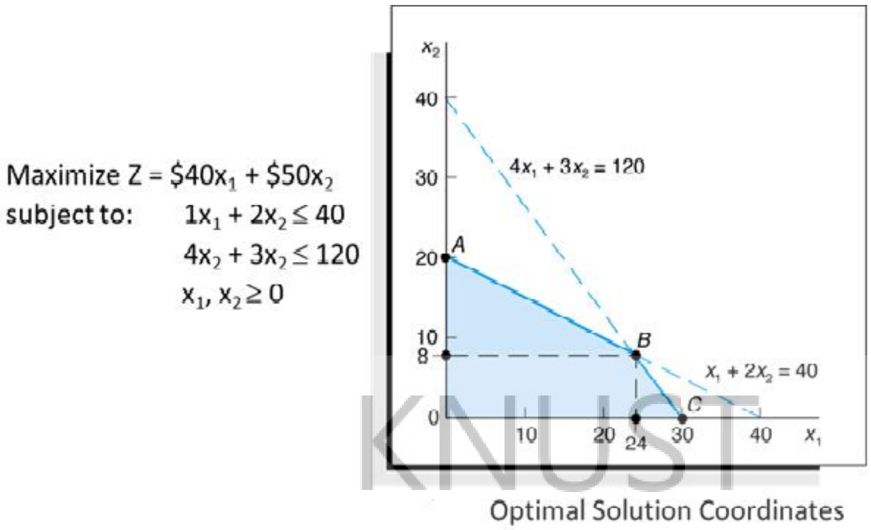
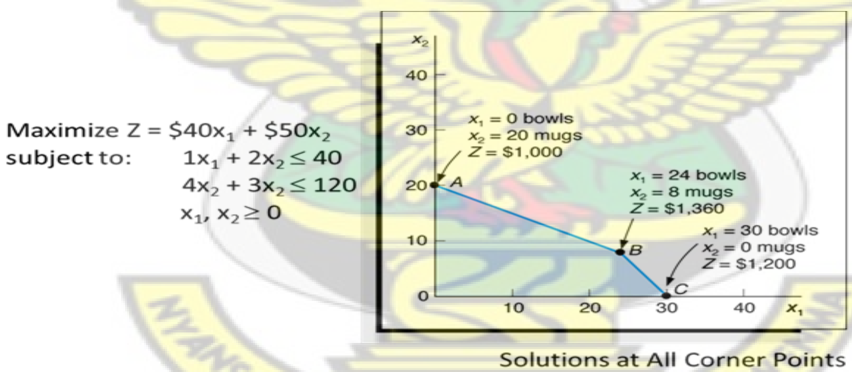


Fig. 3.2 Extreme (Corner) Point Solutions



3.7.2 The Simplex Algorithm

Introduction

George Dantzig 'invented' the simplex method while looking for methods for solving optimization problems. He used a primitive computer in 1947 to achieve his success in developing the simplex method.

The graphical method for solving linear programming problems is not practical if there are more than two variables in the problem. Many business or economics problems may involve thousands or millions of variables. Now new method is introduced to handle these problems more efficiently. The simplex method is an algorithmic approach and is the principal method used today in solving complex linear programming problems for the last four decades. Computer programs are written to handle these large problems using the simplex method.

3.7.2.1The Standard form for a Linear Program

A standard maximum problem is a linear program in which the objective is to maximize an objective function of the form:

Max. C^TX

Subject to

$AX \leq b$ or

Min C^TX

Subject to

$AX \leq b$

$0 \leq X$ thus all variables must be non-negative

X is the vector of variables to be determined.

A is a known matrix of coefficients and $(.)^T$ is a matrix transpose.

C and b are vectors of known coefficients.

Every linear problem can be converted to a standard for as

Max $c_1x_1+c_2x_2+c_3x_3+..........+c_nx_n$

s.t

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \leq b_2$$

.

.

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \leq b_m$$

Where $x_i \geq 0$ for i =1, 2, . . ., n

Table 3.4 Formulating simplex tableau

	C_J	C_1	C_2	$\dots$	C_n	0	0	$\dots$	0	
c_B	B. V.	x_1	x_2	$\dots$	x_n	s_1	s_2	$\dots$	s_n	RHS
0	s_1	a_{12}	a_{12}	$\dots$	a_{1n}	1	0	$\dots$	0	b_1
0	s_2	a_{21}	a_{22}	$\dots$	a_{2n}	0	1	$\dots$	0	b_2
.	.	.	.		.	.	.			
. .	. .	. .	. .	. .	. .	. .	. .	. .	. .	. .
0	s_m	a_{m1}	a_{m2}	$\dots$	a_{mn}	0	0	$\dots$	1	b_m
	z_j	0	0	$\dots$	0	0	0	$\dots$	0	0
	c_j $- z_j$	c_1	c_2	$\dots$	c_m	0	0	0	0	

C_B is the objective function coefficients for each of the basic variables.

Z_j is the decrease in the value of the objective function that will result if one unit of the variable corresponding to the $j^{th.}$ column of the matrix formed from the coefficients of the variables in the constraints is brought into the basis (thus if the variable is made a basic variable with a value of one).

C_j-Z_j called the Net Evaluation Row, is the net change in the value of the objective function if one unit of the variable corresponding to the j^{th} column of the matrix (formed from the coefficient of the variables in the constraints), is brought into solution.

From the C_j- Z_j row we locate the column that contains the largest positive number and this becomes the Pivot Column. In each row we now divide the value in the RHS by the positive entry in the pivot column (ignoring all zero or negative entries) and the smallest one of these ratios gives the pivot row. The number at the intersection of the pivot column and the pivot row is called the PIVOT. We then divide the entries of that row in the matrix by the pivot and use row operation to reduce all other entries in the pivot column, apart from the pivot, to zero.

3.7.2.2 The Stopping Criterion

When all the entries in the net evaluation row, that is, Cj-$Z j$, are all negative or zero then the optimal solution to the linear program problem is reached.

3.7.2.3 Summary

Pivoting around a selected element means to make all the entries above and below it 0.

1. Verify the solutions in the original inequalities and objective function is not easy when dealing with 3 or more values. Although the solutions for 2 variable problems can actually be verified, it should be just accept that the theory works for higher dimensional problems.
2. Slack variables: These are the 'extra' variables put into the table (tableau). They will form a diagonal of 1's.

3.7.3 Branch and Bound

The most widely used method for solving integer programs is branch and bound. Sub problems are created by restricting the range of the integer variables. For binary variables, there are only two possible restrictions: setting the variable to 0, or setting the variable to 1. More generally, a variable with lower bound l and upper bound u will be divided into two problems with ranges l to q and q+1 to u respectively. Lower bounds are provided by the linear-programming relaxation to the problem: keep the objective function and all constraints, but relax the integrality restrictions to derive a linear program. If the optimal solution to a relaxed problem is (coincidentally) integral, it is an optimal solution to the subproblem, and the value can be used to terminate searches of subproblems whose lower bound is higher.

3.7.3 .1 Solving Integer Programming with Branch-and-Bound Technique

This method is also called divide and conquer . This is because a large problem is divide into a few smaller ones. (This is the "branch" part.) The conquering part is done by estimate how good a solution we can get for each smaller problems (to do this, we may have to divide the problem further, until we get a problem that we can handle), that is the "bound" part. We will use the linear programming relaxation to estimate the optimal solution of an integer programming.

For an integer programming model P , the linear programming model we get by dropping the requirement that all variables must be integers is called the linear programming relaxation of P .

The steps are:

1. Divide a problem into subproblems

2.Calculate the LP relaxation of a subproblem

3. The LP problem has no feasible solution, done;

4. The LP problem has an integer optimal solution; done. Compare the optimal solution with thebest solution we know (the incumbent).

5.The LP problem has an optimal solution that is worse than the incumbent, done.

6.The LP problem has an optimal solution that are not all integer, better than the incumbent. In this case the subproblem would be further divided and the steps repeated.

3.7.4 Solver Technology for Integer and Mixed-Integer Programming problems

The various technologies for solving Integer and mixed integer programming problems are discussed below: Thousands or tens of thousands of linear programs might be solved in the course of branch-and-bound. Clearly a faster linear programming code can result in faster integer programming solutions. Some possibilities that might be offered are primal simplex, dual simplex, or various interior point methods. The choice of solver depends on the problem size and structure for instance, interior point methods are often best for very large, block-structured models and can differ for the initial linear relaxation (when the solution must be found "from scratch") and subproblem linear relaxations when the algorithm can use previous solutions as a starting basis. The choice of algorithm can also be affected by whether constraint and/or variable generation are being used.

3.7.4 .1 Branch and Bound

The standard **Microsoft Excel Solver** uses a basic implementation of the Branch and Bound method to solve MIP problems. Its speed limitations make it suitable only for problems with a small number (perhaps 50 to 100) integer variables.

The **Premium Solver** and **Premium Solver Platform** use an extended Branch and Bound method that supports the alldifferent constraint as a native type, as well as reduced cost fixing for integer variables. It also uses more sophisticated rules for choosing the next node to explore and the next variable to branch upon, based upon pseudocosts which are estimates of the change in the objective that will result from branching on a given variable.

The **Large-Scale GRG Solver,Large-Scale SQP Solver**, **KNITRO Solver**, **MOSEK Solver**, and **LGO Global Solver** make use of the Premium Solver Platform's Branch and Bound method to handle integer variables and the all different constraint.

The **Large-Scale LP Solver** an integrated Branch and Bound plus Cut Generation strategy, often called Branch and Cut. It supports the all different constraint by generating an equivalent matrix of 0-1 variables and incorporating these into the problem. Its Branch and Bound method uses pseudocosts, degradation factors and strong branching, and it implements a number of cuts.

The **XPRESS Solver Engine** uses an integrated and highly tuned Branch and Cut strategy. It uses a variety of node selection and branch variable selection strategies, including pseudocosts, degradation factors and strong branching, and offers many user options for controlling the search strategy. Like the Large-Scale LP Solver, it supports the all different constraint by generating an equivalent matrix of 0-1 variables and incorporating these into the problem.

The **Gurobi Solver Engine** also uses an integrated and highly tuned Branch and Cut strategy, with a variety of node selection and branch variable selection strategies. It was designed to take maximum advantage of multi-core processors by parallelizing the Branch and Bound search. Like the XPRESS Solver Engine, it supports the all different constraint by generating an equivalent matrix of 0-1 variables and incorporating these into the problem.

3.7.4 .2 Strong Branching

Strong Branching is a method used to estimate the impact of branching on each integer variable on the objective function, by performing a few iterations of the Dual Simplex method. Such pseudo costs are used to guide the choice of the next subproblem to explore, and the next integer variable to branch upon, throughout the Branch and Bound process.

The **Large-Scale LP Solver**, **XPRESS Solver Engine** and **Gurobi Solver Engine** use Strong Branching techniques in their own Branch and Bound methods.

3.7.4 .3 Preprocessing and Probing

Preprocessing and probing strategies exploit the special properties of 0-1 or binary integer variables. For example, they use the constrained settings of certain 0-1 variables to determine settings for other 0-1 variables, without solving an optimization subproblem.

The standard **Microsoft Excel Solver** and the **Premium Solver** do not employ any such strategies. The **Premium Solver Platform** uses several Preprocessing and Probing methods including feasibility testing, optimality fixing, bounds improvement and variable reordering for Branch and Bound.

The **Large-Scale LP Solver**, **Large-Scale SQP Solver**, and **MOSEK Solver** make full use of the Premium Solver Platform's Preprocessing and Probing methods.

The **XPRESS Solver Engine** and **Gurobi Solver Engine** both use a variety of Preprocessing and Probing strategies including most of the logical preprocessing methods of the Premium Solver Platform, reduced cost fixing, and probing at the top node.

3.7.4 .4 Cut Generation

Cut Generation involves the automatic generation of additional constraints, or "cuts," that reduce the size of the feasible region for the optimization subproblems that must be solved, without eliminating any potential integer solutions.

The LP/Quadratic Solver in the **Premium Solver Platform** can generate both Gomory Cuts and Lifted Cover Inequalities at the root node, using a "Cut and Branch" framework. The

Large-Scale SQP Solver and the **MOSEK Solver** can generate Lifted Cover Inequalities at the root node.

The **Large-Scale LP Solver** uses a wide range of Cut Generation methods. It can generate Lift and Cover, Rounding, Knapsack, Gomory, Clique and "Odd Hole" cuts in several passes at any node in the Branch and Bound tree.

The **XPRESS Solver Engine** employs sophisticated Cut Generation methods in an integrated Branch and Cut framework. It can generate both Gomory Cuts and Lifted Cover Inequalities at any node. User options make it possible to control cut frequency and the depth of nodes eligible for cut generation.

The **Gurobi Solver Engine** also employs many sophisticated Cut Generation methods in an integrated Branch and Cut framework. It gives users control of the overall degree and frequency of cut generation, but wherever possible it makes an automatic choice of the best methods for a specific problem.

3.7.4 .5 Integer Heuristics

Heuristicsare "rules of thumb" that may often, but not always, succeed in achieving a given result. The **Evolutionary Solver** and the **LP/Quadratic Solver** in the Premium Solver Platform, and the **Large-Scale SQP Solver** and **MOSEK Solver** each use heuristics to attempt to find an integer feasible solution, or "incumbent," early in the Branch and Bound search. Such an incumbent can be used to prune the search tree and save time later in the search.

The **XPRESS Solver Engine** and **Gurobi Solver Engine** both make sophisticated use of integer heuristics. User options make it possible to control the type and frequency of application of these heuristic rules.

3.7.4 .6 Nontraditional Methods

The Evolutionary Solver built into the **Premium Solver Platform** and the **OptQuest Solver** use "nontraditional methods" to handle integer variables and the alldifferent constraint. In both of these solvers, integer variables and permutations are represented directly, and candidate solutions are generated that always satisfy integer and alldifferent constraints. The Evolutionary Solver uses several different integer- and permutation-preserving mutation and crossover operators to generate new candidate solutions

3.7.5 Interior Point Methods

Introduction

All forms of the simplex methods reach the optimum solution by traversing a series of basic solutions. Although the simplex method performs well in practice a theory on the performance of the simplex method was not available. In 1972, Klee and Minty showed by examples that for certain linear programs the simplex method will examine every vertex. These examples proved that in the worst case, the simplex method requires a number of steps that is exponential in the size of the problem. In view of this result, many researchers believed that a good algorithm, different from the simplex method, might be devised whose number of steps would be polynomial rather than exponential in the program's size, that is, the time required to compute the solution would be bounded above by a polynomial in the size of the problem. Indeed, Khachiyan (1979), discovered the first polynomial algorithm for solving linear programming known as Khachiyan'selLiPSoid method. The method is quite different in structure than the simplex method, for it constructs a sequence of shrinking elLiPSoids each of which contains the optimal solution set and each member of the sequence is smaller in volume than its predecessor by at least a certain fixed factor. Khachiyan's elLiPSoid method showed that polynomial time algorithms for linear programming do exist. It left open the question of whether one could be find an algorithm that is faster in practice than the simplex method. But unfortunately, practical experience of Khachiyan's elLiPSoid method was disappointing. In almost all cases, the simplex method was much faster than the elLiPSoid method.

Karmarkar (1984) found a new polynomial time algorithm, an interior-point method, with the potential to improve the practical effectiveness of the simplex method. The interior-point method algorithm is designed for dealing with big problems having many hundreds or thousands of functional constraints.

3.7.5.1 Primal Dual Methods

It is one of the three main categories of the interior point methods. The primal dual algorithm operates simultaneously on the primal and the dual linear programming. They find the solutions

$$(x^*, y^*, s^*)\ of$$

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ S^k & 0 & A^k \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_s \end{bmatrix} = \begin{bmatrix} r_P^k \\ r_D^k \\ -X^k s^k + \gamma \mu_k e \end{bmatrix}$$

by applying variants of Newton's method to the above and modifying the search directions and the step lengths so that inequalities $(x, s) \geq 0$ are satisfied strictly at every iteration. X , $S \in R^{nxn}$ are diagonal matrices of , $x_i s_i$ respectively and $e \in R^n$ is a vector of ones.

The Primal Problem

Given the linear programming problem in the standard form:

(P) minimize $c^T x$

subject to $Ax = b$, $x \geq 0$

where $c \in R^n$, $_{m\,n} A \in R^{nxn}$ and $b \in R^m$ are given data, and $x \in R^n$ is the decision variable.

The dual (D) to the primal (P) can be written as:

(D) maximize $b^T y$

subject to $A^T y + s = c$, $s \geq 0$

with variables $y \in R^m$ and $s \in R^m$

The Centering Parameter (σ)

It balances the movement towards the central path against the movement toward optimal solutions. If $\sigma = 1$, then the updates move towards the center of the feasible region. If $\sigma = 0$, then the update step is in the direction of the optimal solution.

The Duality Gap (μ)

It is the difference between the primal and dual objective functions. Theoretically, these two quantities are equal and so give a result of zero (0) at optimality. In practice however, the algorithm drives the result down to a small amount. This is given by the equation

$$\mu \equiv \frac{1}{n}(X^T S) = C^T x - b^T Y$$

While $\mu \geq \varepsilon$, Newton's method is applied until $\mu \leq \varepsilon$ when the algorithm terminates. ε is a positive fixed number.

The Primal-Dual Algorithm

Initialization

1.Choose$\beta\gamma \in (0,1)$ and $(\varepsilon_p, \varepsilon_D, \varepsilon_G,) > 0$

Choose (x^0, y^0, s^0) such that $(x^0, s^0) > 0$ and $\|X^0 s^0 - \mu_0 e\| \leq \beta\mu_0$

Where

$$\mu_0 = \frac{(x^0)^T s^0}{n}$$

1. Set $k = 0$
2. Set $r_p^k = b - Ax^k,\ r_D^k = c - Ak^T - s^k,\ \ r_k = \frac{(x^k)^T s^T}{n}$
3. Check the termination, if $\|r_P^k\| \leq \varepsilon_P, \|r_p^k\| \leq \varepsilon_D, (x^k)^T s^k \leq \varepsilon_G$, then terminate.
4. Compute the direction by solving the system

$$\begin{bmatrix} A & 0 & 0 \\ 0 & A^T & I \\ S^k & 0 & A^k \end{bmatrix} \begin{bmatrix} d_x \\ d_y \\ d_s \end{bmatrix} = \begin{bmatrix} r_P^k \\ r_D^k \\ -X^k s^k + \gamma\mu_k e \end{bmatrix}$$

5. Compute the step size

$\alpha = \max\{\alpha' : \|X(\alpha)s(\alpha) - \mu(\alpha)e\| \leq \beta(\alpha), \nabla\alpha\epsilon[0, \alpha']\}$,

$where: x(\alpha) = x^k + ad_x,\ s(\alpha) = s^k + ad_z\ and\ \mu(\alpha) = \frac{x^T(\alpha)s(\alpha)}{n}$.

6. Update $x^{k+1} = x^k + \alpha_k d_x,\ y^{k+1} = y^k + \alpha d_y, s^{k+1} = s^k d_s$

7. Set $k = k + 1$, and go to step 3.

CHAPTER FOUR

DATA COLLECTION, ANALYSIS AND RESULTS

4.0 Introduction

This research aimed at finding the optimum profit in trading six commodities between Techiman and Obuasi central markets using the linear programming technique to formulate the trader's problem. The main goal of this study is to determine the optimal quantity of the six commodities to trade in other the optimize the profit. The commodities use in this study were Maize, Groundnut, Cassava(Gari), Millet, Beans, and Yam.

Linear Programming(LP) models were designed to reflect various quantities of commodities traded between the two markets, current market prices and the demand of the commodities in the Obuasi market.

The objective of the study was to model quantities of commodities traded at Obuasi as a Linear Programming problem and determine the optimal quantities of commodities using Integer Linear Programming after satisfying a set of constraints. The variables in the models were the quantities of the commodities while the cost of each commodity was the Parameter.

4.1 Source and Data Collection

For the purpose of this research work a field survey was conducted in two market places thus,Techiman and Obuasi Central Markets on six agricultural commodities which are frequently traded. The data collected for this study were based on the whole sale prices of the commodities, cost of loading per 100kg, cost of royalties per 100kg and the cost of transporting 100kg of commodities from the Techiman market to Obuasi market.

The commodities used in ration formulation for the optimal commodity trading include maize (x_1), Groundnut (x_2) Cassava Gari (x_3) Millet (x_4) Beans (x_5) and Yam (x_6).

Table 4.1 shows the average demand of the six agricultural commodities under consideration in Obuasi central market.

Table 4.1 Average commodity demand in Obuasi central market.

Commodity	Variables representing Commodities	Demand Kg/day	Percentage of demand
Maize	X_1	139	24%
Groundnut	X_2	72	12.41%
Cassava (Gari)	X_3	98	16.89%
Millet	X_4	42	7.24%
Beans	X_5	105	18.1%
Yam	X_6	124	21.36%

Source: Field Survey in the Obuasi central market January –June 2013.

The table 4.1 above gives information about the quantity of the six commodities traded in Obuasi market, it is observed that, among the six agricultural commodities Maize is highly demanded per day with a quantity of 139kg while Millet was least demanded with an average quantity of 42kg per day.

Let

C.P = Cost Price

S.P = Selling Price

T.C.P = Total Cost Price (Cost Price+ Royalties+ Loading+ Transportation)

The table below shows the wholesale prices and other cost of the six commodities in the Techiman and Obuasi central markets

Table 4.2 Average Prices of the six agricultural commodities in the Obuasi and Techiman central markets.

Commodity	C.P atTechiman GH₵/Kg	S.P atObuasiGH₵/Kg	C.P 100Kg at Techiman	S.P/100kgatObuasi	Royalties Per 100kg	Transportation Per 100kg	LoadingAndoffLoading Per 100Kg
Maize (white grain)	0.72	1.32	72	132	0.5	20	1
Groundnuts (edible)	3.10	3.65	310	365	0.5	20	1
Cassava (Gari)	1.03	2.05	103	205	0.5	20	1
Millet (Saniograin)	0.85	1.09	85	109	0.5	20	1
Beans (white)	0.70	1.28	70	128	0.5	20	1
Yam tuber	0.4	1.12	40	112	0.5	20	1

Source: Field Survey in the Obuasi central market January –June 2013

It could be observed in the above table 4.2 that, it will be good for a trader trading these six agricultural commodities between the two markets to trade from Techiman to Obuasi on Yam, Gari, Maize, Beans, Millet and Groundnuts, because the spot prices show that it is cheap to buy from Techiman and sell at Obuasi.

Table 4.3 Total Cost Prices and Selling Prices of the six agricultural commodities in Techiman and Obuasi central markets respectively .

Commodity	T.C.P/ bag at Techiman GH¢/100Kg	S.P/bag at Obuasi GH¢/100Kg ($S.P_i$)	Profit /100Kg (GH¢)
Maize (white grain)	93.5	132	38.5
Groundnuts (edible)	331.5	365	33.5
Cassava (Gari)	124.5	205	80.5
Millet (Sanio grain)	106.5	109	2.5
Beans (white)	91.5	128	36.5
Yam tuber ((Pona)	61.5	112	50.5

Other findings:

- The average trading cost of the traders in the commodity market was GH¢ 4500.00
- At least one 100kg of each of the commodities is traded in the Obuasi market any day since there is a demand for each of the six commodities

4.2 Problem Formulation

4.2.1 Assumptions

a. The first assumption of this model is that, the quantities of each commodity are directly proportional to the cost. This means that, discounts were not considered in the formulation of this model.

b. The second assumption of this model is no lead-times when transporting the commodity from one market to the other. Implicitly, assuming that at any given time what is bought in one market is sold at the other market. Although in reality transporting of physical commodities cannot be done immediately, the second assumption simplifies the problem and makes it more tractable.

4.2.2 Objective Function

The objective function is built up from the sum of profits for the various commodities.

Let

$x_i=[x_1, x_2,x_3,x_4,x_5,x_6]$ be quantity of commodities

$S.P_i$= selling price and are formed in Table 4.3 Column 3

Table 4.4 Selling price of the commodities

Commodity	Maize (white grain)	Groundnuts (Edible)	Cassava (Gari)	Millet (Sanio grain)	Beans (white)	Yam tuber (Pona)
S.P(x_i)	132	365	205	109	128	112

The above table 4.4 gives information about the selling prices of the six agricultural products under consideration in this work

TCP=Total cost price and are formed in Table 4.3 Column 2

TCP is obtained by ∑(Actual cost of the commodity + Royalty + Transportation+ Loading and offloading)

Table 4.5 Total cost price of the commodity

Commodity	Maize (white grain)	Groundnuts (Edible)	Cassava (Gari)	Millet (Sanio grain)	Beans (white)	Yam tuber (Pona)
TCP(x_i)]	93.5	331.5	124.5	106.5	91.5	61.5

The table 4.5 above gives information about the cost of each of the six agricultural commodities under consideration in this work. The total cost price (TCP) is the summation of the actual cost price of the commodity at Techiman market, royalty, transportation, Loading and Off loading

The profit = ∑[SP(x_i)-TCP(x_i)]

The profits for trading each of the commodities are formed in Table 4.3 column 4, by subtracting the TCP of each commodity from the SP.

Table 4.6 Commodities profit

Commodity	Maize (white grain)	Groundnuts (Edible)	Cassava (Gari)	Millet (Sanio grain)	Beans (white)	Yam tuber (Pona)
Profit	38.5	33.5	80.5	2.5	36.5	50.5

The profit of the commodities are obtained by subtracting the TCP from the SP as shown in the above table Table. 4.6

Combining with the quantities of commodity to be sold the objective function of the proposed model is formulated as;

Max Z=∑[SP(x_i)-TCP(x_i)], is the Objective function]

Substituting the various SP and TCP in the tables 4.0 and 4.3 into the model, we have

$$Z = [132 - 93.5]x_1 + [365 - 361.5]x_2 + [205 - 124.5]x_3 + [109 - 106.5]x_4 + [128 - 91.5]x_5 + [112 - 61.5]x_6$$

This function is simplifies to

Maximum $Z = 38.5x_1 + 33.5x_2 + 80.5x_3 + 2.5x_4 + 36.5x_5 + 50.5x_6$

Where

Z = the total profit

SP= Selling Price

TCP= Total Cost Price (Cost price +Royalties + Loading +transportation)

4.2.3 The problem constraints:

1. Total funds constraints

The average fund for trading the six commodities was found to be Gh¢ 4500. The total cost price of the commodities is found Table 4.5, thence the total funds constraints is give as:

$$93.5x_1 + 331.5x_2 + 124.5x_3 + 106.5x_4 + 91.5x_5 + 61.5x_6 \leq 4500$$

2 The demand Constraints:

- The quantity of Millet and that of the Groundnut traded per day is always less than or equal to the quantity of Maize traded perday.

Millet(x_2) + Groundnut(x_4) ≤ Maize(x_1)

$x_2 + x_4 \leq x_1$

$-x_1 + x_2 + x_4 \leq 0$

- Maize represent 24% of all the six commodities (see Table 4.1)

Maize(x_1) ≥ 24% (of the quantities of the six commodities)

$x_1 \geq 0.24(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)$

$x_1 \geq 0.24x_1 + 0.24x_2 + 0.24x_3 + 0.24x_4 + 0.24x_5 + 0.24x_6$

$0.76x_1 \geq \quad 0.24x_2 + 0.24x_3 + 0.24x_4 + 0.24x_5 + 0.24x_6$

Non-negativity constraints

$x_1 \geq 1$, $x_2 \geq 1$, $x_3 \geq 1$, $x_4 \geq 1$, $x_5 \geq 1$, $x_6 \geq 1$

The maximization problem and its constraints can be rewritten as:

Max Z= $38.5x_1 + 33.5x_2 + 80.5x_3 + 2.5x_4 + 36.5x_5 + 50.5x_6$

Subject to:

$93.5x_1 + 331.5x_2 + 124.5x_3 + 106.5x_4 + 91.5x_5 + 61.5x_6 \leq 4500$

$$-x_1 + x_2 + x_4 \leq 0$$

$$0.76x_1 \geq \quad 0.24x_2 + 0.24x_3 + 0.24x_4 + 0.24x_5 + 0.24x_6$$

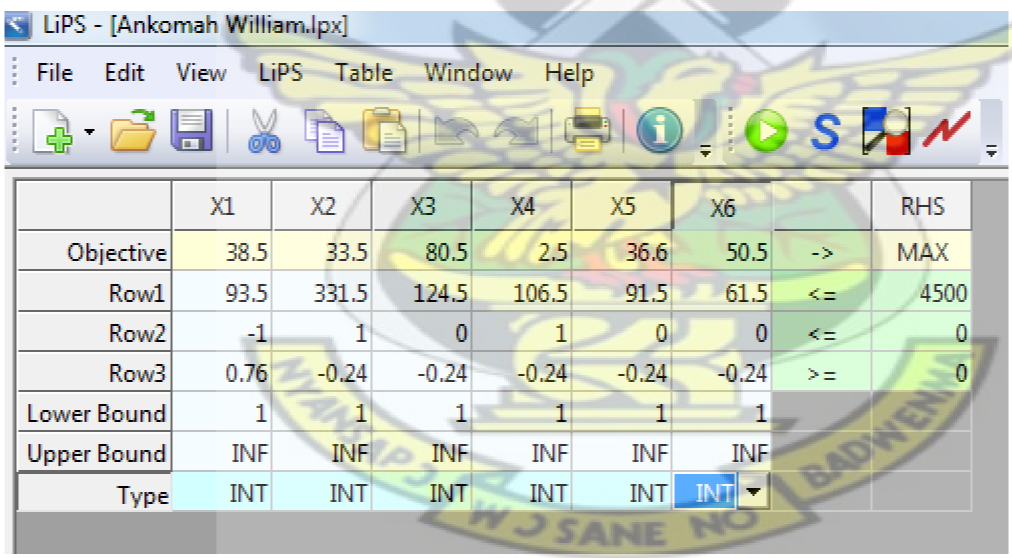
$x_1 \geq 1$, $x_2 \geq 1$, $x_3 \geq 1$, $x_4 \geq 1$, $x_5 \geq 1$, $x_6 \geq 1$

4.3 Computational procedure

The traders problem was developed as a Linear integer programming problem. The Linear Program Solver (LiPS) software was used in arriving at the optimal solution. According to Melnick (2012), The LiPS uses Branch and Bond method in arriving at the optimal solution , it is intended for solving linear, integer and goal programming problems.

The LiPS software allows the user to input data into the application interface by typing data directly into Ordered Sizes grid. Results from the computation can also be saved to a text file by clicking the "Save Output button.

Fig. 4.1 LiPS User Interface



	X1	X2	X3	X4	X5	X6		RHS
Objective	38.5	33.5	80.5	2.5	36.6	50.5	->	MAX
Row1	93.5	331.5	124.5	106.5	91.5	61.5	<=	4500
Row2	-1	1	0	1	0	0	<=	0
Row3	0.76	-0.24	-0.24	-0.24	-0.24	-0.24	>=	0
Lower Bound	1	1	1	1	1	1		
Upper Bound	INF	INF	INF	INF	INF	INF		
Type	INT	INT	INT	INT	INT	INT		

It could be observed from fig. 4.1 above that, the variable to be determine their quantities were set to integers with the lower bound equal to one and upper bound set to infinity as show on fig. 4.1

Data input to the LiPS are as follows:

max: 38.5*X1 + 33.5*X2 + 80.5*X3 + 2.5*X4 + 36.5*X5 + 50.5*X6;

Row1: 93.5*X1 + 331.5*X2 + 124.5*X3 + 106.5*X4 + 91.5*X5 + 61.5*X6 <= 4500;

Row2: -X1 + X2 + X4 <= 0;

Row3: 0.76*X1 - 0.24*X2 - 0.24*X3 - 0.24*X4 - 0.24*X5 - 0.24*X6 >= 0;

4.4 Computer Specification

After formulating the problem the LiPS software was installed on a computer with the specifications below, for the purposes of computing for the optimal profit.

- Operating System: Windows 7 Home Basic
- Processor type: Intel Core 2 Duo
- Processor speed: 2.00 Hz
- RAM: 1.00GB
- System type: 64 bits operating system

4.5 Results

The result generated using the Linear Programming Solver software is displayed in the appendixes but a portion of it is shown table 4.7 below for the purposes of our discussions.

Table 4.4 below, displays the summary of the output from the LiPS solver.

Table 4.7 Result Variable (LiPS output)

Variable	Value	Obj. Cost	Integer	Reduced cost
Maize (X_1)	12	38.5	YES	0
Groundnut (X_2)	1	33.5	YES	0
Cassava (Gari)(X_3)	3	80.5	YES	0
Millet(X_4)	12	2.5	YES	0
Beans (X_5)	1	36.5	YES	0
Yam (X_6)	21	50.5	YES	0

The above table 4.7, displays among others the quantities of the commodities to be traded for an optimal profit of GH₵ 1864.00.

4.6 Discussions

The optimal solution was reached after 10 iterations in 0.11s by the LiPS software. The best quantities of commodities to trade with an average capital of Gh¢ 4500.00 between Techiman and Obuasi market is given in the Table 4.4 above, this would results in optimum net profit of GH₵ 1864.00. per day when the following quantities of commodities are traded.

Maize =12 Groundnut=1 Cassava (Gari)= 3, Millet=12, Beans=1 Yam= 21.

The result variable table 4.4 (output from LiPS) consists the following columns:

- Variable column

  The variable column contains the names of the structural variables which represent the six agricultural commodities under study thus Maize, Groundnut, Cassava (Gari), Millet, Beans and Yam;

- Value column

  The Value column contains the quantities of each of the six commodities to trade in for the optimal profit.

- Obj. Cost Column

  The Obj. Cost Column contains the values of the objective function coefficients.

- Integer column

  The Integer column indicates whether the values are integers or continuous variables. It could be seen in the LiPS output in table 4.4 that all the values are integers.

- Reduced Cost

  The reduced cost column in the LiPS output table 4.4 for a basic variable indicates how much the coefficient for the variable can be increased before the optimal solution would change and this variable would become a non basic variable.

From the output, before the optimal value could be any better than GH₵ 1864.00 the reduced costs for the variables (which are non basic in the optimal solution) should be increased by 0 each respectively.

When any variable x_1, $x2$, $x3$, x_4, x_5, x_6 is a basic variable in the optimal solution, its reduced cost automatically is 0. This suggests that its coefficient in the objective function is too small to justify undertaking the activity it represents.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATION

5.0 Introduction

This study was to determine the quantity of commodity to trade for an optimal profit, the problem was formulated using Linear Programming (LP) technique and solved using branch and Bound method.

The objectives of this study was to model quantity of commodities traded at Obuasi as a Linear Programming problem (LPP) and determine optimal quantity of commodities using Integer Linear Programming (ILP) for profit maximization. Appropriate literature was reviewed to gather information on the practices of commodity trading in a market, as well as literature on price formulations of commodities. The decision variables, objective function and problem constraints were defined and a mathematical model of the traders' problem in Obuasi and Techiman market was developed and parameterized using data from Obuasi and Techiman central markets

Model solution and post-optimality analysis results were obtained. Six (6) decision variables and three (3) constraints were identified. The optimal solution of the linear programming model for maximization of profit was obtained. Using a real data on the model the profit was found to be GH₵ 1864.00 with the following quantities of commodities Maize =12 Groundnut=1 Cassava (Gari) = 3, Millet=12, Beans=1, Yam=21 are purchased from the Techiman market and sold at Obuasi market with the average capital of GH₵ 4500.00

The model will be very useful to the commodity traders association in Ghana.

5.1 Recommendation

Profit maximization is so important to the sustainability of trading not only in Ghana but the whole world. This study has demonstrated how the application of a linear programming approach to determine the optimal quantity of commodity to trade will lead to maximization of profit in this sector as opposed to the use of relatively inefficient methods such as the trial and error method.

This model was based on the demand of commodities in Obuasi market and the price spread of the commodities in the Techiman and Obuasi markets.

The general model could be extended to integrate the inventory aspect of trading. This model could also be extended to forecast profit of trading with the aid of a chart.

From the conclusion it is realized that using scientific methods to determine quantities of commodity trade will increase profits margin. Hence it is recommend that traders trading between Obuasi and Techiman should adapt this model to help them decide on the quantity of each commodity to trade.

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Appendix: Output from LiPS Software

Variable	Value	Obj. Cost	Integer	Reduced cost
Maize (X_1)	12	38.5	YES	0
Groundnut (X_2)	1	33.5	YES	0
Cassava (Gari) (X_3)	3	80.5	YES	0
Millet(X_4)	12	2.5	YES	0
Beans (X_5)	1	36.5	YES	0
Yam (X_6)	21	50.5	YES	0

COST Range

Variable	Current COST	Min COST	Max COST	Reduced Cost
X1	38.5	-6.72944	69.9654	0
X2	3.5	-infinity	216.113	0

X3	80.5	-infinity	85.7084	0
X4	2.5	-infinity	74.3688	0
X5	36.5	-infinity	64.9192	0
X6	50.5	40.519	+infinity	0

RHS Range

Constraint	Current RHS	Min RHS	Max RHS	Dual Price
Row 1	4500	984.5	+infinity	0.629974
Row 2	0	-12.196	+infinity	0
Row 3	0	-13.719	29.5617	-30.3192

Certification

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

William Ankomah	---------------------	--------------
ID: 20249896	Signature	Date

Certify by:

K.F. Darkwah	--------------------	-------------
Supervisor	Signature	Date

Certified by:

Prof. S.K. Amponsah	---------------------	---------------
Head of Dept. Name	Signature	Date

and

Certified by

Prof. I.K. Dontwi	-------------------------------	--------------------
Dean, IDL	Signature	Date

Dedication

This thesis is dedicated to my two children, Paapa Anyemadu Ankomah and Adwoa Nuako Ankomah

ABSTRACT

The goal of every commodity trader is to maximize his/ her profit, but what should traders do to maximize their profit? In this thesis the wholesale spot prices of six agricultural commodities in Obuasi and Techiman Central markets were examined. The trader purchases commodities from one market place and sells in the other market taking into consideration the spot price of the commodities in questions. In this thesis we are interested in finding the volume of the six commodities to be purchased from Techiman and sold in Obuasi central market to make maximum profit knowing the demand of those commodities. Linear programming Solver (software) was used to solve real trader's problem of a trader in the presence of some constraints

ACKNOWLEDGEMENT

The fear of the Lord is undeniably the beginning of Wisdom. At the outset I thank and praise the Almighty whose blessings and grace had bestowed in me the power and confidence to successfully complete this research work.

My gratitude is highly expressed to the lecturers in the Department of Mathematics for the guidance given to me to accomplish this work. I particularly owe an appreciable gratitude to my supervisor, Mr. K.F. Darkwah for his guidance and constructive suggestions leading to the realization of this work. I thank Mrs. Georgette Adams for editing the grammar in this thesis. I am also deeply indebted to my father Mr. John Carr who graciously gave me perfect education and always encourages me that the sky should always be my limit. I would like to extend my heartfelt thanks and gratitude to him.

Motivation will almost always beat mere talents. Hence I would be failing in my duty if I do not thank my well-wishers: Elder Charles Nti (presiding elder, C.O.P Maranatha Assembly-Akaporiso District) ,Mr. Peter Amponsah (former headmaster, Akrofuom Senior High School) for motivating me to do the work with great confidence.

My sincere thanks to my adorable wife Mrs. Charlotte Ankomah who rendered her support both mentally and physically throughout the writing of this thesis.

Finally, thanks to all the people who were directly and or indirectly involved in bringing out this research work. May God bless you all.

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