KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

ANALYSIS OF LOSS SEVERITY ON BANK LOANS: A CASE STUDY OF GHANAIAN BANKS

BY

JOHN OKAE-ASANTE

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DECLARATION

I hereby declare that this submission is my own work towards the award of the M. Phil degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

John Okae-Asante		
Student(PG8735812)	....	....
Student	Signature	Date

Certified by:

Yao Elikem Ayekple	....	....
Supervisor	Signature	Date

Certified by:

Prof. S. K. AMPONSAH	....	....
Head of Department	Signature	Date

DEDICATION

I dedicate this work to all Pioneering students of MSc Actuarial Science, KNUST who have put in a lot of efforts and sacrifice to the development of the Actuarial profession in Ghana.

ABSTRACT

In this thesis, we explore an actuarial approach to credit risk modeling. We use CREDITRISK+ to module loss severity on bank loans and use various tools such as the Poisson distribution to module default event. We also use various mathematical techniques in analyzing Credit Risk. We focus on modeling as well as quantitative analysis of bank loan portfolio. We start with a Credit Risk management problem. More specifically, we consider credit portfolio of multiple obligors subject to possible default. We propose a new structural module for the loss given default, which takes into account the severity of default. Then we study the behavior of the loss given default under the assumption that the losses of the obligors follow the well-known Poisson distribution. We then proceed to derive the distribution of default losses. Finally, we consider a credit portfolio of banks and analyze the data based on the CreditRisk+ framework to generate a loss distribution for the portfolio.

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CHAPTER 1

Introduction

1.1 Background of the Study

The purpose of this study is to conduct analysis of loss severity on bank loans. As a major component of risk management, risk analysis can provide management tools to product investors against a decline in value of assets and enhance the shareholder's value. Mathematically rigorous and applicable models for complex structures are used to analyse the risk associated with the daily business of banks all over the world. In this section we provide a summary of the scope of this research work. We provide here with the statement of the problem and also the research questions to be used throughout this thesis. The objective of this study is as well spelt.

The worldwide 'credit crunch' which started in 2006 with sub-prime mortgages in the United States, has highlighted the fundamental importance of the credit decision. As the problems in these mortgages have unfolded, it has demonstrated that unsound credit decisions were made and lessons as to how to effectively manage credit risk were either ignored or never learned. It shows that poor lending decisions, whether by a financial institution or a corporate, can lead to significant losses.

Even though one of the major causes of serious banking problems continues to be ineffective credit risk management, the provision of credit remains the primary business of every bank in the world. For this reason, credit quality is considered a primary indicator of financial soundness and health of banks. Interest that are charged on loans and advances form sizeable part of banks' asset. Default of loans and advances poses serious setbacks not only for borrowers and lenders but also

to the entire economy of a country. Studies of banking crises all over the world have shown that poor loans (asset quality) are the key factors of bank failures.
To protect depositors and the financial system overall, the 1998 Capital Accord ('Basel I') placed restrictions on the exposure a bank could have in relation to its capital. Thus, it restricted how much a bank could lend in total with a goal to decrease the probability that, in an extreme downturn of the economy, depositors would lose their money and (since banks lend to other banks) the banking system would collapse. As a result of the likely huge and widespread of economic impact in connection with banks failure, the management of credit risk is a topic of great importance since the core activity of every bank is credit financing.

1.2 Problem Statement

While a bank can fail for any number of reasons, one major cause of bank failure is weak risk management. Effective management of a bank requires effective management of the bank's credit risk as poor risk management could have an adverse effect on the bank's profitability A brief look at any typical banking portfolio will be sufficient to convince people that defaulting obligors belong to the daily business of banking the same way as credit applications or ATM machines. Banks therefore started to think about ways of loan insurance many years ago, and the insurance paradigm will now directly lead us to the first central building block credit risk management.
How do the banks appraise loan propositions prior to lending funds? What monitoring mechanisms have been built into the credit risk management practices of banks to minimize bad debts?
Banks use a variety of means to reduce and control credit risk. One way banks reduce credit risk is by using "risk-based pricing," in which banks charge higher rates to borrowers with more perceived credit risk. Another way is with "covenants," whereby banks apply stipulations to a loan, such as borrowers must periodically report on their financial condition, or such that borrowers must repay

the loan in full after certain events (like changes in the borrower's debt-to-equity ratio or other debt ratios). Another method is diversification, which can reduce credit risk to banks as well as a diversified borrower pool is less likely to default simultaneously, leaving the creditor without hope of recovery. Besides these, many firms utilize credit insurance or credit derivatives, such as "credit default swaps," in an attempt to transfer risk to other firms. The above methods used by banks are focused on individual risks. The entire credit portfolio is also subject to credit risk. The portfolio contains both retail obligors and corporate obligations and hence is heterogeneous. A portfolio approach to risk management is therefore very essential to a bank. It can help achieve.

1.3 Objectives of the Study

The purpose of this study is to conduct a quantitative analysis of loss severity on bank loans. As a major component of risk management, risk analysis can provide management tools to protect investors against a decline in value of assets and enhance the shareholder value. In this study, mathematical rigorous and applicable models for complex structures are used to analyze the risk associated with the daily business of banks all over the world.
Quantities of interest in this project include:

1. Finding the distribution of losses in a portfolio of credit exposures

2. Determining the expected loss and Value at Risk for the credit portfolio

3. Determine the economic capital for the portfolio

1.4 Significance of the Study

The study will serve as an analytical tool for stakeholders in the financial industries. Another significance of the study is to serve as a reference material for future research studies in the field of credit risk. It is my hope that the findings and recommendation of the study will serve as a yardstick in credit risk management.

1.5 Methodology

The methodology that will be used in this study is similar to methods used to model claim distribution in the insurance industry. It is an actuarial approach to risk modelling. We apply the creditrisk+ modelling technique to data obtained from a Ghanaian bank. The creditrisk+ model is highly analytic and mathematically tractable. It makes no assumption on the causes of default hence models default as a Bernoulli trial. Obligors would be grouped into sectors and a sectorial analysis would also be done.

1.6 Scope and Limitation of Study

The study focuses on credit risk management in financial institutions. The analysis is performed using data derived from the financial statement of banks supervised by the Central Bank during the most recent six year period. The credit portfolio of some commercial banks are studied and critically analyzed.

1.7 Organisation of the Thesis

To facilitate reading and understanding of this report, the study has been structured into five distinct chapters.
Chapter one is the introduction of research proposal and featured background information, problem statement, purpose and rationale of the study, research questions, as well scope and limitations of the study. Chapter two featured the literature review related to the topic of the study Chapter three focuses on the details of the research methodology as well as outlined credit risk management problem. More specifically, we consider a credit portfolio some commercial

banks in Ghana. Chapter four presented the analyzed data together with their interpretation as well as discussion of findings. Chapter five summarized the study made recommendations and drew very useful conclusions.

CHAPTER 2

Literature Review

2.1 Introduction

Over the last decade, a number of the world's major banks have developed sophisticated systems to quantify and aggregate credit risk across geographical and product lines. The initial interest in credit risk models stemmed from the desire to develop more rigorous quantitative estimates of the amount of economic capital needed to support a bank's risk-taking activities. As the outputs of credit risk models have assumed an increasingly large role in the risk management processes of large banking institutions, the issue of their potential applicability for supervisory and regulatory purposes has also gained prominence.

This chapter provides a description of the state of practice in credit risk modeling, and assesses the potential uses of credit risk models for supervisory and/or regulatory purposes, including the setting of regulatory capital requirements. This review highlighted the wide range of practices both in the methodology used to develop the models and in the internal applications of the model's output. Credit risk modeling methodologies allow a tailored and flexible approach to price measurement and risk management. Models are, by design, both influenced by and responsive to shifts in business lines, credit quality, market variables and the economic environment.

Furthermore, models allow banks to analyses marginal and absolute contributions to risk, and reflect concentration risk within a portfolio. These properties of models may contribute to an improvement in a bank's overall credit culture.

The degree to which models have been incorporated into the credit management and economic capital allocation process varies greatly between banks. While some banks have implemented systems that capture most exposures throughout the organisation, others only capture exposures within a given business line or legal entity. Additionally, banks have frequently developed separate models for corporate and retail exposures, and not all banks capture both kinds of exposures. The internal applications of model output also span a wide range, from the simple to the complex. For example, only a small proportion of the banks surveyed by the Task Force is currently using outputs from credit risk models in active portfolio management; however, a sizeable number noted they plan to do so in the future. Current applications included:

1. setting of concentration and exposure limits
2. setting of hold targets on syndicated loans
3. risk-based pricing
4. improving the risk/return profiles of the portfolio
5. evaluation of risk-adjusted performance of business lines or managers using risk-adjusted return on capital ("RAROC")
6. economic capital allocation. Institutions also rely on model estimates for setting or validating loan loss reserves, either for direct calculations or for validation purposes

2.2 Overview of Conceptual Approaches to Credit Risk Modelling

In surveying a significant number of credit risk models arrange of conceptual modelling approaches was reviewed. In this section, we will make a list of these approaches, but aim to discuss key elements of the different methodologies we reviewed.

We begin this section by introducing the concepts of economic capital allocation and the probability density function of credit losses, then move on to a discussion of various constituent elements of credit risk models. These are:

i choice of time horizon and review of default-mode and mark-to-market approaches to measuring credit loss

ii probability density functions

iii conditional/unconditional models

iv approaches to credit aggregation

v approaches to dependence between default events (default correlations, etc.).

The choices of conceptual methodology that a bank makes when building a credit risk model are largely subjective ones, based on considerations such as the characteristics of the bank's loan portfolio and its credit culture. While this section raises many conceptual issues regarding the various approaches, the question of the materiality of each is an empirical one.

2.3 Economic Capital Allocation For Credit Risk

2.3.1 Probability density function of credit losses

When estimating the amount of economic capital needed to support their credit risk activities, many large sophisticated banks employ an analytical framework that relates the overall required economic capital for credit risk to their portfolio's probability density function of credit losses (PDF), which is the primary output of a credit risk model. A bank would use its credit risk modelling system to estimate such a PDF. An important property of a PDF is that the probability of credit losses exceeding a given amount X (along the x-axis) is equal to the (shaded) area under the PDF to the right of X. A risky portfolio, loosely speaking, is one whose PDF has a relatively long and fat tail.

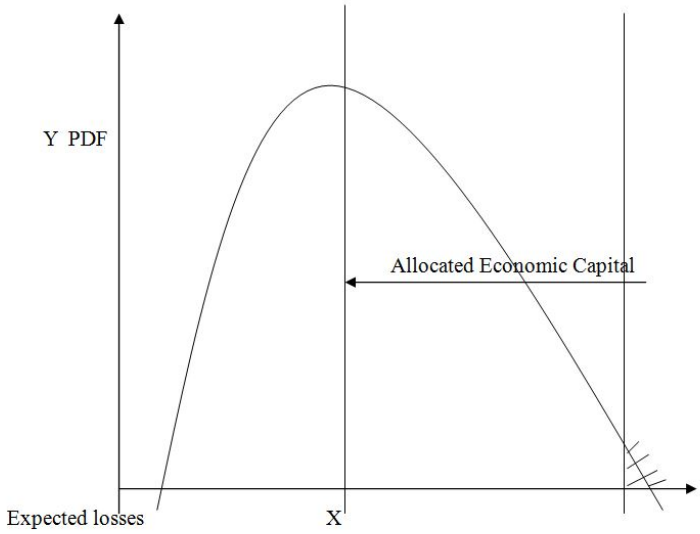

Figure 2.1: PDF of Credit Losses

The expected credit loss (shown as the left-most vertical line)shows the amount of credit loss the bank would expect to experience on its credit portfolio over the chosen time horizon. Banks typically express the risk of the portfolio with a measure of unexpected credit loss (i.e. the amount by which actual losses exceed the expected loss) such as the standard deviation of losses or the difference between the expected loss and some selected target credit loss quartile. The estimated economic capital needed to support a bank's credit risk exposure is generally referred to as its required economic capital for credit risk. The process for determining this amount is analogous to value at risk (VAR) methods used in allocating economic capital against market risks. Specifically, the economic capital for credit risk is determined so that the estimated probability of unexpected credit loss exhausting economic capital is less than some target insolvency rate. Capital allocation systems generally assume that it is the role of reserving policies to cover expected credit losses, while it is that of economic capital to cover unexpected credit losses. Thus, required economic capital is the additional amount of capital necessary to achieve the target insolvency rate, over and above that needed for coverage of expected losses. In Exhibit 1, for target insolvency rate equal to the shaded area, the required economic capital equals the distance between the two dotted lines. Broadly defined, a credit risk model encompasses all of the policies, procedures and practices used by a bank in estimating a credit portfolio's PDF.

2.3.2 Measuring Credit Loss

In general, a portfolio's credit loss is defined as the difference between

• the portfolio's current value and

• Its future value at the end of some time horizon

The estimation of the current portfolio's PDF involves estimating

• The portfolio's current value and

• The probability distribution of its future value at the end of the planning horizon. The precise definitions of current and future values – and, hence, virtually all of the operational details of the credit risk model – follow from the specific concept of credit loss that is of interest to the model-builder. Within the current generation of credit risk models, banks employ either of two conceptual definitions of credit loss, termed the default mode (DM) paradigm or the mark-to-market (MTM) paradigm.

2.3.3 Time Horizon

A bank's decision on the time horizon over which it monitors credit risk can follow one of two approaches. First is the "liquidation period" approach, in which each facility is associated with a unique interval, coinciding with the instrument's maturity or with the time needed for its orderly liquidation. Alternatively, an institution may choose to apply a common time horizon across all asset classes. Most of the banks surveyed adopt a one-year time horizon across all asset classes. A minority utilize a five-year approach or modeled losses over the maturity of the exposure. A small number use other horizons, while some noted they might run their models for more than one horizon. A number of vendor models allow users to select an asset-specific (or portfolio-specific) holding period horizon based on the unique structure of each underlying exposure. The considerations mentioned for the choice of a modelling horizon of one year were that this reflected the typical interval over which:

- new capital could be raised
- loss mitigating action could be taken to eliminate future risk from the portfolio
- new obligor information could be revealed
- default rate data may be published
- internal budgeting, capital planning and accounting statements are prepared

For the banks that chose a "hold-to-maturity" approach, the considerations included the following:

- exposures were intended to be held to maturity
- there were limited markets in which the credits could be traded.

2.3.4 Discounted Cash Flow vs. Risk-neutral Valuation Approaches Default Mode Paradigm

Within the DM paradigm, a credit loss arises only if a borrower defaults within the planning horizon. To illustrate, consider a standard term loan. In the absence of a default event, no credit loss would be incurred. In the event that a borrower defaults, the credit loss would reflect the difference between the bank's credit exposure (the amount it is owed at the time of default) and the present value of future net recoveries (cash payments from the borrower less workout expenses). The current and future values of credit instruments in the DM paradigm are defined in a manner consistent with the underlying two-state (default vs. non-default) notion of credit losses. For a term loan, the current value would typically be measured as the bank's credit exposure (e.g., book value). The (uncertain) future value of the loan, however, would depend on whether or not the borrower defaults during the planning horizon. If the borrower does not default, the loan's future value would normally be measured as the bank's credit exposure at the end of the planning horizon, adjusted so as to add back any principal payments made over the period. On the other hand, if the borrower were to default, the loan's future value would be measured as one minus its loss rate given default (LGD). The lower the LGD, the higher the recovery rate following default. Note that at the time the credit risk model is being used to estimate the portfolio's PDF– the beginning of the planning horizon – the current values of credit instruments are assumed to be known, but their future values are uncertain. Within DM-type credit risk models,therefore, for each separate credit facility (e.g. loan vs. commitment vs. counter party risk) a bank must impose or estimate the joint probability distribution with respect to three types of

2.3.5 The Mean/Standard Deviation Approach

To illustrate the above concepts, it is useful to relate the above variables to the mean and standard deviation of a portfolio's credit losses. Some systems for allocating economic capital against credit risk typically assume that the shape of the PDF is well-approximated by some family of distributions (e.g. the beta distribution) that could be parameterized by the mean and standard deviation of the portfolio's losses. Market practitioners generally term this methodology the unexpected losses (UL) approach. Under the UL approach, the economic capital allocation process generally simplifies to setting capital at some multiple of the estimated standard deviation of the portfolio's credit losses. Within the DM paradigm, the UL approach requires estimates of a portfolio's expected and unexpected credit loss. A portfolio's expected credit loss) μ over the assumed time horizon equals the summation of the expected losses for the individual credit facilities:

$$\mu = \sum_{i=1}^{n} EDFLEELGD \tag{2.1}$$

where for the i facility, LGDi is the expected loss rate given default, EDFiis the facility's expected probability of default (often termed the expected default frequency or EDF), and LEEi is the bank's expected credit exposure (often termed the loan equivalent exposure or LEE). The portfolio's standard deviation of credit losses (σ) can be decomposed into the contribution from each of the individual credit facilities:

$$\sigma = \sum \sigma_i \rho_i \tag{2.2}$$

Where σ_i denotes the stand-alone standard deviation of credit losses for the i^{th} facility, and π denotes the correlation between credit losses on the it facility and those on the overall portfolio. The parameter π captures the i^{th} facility's correlation and diversification effects with the other instruments in a bank's credit portfolio. Other things being equal, higher correlations among credit instruments – represented by higher π – lead to a higher standard deviation of credit losses for the portfolio as a whole.

Under the further assumptions that;

• facility's exposure is known with certainty,

• Customer defaults and LGDs are independent of one another

• LGDs are independent across borrowers, the stand-alone standard deviation of credit losses for the it facility can be expressed as

$$\sigma = LEE_i \sqrt{EDF_i(1 - EDF_i)LGD} + EDF_i VOL \tag{2.3}$$

where VOL is the standard deviation of the facility's LGD. These equations provide a convenient way of summarizing the overall portfolio's credit risk (within the DM framework) in terms of each instrument's VOL, ,LGD , EDF, ρ and LEE.

They also serve to highlight those aspects of the credit risk modelling process that determine its overall reliability, namely:

• The accuracy of parameter estimates as representations of the future.

• the validity of the model's underlying assumptions, such as assumptions of independence among random variables, assumptions that certain variables are known with certainty, and the distributional assumption that maps UL to a target credit loss quantile

2.3.6 Internal Risk Rating Systems, EDFs and Rating Transition Matrices

As illustrated by the UL approach, the EDF – the probability of a particular credit facility defaulting during the time horizon – is a critical model input. This is true not only for DM-type credit risk models, but also for MTM-type models (discussed below). Within most credit risk modelling systems, a customer's internal credit risk rating (as determined by a bank's credit staff) is a key – if not the sole – criterion for determining the EDFs applicable to the various credit facilities associated with that customer; generally all the customer's facilities are presumed to default concurrently, or not at all. Most of the large, internationally active banks reviewed by the Task Force assign risk ratings to each large corporate customer. Each large corporate customer, for example, might be placed into one of, say, 10 possible risk rating categories or buckets. In general, the process of arriving at a credit rating for a customer or facility can be described as containing one or more of the following three elements:

• The traditional "spreading of numbers" in which financial and other characteristics of the customer (e.g. country and business sector code) are incorporated into a relatively subjective approach to determining grades

• The use of vendor-supplied commercial credit scoring models

• The use of internally developed credit scoring models. Increasingly, banks are also assigning internal risk ratings, or their equivalent, to small- and middle-market business customers, and even to individual retail customers based on credit scoring models and other information.

Often, a bank will establish a concordance schedule that relates its internal risk rating categories to some external rating standard, such as S&P's or Moody's ratings for corporate bonds. For example, a grade-1 loan may be deemed roughly equivalent to an S&P bond rating from AA to AAA, a grade-2 loan equivalent to a bond rating of single-A, and so on. Under such a scheme, the worst internal grade, say grade-10, would typically correspond to the "worst state", termed the "default" state. Given this concordance, an EDF can be interpreted as representing a loan's probability of migrating from its current internal rating grade to default within the credit model's time horizon. The likelihood of a customer migrating from its current risk rating category to any other category within the time horizon is frequently expressed in terms of a rating transition matrix. Given the customer's current credit rating (delineated by each row), the probability of migrating to another grade (delineated by the columns) is shown within the intersecting cell. Thus, in the exhibit, the likelihood of a BBB-rated loan migrating to single-B within one year would be 0.32%. Since under the DM paradigm only rating migrations into the default state lead to changes in the values of loans, only the last column of this matrix would be relevant. Within the MTM paradigm (discussed below), however, the other columns of the transition matrix also play a critical role.

2.3.7 Mark-to-market paradigm

In contrast to the DM paradigm, within the MTM paradigm a credit loss can arise in response to deterioration in an asset's credit quality short of default. In effect, the MTM paradigm treats the credit portfolio as being marked to market (or, more accurately, marked to model) at the beginning and end of the planning horizon, with the concept of credit loss reflecting the difference between these valuations. MTM-type models recognize that changes in an asset's creditworthiness, and its potential impact on a bank's financial position, may occur due to events short of default. Hence, in addition to EDFs, these models must also incorporate (through the rating transition matrix described above) the probabilities of credit rating migrations to non-default states. Given the rating transition matrix associated with each customer, Monte Carlo methods are generally used to simulate migration paths for each credit position in the portfolio.

For each position, the simulated migration (and the risk premium associated with the instrument's end-of-period rating grade) is used, in effect, to mark the position to market as of the end of the time horizon. Most MTM-type credit models employ either a discounted contractual cash flow(DCCF) approach or a risk-neutral valuation (RNV) approach for purposes of modelling the current and future (mark-to-market) values of credit instruments. The DCCF methodology is commonly associated with J.P. Morgan's Credit Metrics framework. The current value of a loan that has not defaulted is represented as the present discounted value of its future contractual cash flows. For a loan having a particular internal risk rating (comparable to, say, BBB), the credit spreads used in discounting the contractual cash flows would equal the market-determined term structure of credit spreads associated with corporate bonds having that same grade. The current value of a loan would be treated as known, while its future value would depend on its uncertain end-of-period risk rating and the term structure of credit spreads associated with that rating. Thus, the value of a loan can change over the time horizon, reflecting either a migration of the borrower to a different risk rating grade or a change in the market-determined term structure of credit spreads. One of threatening grades to which a loan can migrate over the planning horizon is "default". Obviously, the present value of a defaulted loan would not be based on the discounting of contractual cash flows. Rather, as with DM-type models, in the event of default, the future value of a loan would be given by its recovery value, equal to one minus the LGD.

2.3.8 Risk-neutral Valuation Approach

Although it is easily understood and implemented, the DCCF approach is not fully consistent with modern finance theory. Typically, identical discount rates are assigned to all loans to firms having the same internal risk rating or EDF. Consequently, if a firm has not defaulted as of the planning horizon, the future values of its loans do not depend on the expected LGDs of the loans. Senior and subordinated loans to a single firm would have the same future discount price, regardless of differences in expected recovery in the event of future default. Furthermore, finance theory holds that the value of an asset depends on the correlation of its return with that of the market. Under DCCF, however, loans to two identically rated firms receive the same discount rates, even if the two firms are not equally sensitive to the business cycle or to other systematic factors. To avoid these problems, the RNV approach imposes a structural model of firm value and bankruptcy based on the work of Robert Merton. In this framework, a firm goes into default when the value of its underlying assets falls beneath the level needed to support its debt. Instead of discounting contractual payments, the RNV method discounts contingent payments: if a payment is contractually due at date t, the payment actually received by the lender will be the contractual amount only if the firm has not defaulted by date t; the lender receives a portion of the loan's face value equal to 1-LGD if the borrower defaults at date t,and the lender receives nothing at date t if the borrower has defaulted prior to date t. A loan can thus be viewed as a set of derivative contracts on the underlying value of the borrower's assets. The value of the loan equals the sum of the present values of these derivative contracts. The discount rate applied to the contract's contingent cash flows is determined using the risk-free term structure of interest rates and the risk-neutral pricing measure. Intuitively, the risk-neutral pricing measure can be thought of as an adjustment to the probabilities of borrower default at each horizon, which incorporates the market risk premium associated with the borrower's default risk. The size of the adjustment depends on the expected return and volatility of the borrower's asset value. If asset return is modeled consistent with the Capital Asset Pricing Model (CAPM) framework, then the expected return can be expressed in terms of the market expected return and the firm's correlation with the market. Thus, consistent with standard finance theory, the pricing of loans under RNV adjusts not only for the EDF and LGD of the borrower, but also for the correlation between borrower risk and systematic risk.

The dichotomy between the DCCF and RNV approaches to pricing may be sharper in theory than in practice. In each methodology, a loan's value is constructed as a discounted present value of its future cash flows. The approaches differ mainly in how the discount factors are calculated. The DCCF method takes a non parametric approach to estimating these discount factors. Public issuers of debt are grouped into rating categories. Credit spreads on the issuers are then averaged within each "bucket". Alternatively, the RNV method is highly structural – it imposes a model that prices each loan simultaneously in a single unified framework. In practice, the calibration of the market risk premium in the model typically makes use of credit spreads from the debt market. Econometric theory shows that highly structural estimators make efficient use of available data but are vulnerable to model misspecification; whereas non parametric estimators make minimal use of modelling assumptions but perform poorly where data are scant or noisy. The two approaches will, in general, assign different values to any given loan. Nonetheless, if debt markets are reasonably efficient and the assumptions of the RNV model are approximately valid, then the two methods ought to produce similar aggregate values for well-diversified portfolios.

2.4 Probability Density Functions

2.4.1 Measurement

Each model examined aims to quantify a portfolio's credit risk via the concept of PDF of credit losses over a chosen time horizon. Many models reviewed seek to estimate explicitly the full PDF; statistics such as the mean and standard deviation or a chosen target credit loss quantile can then be calculated readily. Examples of this approach include the vendor models Credit Risk, Portfolio Manager, Credit Portfolio View, and Credit Metric in its Monte Carlo formulation. Other proprietary and vendor models (including the unexpected losses approach and Credit Metric in its analytical formulation) aim only to generate the first two moments of the distribution, i.e. its mean and standard deviation; the full PDF remains implicit in the model. There seem to be two main reasons for this technique:

• For purposes of analytical simplicity or computational speed, the model seeks to establish only the mean and standard deviation from the outset; no particular functional form for the PDF is assumed;

• due to data or computational constraints, the full PDF is available for some but not all sub-portfolios; for the other sub-portfolios, only the means and standard deviations are calculated; consequently, only the mean and standard deviations are calculated for the total portfolio.

A consensus within the industry about a "standard" shape of the PDF has yet to emerge. This stands in contrast with market risk models, where the normal distribution is frequently used as a standard or benchmark. Observed portfolio credit loss distributions are markedly on-normal. They are typically skewed towards large losses, and leptokurtic (i.e. for a given mean and standard deviation, the probability of large losses occurring is greater than would be the case if the distribution were normal). One reason why no industry "standard" portfolio credit loss PDF has emerged is that the modelling of losses from individual credit exposures is more difficult than is the case for market risk, and a wide range of simplifying assumptions is made. Individual losses might be assumed to be binary, or else to follow one of a range of continuous distributions. The portfolio PDF that results from aggregating these individual credit exposure losses will depend strongly upon these assumptions (and upon assumptions made in estimating credit correlations).

2.5 Top-down and Bottom-up Approaches

Within most credit risk models, broadly the same conceptual framework is used indwelling individual-level credit risk for different product lines; differences in implementation arise primarily in the ways the underlying parameters are estimated using available data (see Part III for a discussion of parameter estimation). For most of the banks surveyed, credit risk is measured at the individual asset level for corporate and capital market instruments (a so-called "bottom-up" approach), while aggregate data is used for quantifying risk in consumer, credit card or other retail portfolios (a so-called "top-down" approach). However, while the literature on credit risk models tends to make a distinction between these two approaches, the differences are less clear-cut in practice. For example, different models may be classified as "bottom-up" given their use of borrower-specific information to "slot "loans into buckets, even though underlying parameters may be calibrated using aggregate data. Models adopting a bottom-up approach attempt to measure credit risk at the level of each loan based on an explicit evaluation of the creditworthiness of the portfolio's constituent debtors. Each specific position in the portfolio is associated with a particular risk rating, which is typically treated as a proxy for its EDF and/or probability of rating migration. These models could also utilize a micro approach in estimating each instrument's LGD. The data is then aggregated to the portfolio level taking into account diversification effects. For retail customers, the modeling process is conceptually similar; however, due to the sheer number of exposures, models tend to adopt a more top-down empirical approach. In this instance loans with similar risk profiles, such as credit scores, age and geographical location, are aggregated into buckets, and credit risk is quantified at the level of these buckets. Loans within each bucket are treated as statistically identical. In estimating the distribution of credit losses, the model-builder would attempt to model both the (annual) aggregate default rate and the LGD rate using historical time-series data for that risk segment taken as a whole, rather than by arriving at this average through the joint consideration of default and migration risk factors for each individual loan in the pool.

2.6 Overview of Correlations Between Credit Events

While no bank with a diversified portfolio would expect all – or even nearly all – of its obligors to default at once, experience shows that the factors affecting the creditworthiness of obligors sometimes behave in a related manner. Consequently, in measuring credit risk, the calculation of a measure of the dispersion of credit risk (i.e. its standard deviation, or indeed the full PDF) requires consideration of the dependencies between the factors determining credit-related losses, such as correlations among defaults or rating migrations, LGDs and exposures, both for the same borrower and among different borrowers. Various models achieve these in very different ways and authors have sought to draw comparisons and contrasts between the methodologies.

2.7 Cross-Correlations between Different types of Credit Events

At least in theory, across different bank customers, one might expect to observe significant correlations among:

- Default events/rating migrations
- LGDs
- Exposures

For example, the financial condition of firms in the same industry or within the same country may reflect similar factors, and so may improve or deteriorate in a correlated fashion. Similarly, for firms within the same industry, LGDs, as well as exposures due to drawdowns of credit lines, ay tend to increase (decrease) relative to their long-run averages in periods when the average condition of firms in that sector is deteriorating (improving). While banks tend to be well aware of these potential relationships, their ability to model such correlations is often limited in practice. In general, owing to data limitations, credit risk models do not attempt to explicitly model correlations between different types of risk factors. Specifically, correlations between defaults/rating migrations and LGDs, between defaults/rating migrations and exposures and between LGDs and exposures are typically assumed to equal zero. According to the Task Force findings in virtually all credit risk models the only correlation effects considered at present are the correlations between defaults/rating migrations of different customers. Broadly, banks have adopted either a structural approach or a reduced-form approach for handling default/rating migration correlations.

2.8 Correlations Among Defaults or Rating Migrations

As exemplified by the Credit Metrics and Portfolio Manager modeling frameworks, under the structural approach the model-builder would typically posit some explicit microeconomic model of the process determining defaults or rating migrations of individual customers. A customer might be assumed to default if the underlying values of its assets falls below some threshold, such as the level of the customer's liabilities. Within the MTM framework, the change in the value of a customer's assets in relation to various thresholds is often assumed to determine the change in its risk rating over the planning horizon. For example, given a customer's current risk rating (say, equivalent to BBB), an extremely large positive change to its net worth (appropriately scaled) might correspond to an upgrade to AAA, while an extremely large negative realization might generate a downgrade to default, etc. In general, the random variable assumed to determine the change in a customer's risk rating, including default (e.g. customer asset value or net worth) is called the migration risk factor. Thus, within structural models, it is the correlations between migration risk factors (across borrowers) that must be specified (estimated or assumed) by the model-builder. In turn, these correlations between migration risk factors determine, implicitly, the correlations among borrower's defaults or rating migrations.

2.9 Theoretical Framework

The project results provided in the literature can be explored to determine the most commonly used frameworks among banks and financial institutions. One survey we could find in the literature is the survey of Fatemi and Fooladi (2006) performed among the 21top banking firms in the United States. The results of this survey shows that most of these firms use or are planning to use the Credit Metrics framework of J. P. Morgan or the Portfolio Manager framework of KMV, and some use the Credit Risk model of Credit Suisse First Boston. Another survey whose results are presented by Smithson et al. (2002)and which was carried out by Rutter Associates in 2002 among 41 financial institutions around the world reveals that 20 per cent of the institutions that use a credit risk model (85per cent) use Credit Manager, which is the application service based upon Credit Metrics, 69per cent use Portfolio Manager, and the remaining use their own internal models. Moreover, ECB (2007) states that most central banks use a model based on the Credit Metrics framework. Besides these surveys, there are also surveys performed among Turkish banks. One of these surveys is the survey of Anbar (2006) that was carried out among 20 banks in Turkey in 2005. The results show that only 30 per cent of these banks use a credit risk model or software but 33 per cent of those who use a credit risk model use Risk Metrics, which was developed by J. P. Morgan and is philosophically very similar to Credit Metrics, while the remaining banks use their own models. Another survey, which was carried out by Oktay and Temel (2007) in 2006 among 34 commercial banks in Turkey, shows that most of the banks participated in the survey use Portfolio Manager, Credit Metrics and/or Credit Risk. Besides these models, the Credit Portfolio View framework developed by McKinsey&Company is another model we frequently run in the literature. The following subsections explain the methodologies of these models.

2.10 Credit Metrics

In this section, we first give a brief review of the literature over Credit Metric methodology. Then, we discuss the statistical basis of the model and look into the calibration of the necessary parameters. Finally, the drawbacks of the methodology mentioned in the literature are stated.

2.11 Framework

Credit Metrics framework proposed by J.P. Morgan is one of the portfolio credit value-at-risk models. Guptonet al. (1997) give details of Credit Metrics as follows. Credit Metrics essentially utilizes the fact that if asset returns (percent changes in assets value) of a firm, namely an obligor, fall below a certain threshold, then that firm defaults. In fact, Credit Metrics is not a pure-default or default-mode model, meaning a model which accepts loss only in case of a default. It combines the default process with credit migrations which correspond to rating transitions. These kinds of models are called Mark-to-Market Models. So, by applying forward yield curves for each rating group, Credit Metrics is able to estimate credit portfolio value and the unexpected loss of a credit portfolio. In that manner, as Crouhyet al. (2000) state that Credit Metrics is an extended version of Merton's option pricing approach to the valuation of a firm assets since Merton only considers default. Although Credit Metrics offers the estimation of joint credit quality migration likelihoods as a way of observing the correlation structure, it is not very practical to do so when dealing with extremely large credit portfolios. In addition, it will require a huge dataset. For these reasons, it handles the correlations between obligors by introducing multi factor model. In a multi factor model, a latent variable triggers a change (default orating transition) in the credit worthiness of a firm. Furthermore, in Credit Metrics framework, the asset return of a firm is used as a latent variable driven both by systematic risk factors, such as country index, industry index and regional index, and by a firm specific factor (non-systematic or idiosyncratic risk factor). However, Credit Metrics proposes the use of equity returns to reveal the correlation structure of a credit portfolio instead of asset returns while asset returns are not always observable in the market. Yet, equity correlations are not equal to asset correlations. This assumption does not take into account the firm leverage effect on asset values (Jacobs, 2004). By examining the effect of different systematic factors (for instance, country-industry index) over the changes in equity of a firm and time series data belonging to those systematic factors, one can determine the correlation structure as explained. Correlations between firms are explained through systematic factors in Credit metrics. These factors are more like global parameters that may affect the firms, which have direct or indirect connections or relations, concurrently. Non-systematic factors do not contribute any correlation between firms while they are firm specific. Moreover, Credit Metrics framework assumes that normalized asset returns are distributed standard normally and so are systematic and non-systematic risk factors. After the simulation of asset returns and thus rating changes, at the beginning of each period Credit Metrics re-evaluate the future cash flows (monthly or yearly payments of a debt or coupon payments of a bond) in order to find the current portfolio value. In revaluation of future cash flows, Credit Metrics uses spot rates, which are also called forward zero-coupon rates. Spot rates or forward zero-coupon rates are nothing but the geometric mean of the forward rates, which are agreed interest rates between two parties for the upcoming years of the payment horizon (Choudhry, 2003).

2.12 Link to Statistical Models

At first glance, Credit Metrics methodology looks like a qualitative-response model, such as an ordered probit model. Duffie and Singleton (2003) explains that these models link the ratings and thus the rating changes to an underlying variable worthy to explain the credit-worthiness of an obligor (or a bond). Again, the boundaries around this underlying variable are determined in order to be able to trigger a rating change. However, these boundaries are specific to each obligor not to each rating group as in Credit Metrics. In addition, these factors are far from being external factors and thus different from the Credit Metrics' correlation structure, which triggers correlated transitions. For instance, Duffie and Singleton (2003) gives asset return coefficient beta of the obligor, and its balance sheet information as examples to possible factors of an ordered probit model. Moreover, a probit model uses the cdf of the standard normal distribution to map the value of the latent variable, to an event probability(Rachevet al., 2007). In other words, a probit model finds default probabilities without the use of a default threshold. In fact itself, acts as a default threshold in probit models. Glasserman and Li (2005) call the multi factor model of Credit Metrics as the normal copula model and use this model in their simulation studies whereas Kalkbreneret al.(2007) call this group of models as Gaussian multi factor models. Both ideas are based on the same assumption that systematic factors, which affect the asset value of a firm, have multivariate normal distribution and so do the asset values of different firms or obligors. The statistical models that Glasserman and Li (2005) and Kalkbreneret al. (2007) use and the Credit Metrics framework in default mode are very similar models. However, the latent variable in the statistical model used by Glasserman and Li (2005) triggers a default if it crosses a barrier, which is again calculated from the long-term average default probabilities as done in Credit Metrics but these probabilities are assumed to be firm specific (but obviously these Probabilities can be chosen again with respect to the ratings of obligors to obtain a Credit Metrics-wise model). Yet, all these models are asset based models. Next, Credit Metrics essentially uses a rating transition matrix, which is nothing but the transition matrix of a Markov Chain Process. Therefore, in a sense, Credit Metrics tries to model the intensities to default and to other states, namely ratings. Nonetheless, we cannot call it simply as a default-intensity (also called reduced-form) model while it does not use these intensities in valuing the bond or the credit exposure. Besides, it does not use a tractable stochastic model for the intensities. Instead, Credit Metrics merely makes use of rating-specific historical averages of the transitions in order to explain a steady-state Markov Chain. Although Credit Metrics has an inspiration from asset-value models, or in other words structural models, it does not utilize the log-normality assumption of asset returns in any estimation procedure within the credit portfolio simulation. As a result, we choose to call Credit Metrics merely as a historical method explained by Schmid (2004).

2.13 Drawbacks from Literature

One important drawback of this framework is the usage of average transition probabilities, which are calculated as historical average of migration and default data(Crouhyet al., 2000). As a result, every obligor within the same rating group has the same transition and default probabilities. Furthermore, if and only if an obligor migrates to another rating, default probability of that firm is adjusted accordingly. This is a rather discrete modeling of default. In fact, as Crouhyet al. (2000) also argues, default rates are continuous over time whereas ratings are not. Moody's KMV strongly opposes this conjecture of J. P. Morgan. Similarly, Schmid (2004) also counters the fact that Credit Metrics disregard default rate volatilities. He then emphasizes that default intensities are also correlated with business cycles and industries to which the obligors belong. Yet, Schmid (2004) asserts, there is not enough information in the market to estimate transition matrices with respect to business cycles but the estimation of different matrices is possible for industries. Another disadvantage of this framework is that it requires a wide data set since it is not a default only model but a Mark-to-Market model. Even when the required data are achievable, it is most likely that some of the estimates will have statistically low significance levels (Schmid, 2004). In addition, Schmid (2004) affirms that because of the capability of rating agencies in catching the rating changes, both the probability of maintaining the last rating and the default probability overrate the true probabilities, forcing some other transition probability estimates underestimate their true figures. Altman (2006) sees the recovery rate process adopted by Credit Metrics as another drawback of the framework while it handles recovery only at the time of default and generally uses a beta distribution to assess recovery rates. He provides significant empirical evidence that recovery rates and default probabilities are correlated events, and recovery rate process should be included as a factor in systematic risk of an obligor. Infact, none of the portfolio VaR models apply a similar methodology. As a final disadvantage, it should be added that this methodology is only applicable to the firms with a known rating (Schmid, 2004). Considering that most of the firms in Turkey do not have a rating, this is very essential if implemented in Turkey. In such situations, Schmid (2004) suggests the use of observable financial data of those firms to calculate fundamental financial ratios so that by matching them to the ones of the firms with known ratings, it can be possible to determine the unknown ratings.

CHAPTER 3

Methodology

3.1 Introduction

In this chapter, we shall consider the data collection method for the study and mathematical formulations of the models.

3.2 Types of Data and Method of Data Collection

Secondary data for the study was obtained partly from the Banking Supervision Department of the Bank of Ghana as well as articles in both local newspapers and international journals. In the view of Freund and William (2002), secondary data relates to information that has already been published. Due to the fact that secondary data are already published without specific needs of a decision or a particular purpose such as this research work been taken into consideration, Zikmund and Babin (2010) point out that caution must be exercise when using such information. The information provided through secondary research can be biased, partial and of poor quality. In this study therefore, only audited credit reports covering the period between 2009 and 2014 from the Central Bank were considered. Other reports have been carefully edited and reviewed in consonant with the spirit of the study. From the above reports, the researcher collected information on credit rating, default in payment as well as the recovery rate for corporate department. The researcher employed EXCEL-STATS and R software to help analyse data from the field and this help in calculating descriptive statistics.

3.3 A Portfolio Approach to Managing Credit Risk

Credit risk can be managed through diversification because the number of individual risks in a portfolio of exposures is usually large. Currently, the primary technique for controlling credit risk is the use of limit systems, including individual obligor limits to control the size of exposure, tenor limits to control the maximum maturity of exposures to obligors, rating exposure limits to control the amount of exposure to obligors of certain credit ratings, and concentration limits to control concentrations within countries and industry sectors. C First (1997).

The portfolio risk of a particular exposure is determined by four factors;

i. The size of the exposure: Amount of outstanding with the obligor at the time of default.

ii. The maturity of the exposure(M)

iii. the probability of default of the obligor(PD): Probability that the obligor will default within a given time horizon

iv. Loss given default(LGD): Percentage loss incurred relative to the exposure at default

C First 1997 in their analysis stated that credit limits aim to control risk arising from each of these factors individually. The general effect of this approach, when applied in a well-structured and consistent manner, is to create reasonably well diversified portfolios. However, these limits do not provide a measure of the diversification and concentration of a portfolio.

3.4 CREDITRISK$^+$ Modle

CREDITRISK$^+$ Modle is based on a portfolio approach to modeling credit default risk that takes into account information relating to size and maturity of an exposure and the credit quality and systematic risk of an obligor. Tom Wilde (1999)

The CREDITRISK$^+$ Modle is a statistical model of credit default risk that makes no assumptions about the causes of default. This approach is similar to that taken in market risk management, where no attempt is made to model the causes of market price movements. The research work carried out by Risk April (2003) indicates that since the introduction in 1997, CREDITRISK$^+$ has become one of the most widely used portfolio models. Its advantages include the fact that the portfolio loss distribution can be calculated analytically such that Monte Carlo simulation can be avoided. According to the paper, CREDITRISK$^+$ has become the most popular due to its tractability. The CREDITRISK$^+$ Model considers default rates as continuous random variables and incorporates the volatility of default rates in order to capture the uncertainty in the level of default rates. Often, background factors, such as the state of the economy, may cause the incidence of defaults to be correlated, even though there is no causal link between them. Tom Wilde (1999) The effects of these background factors are incorporated into the CREDITRISK$^+$ Model through the use of default rate volatilities and sector analysis rather than using default correlations as explicit inputs into the model. Mathematical techniques applied widely in the insurance industry are used to model the sudden event of an obligor default. This approach contrasts with the mathematical techniques typically used in finance. In financial modeling one is usually concerned with modeling continuous price changes rather than sudden events. Applying insurance modeling techniques, the analytic CREDITRISK$^+$ Model captures the essential characteristics of credit default events and allows explicit calculation of a full loss distribution for a portfolio of credit exposures.

3.5 Modeling Techniques Used: CREDITRISK $^+$ Model

The economic risk of a portfolio of credit exposures is analogous to the economic risk of a portfolio of insurance exposures. In both cases, losses can be suffered from a portfolio containing a large number of individual risks, each with a low probability of occurring. The risk manager is concerned with assessing the frequency of the unexpected events as well as the severity of the losses. In order to keep model and process errors to a minimum, no assumptions are made about the causes of default. Mathematical techniques applied widely in the insurance industry are used to model the sudden event of an obligor default. In modelling credit default losses one is concerned with sudden events rather than continuous changes. The essential characteristics of credit default events are captured by applying these insurance modelling techniques. This has the additional benefit that it leads to a credit risk model that is analytically tractable and hence not subject to the problems of precision that can arise when using a simulation-based approach. The analytic CREDITRISK $^+$ Model allows rapid and explicit calculation of a full loss distribution for a portfolio of credit exposures.

3.6 Default Event Frequency

Credit default is the risk that an obligor is unable to meet its financial obligations. In the event of a default of an obligor, a firm generally incurs a loss equal to the amount owed by the obligor less a recovery amount which the firm recovers as a result of fore closure, liquidation or restructuring of defaulted obligor. A loan is considered a default when the loan is not paid according to the terms agreed to the promising note. Credit defaults occur as a sequence of events in such a way that it is not possible to forecast the exact time of occurrence of any one default or the exact total number of defaults. In this section we derive the basic statistical theory of such processes in the context of credit default risk. Consider a portfolio consisting of n obligors. In line with the above assumptions, it is assumed that each exposure has a definite known probability of defaulting over a one-year time horizon. Thus

$$Let P_A = Annual\ probability\ of\ default \tag{3.1}$$

To analyze the distribution of losses arising from the whole portfolio, we introduce the probability generating function defined in terms of an auxiliary variable z by

$$F(z) = \sum_{n=0}^{\infty} p(ndefaults) z^n \tag{3.2}$$

An individual obligor either defaults or does not default. The probability generating function for a single obligor is easy to compute explicitly as

$$F_A(z) = 1 - P_A + P_A z = 1 + P_A(z - 1) \tag{3.3}$$

As a consequence of independence between default events, the probability generating function for the whole portfolio is the product of the individual probability generating functions. Therefore

$$F(z) = \prod_A F_A(z) = \prod_A (1 + P_A(z - 1)) \tag{3.4}$$

It is convenient to write this in the form

$$lnF(z) = \sum_A ln(1 + P_A(z - 1)) \tag{3.5}$$

Next suppose that the individual probabilities of default are uniformly small. According to C First (1997), this is characteristic of portfolios of credit exposures. Given that the probabilities of default are small, powers of those probabilities can be ignored. Thus, the logarithms can be replaced using the expression

$$ln(1 + P_A(z - 1)) = P_A(z - 1) \tag{3.6}$$

And in the limit, equation(3.5) becomes

$$F(z) = e^{\sum_A P_A(z-1)} = e^{\mu(z-1)} \tag{3.7}$$

Where we write

$$\mu = \sum_A P_A \tag{3.8}$$

$$F(z) = e^{\mu(z-1)} = e^{-\mu}e^{-\mu z} = \sum_{n=0}^{\infty} \frac{e^{-\mu}\mu^n}{n!} z^n \tag{3.9}$$

Thus if the probabilities of individual default are small, although not necessarily equal, then from equation (3.9)we deduce that the probability of realising n default events in the portfolio in one year is given by;

$$\frac{e^{-\mu}\mu^n}{n!} \tag{3.10}$$

In equation (3.10) we have obtained the well-known Poisson distribution for the distribution of the number of defaults under our initial assumptions. The following should be noted:

• The distribution has only one parameter, the expected number of defaults . The distribution does not depend on the number of exposures in the portfolio.

• There is no necessity for the exposures to have equal probabilities of default; indeed, the probability of default can be individually specified for each exposure if sufficient information is available.

3.7 Default Losses With Fixed Default Rates

Under our initial assumptions, the distribution of numbers of defaults in a portfolio of exposures in one year has been obtained. However, our main objective is to understand the likelihood of suffering given levels of loss from the portfolio, rather than given numbers of defaults. The distributions are different because the same level of default loss could arise equally from a single large default or from a number of smaller defaults in the same year. Unlike the variation of default probability between exposures, which does not influence the distribution of the total number of defaults, differing exposure amounts result in a loss distribution that is not Poisson in general. Moreover, information about the distribution of different exposures is essential to the overall distribution. However, it is possible to describe the overall distribution because its probability generating function has a simple closed form amenable to computation.

3.8 Using Exposure Bands

According to C First(1997) the first step in obtaining the distribution of losses from the portfolio in an amenable form is to group the exposures in the portfolio into bands. This has the effect of significantly reducing the amount of data that must be incorporated into the calculation. Banding introduces an approximation into the calculation. However, provided the number of exposures is large and the width of the bands is small compared with the average exposure size characteristic of the portfolio, the approximation is insignificant. Intuitively, this corresponds to the fact that the precise amounts of exposures in a portfolio cannot be critical in determining the overall risk. Once the appropriate notation has been set up, an estimate of the effect of banding on the mean and standard deviation of the portfolio is given below.

Table 3.1: Definition of Symbols

Reference	Symbol
Obligor	A
Exposure	L_A
Probability of Default	P_A
Expected Loss	λ_A

In order to perform the calculations, a unit amount of exposure L,(the total amount of credit extended to a borrower by a lender. The magnitude of credit exposure indicates the extent to which the lender is exposed to the risk of loss in the event of the borrower default.) denominated in a base currency, is chosen. For each obligor A, define number ε_A and ν_A by writing

$$L_A = L * \nu_A \text{ and } \lambda_A = L * \varepsilon_A$$

Thus ε_A and v_A are the exposure and expected loss, respectively, of the obligor, expressed as multiples of the unit. The key step is to round each exposure size v_A to the nearest whole number. This step replaces each exposure amount L_A by the nearest integer multiple of L. If a suitable size for the unit L is chosen, then, after the rounding has been performed for a large portfolio, there will be a relatively small number of possible values for v_A each shared by several obligors. The portfolio can then be shared into m exposure bands, indexed by j, where $1 \leq j \leq m$. With respect to the exposure bands, we make the following definitions.

Table 3.2: References and Symbols

References	Symbols
Common exposure in Exposure band j in units of L	ν_j
Expected loss in exposure band j in units of L	ε_j
Expected number of defaults in exposure band j	μ_j

The following relations hold, expressing the expected loss in terms of the probability of default events, hence

$$\mu = \frac{\varepsilon_j}{\nu_j} = \sum_{A:\nu_A=\nu_j} \frac{\varepsilon_A}{\nu_A} \tag{3.11}$$

Let μ be the total expected number of default events in the portfolio in one year. Since μ is the sum of the expected number of default events in each exposure band, we have

$$\mu = \sum_{j=1}^{m} \mu_j = \sum_{j=1}^{m} \frac{\varepsilon_j}{\nu_j} \tag{3.12}$$

3.9 Loss Distribution With Fixed Default Rates

We have analyzed the distribution of default events under our initial assumptions. We now proceed to derive the distribution of default losses. Intuitively, the default loss analysis involves a second element of randomness, because some defaults lead to larger losses than others through the variation in exposure amounts over the portfolio. As with default events, the second random effect, is best described mathematically through its probability generating function. Thus, let G (z) be the probability generating function for losses expressed in multiples of the unit L of exposure:

$$G(z) = \sum_{n=0}^{\infty} p(aggregatelosses = nxL) z^n \tag{3.13}$$

The exposures in the portfolio are assumed to be independent. Therefore, the exposure bands are independent and the probability generating function can be written as a product over the exposure bands

$$G(z) = \prod_{m}^{i=1} G_i(z) \tag{3.14}$$

However, by treating each exposure band as a portfolio and using equation(3.9)

$$G_i(z) = \sum_{n=0}^{\infty} p(ndefaults) z^{nvi} = \sum_{n=0}^{\infty} \frac{e^{-\mu} j \mu^n j}{n!} = e^{-\mu j + \mu_j z^{\nu j}} \tag{3.15}$$

Therefore,

$$G(Z) = \prod_{j=1}^{m} e^{-\mu j + \mu_j z^{\nu j}} = e^{-\sum_{j=1}^{m} \mu_j + \sum_{j=1}^{m} \mu_j z^{\nu j}} \tag{3.16}$$

This is the desired formula for the probability generating function for losses from the portfolio as a whole. In the next section, we show how to use the probability generating function to derive the actual distribution of losses under our initial assumptions. First, define a polynomial P(z) as follows:

$$P(z) = \frac{\sum_{j=1}^{m} \mu_j z^{\nu j}}{\mu} = \frac{\sum_{j=1}^{m} \left(\frac{\varepsilon_j}{\nu_j}\right) z^{\nu j}}{\sum_{j=1}^{m} \frac{\varepsilon_j}{\nu_j}} \tag{3.17}$$

The probability generating function in equation (3.17) can now be expressed as;

$$Gz(z) = e^{\mu(P(z)-1)} = F(P(z)) \tag{3.18}$$

This functional form for G(z) expresses mathematically the compounding of two sources of uncertainty arising, respectively, from the Poisson randomness of the incidence of default events and the variability of exposure amounts within the portfolio. Note that G(z) depends only on the data ν and ε. Therefore, to obtain the distribution of losses for a large portfolio of credit risks, all that is needed is knowledge of the different sizes of exposures ν within the portfolio, together with the share ε of expected loss arising from each exposure size. This is typically a very small amount of data, even for a large portfolio.

3.10 Loss Distribution With Fixed Default Rates

In this section, a computationally efficient means of deriving the actual distribution of credit losses is derived from the probability generating function given by equation (3.17). For n, an integer let A_n be the probability of a loss of $n{*}L$, or n units from the portfolio. We wish to compute A_n efficiently. Comparing the definition in equation (3.14) with the Taylor series expansion for G(z), we have

$$p(lossofnL) = \frac{1}{n!}\frac{d^n G(z)}{dz^n} = A_n \tag{3.19}$$

our case G(z) is given in closed form by equation (3.17). Using Leibnitz's formula we have

$$\frac{1}{n!}\frac{d^n G(z)}{dz^n}\Bigg| = \frac{1}{n!}\frac{dz^{n-1}}{dz^{n-1}}\left(G(z).\frac{d}{z}\sum_{m}^{j=1}\mu_j z^{\nu j}\right)\Bigg| \tag{3.20}$$

$$= \frac{1}{n!}\sum_{n-1}^{k=0}\begin{pmatrix} n-1 \\ \\ k \end{pmatrix}\frac{d^{n-k-}}{dz^{n-k-1}}G(z)\frac{d^{k+1}}{dz^{k+1}}\left(\sum_{j=1}^{m}\mu_j z^{\nu j}\right) \tag{3.21}$$

However,

$$\frac{d^{k+1}}{dz^{k+1}}\left(\sum_{j=1}^{m}\mu_j z^{\nu j}\right) = \left\{\begin{matrix} \mu_j(k+1)! \\ \\ 0 \end{matrix}\right\} \tag{3.22}$$

if k=$\nu_j - 1$ for some j, otherwise And by definition

$$\frac{d^{n-k-1}}{dz^{n-k-1}}G(z) = (n-k-1)!A_{n-k-1} \tag{3.23}$$

Therefore

$$A_n = \sum\frac{1}{n!}\begin{pmatrix} n-1 \\ \\ k \end{pmatrix}(k+1)!(n-k-1)!\mu_j A_{n-k-1} = \sum_{j:\nu_j\leq n}\frac{\mu_j\nu_j}{n}A_{n-\nu_j} \tag{3.24}$$

Using the relation $\varepsilon\nu_j x\mu_j$ the following recurrence relationship is obtained

$$A_n = \sum_{j:\nu_j\leq n}\frac{\varepsilon_j}{n}A_{n-\nu_j} This recurrence relationship allows very quick computation of the distributio \tag{3.25}$$

$$A_0 = G(0) = F(P(0)) = e^{-\mu} = e^{-\sum_{j=1}^{m}\frac{\varepsilon_j}{\nu_j}} \tag{3.26}$$

Again, it is worthwhile to note that the calculation depends only on knowledge of ε and ν. In practice, these represent a very small amount of data even for a large portfolio consisting of many exposures.

CHAPTER 4

Data Analysis and Results

4.1 Introduction

This chapter discusses how data collected was used for the intended analysis based on the CreditRisk$^+$ framework considered in chapter three. A sample of 1,000 obligors was obtained from the portfolio for the analysis.

4.2 Descriptive Analysis

The study used secondary data obtained from the Banking Supervision Department (BSD) of the Bank of Ghana. These covered the period between 2009 and 2014. The credit portfolio of all the commercial banks operating in Ghana were used for the study. A sample of 1,000 obligors was obtained by random sampling using the statistical software, XLSTAT. The sample contained obligors from different sectors of the economy as well as different types of credit facilities.

4.3 Sector Analysis

Each obligor in the sample was assigned one of six sectors. It can be assumed that each obligor is subject to one systematic factor. Table 4.1 shows how the sampled obligors are distributed among the sectors. 35% of the obligors were within the consumer category. 267 obligors representing 26.7% of the obligors were in the construction sector. The Telecommunication sector had the lowest number of obligors with 22 obligors representing 2.2% of the sample.

Table 4.1: Descriptive Statistics-Obligors

CATEGORY	FREQUENCY	PERCENTAGE
Construction	267.00	26.7
Consumer	355.00	35.50
Education	60.00	6.00
Retail Trade	232.00	23.2
Service	64.00	6.40
Telecommunication	22.00	2.20

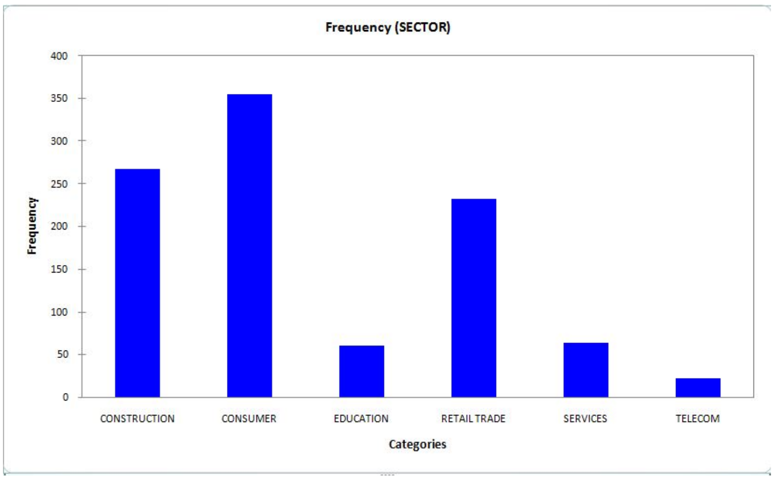

Figure 4.1: Bar-chart of Sectors

4.4 Credit Exposure

A credit exposure is the amount of money given to a borrower by a lender. Credit exposures could be direct or contingent upon some specified events. Table 4.2 below illustrates a descriptive statistics of the credit exposures. The minimum amount was GHS400.00 whiles the maximum amount was GHS20,000,000.00. The wide range of exposures is responsible for the huge variance. This is very typical of a portfolio of credit exposures for obligors with different characteristics and belonging to different sectors of the economy.

Table 4.2: Descriptive statistics - Exposures

STATISTIC	EXPOSURE
No. of observations	1,000.00
Minimum	400.00
Maximum	20,000,000.00
$1^{th}Quatile$	1,000.00
Median	2,000.00
3^{rd} Quartile	9,000.00
Mean	65,766.00
Variance(n-1)	519,708,946,190.19
Standard Deviation(n-1)	720,908.42

4.5 Types of Facilities

Our sample consisted of different types of credit facilities. Figure 4.2 gives a graphical description of how the credit exposures varies with the type of facilities. It was observed that exposures to other banks recorded the highest figure. Table 4.3 describes the distribution of obligors and the type of facilities. Over half (53.6) representing 536 obligors had term loans. Only two obligors representing 0.2% had lease facilities.

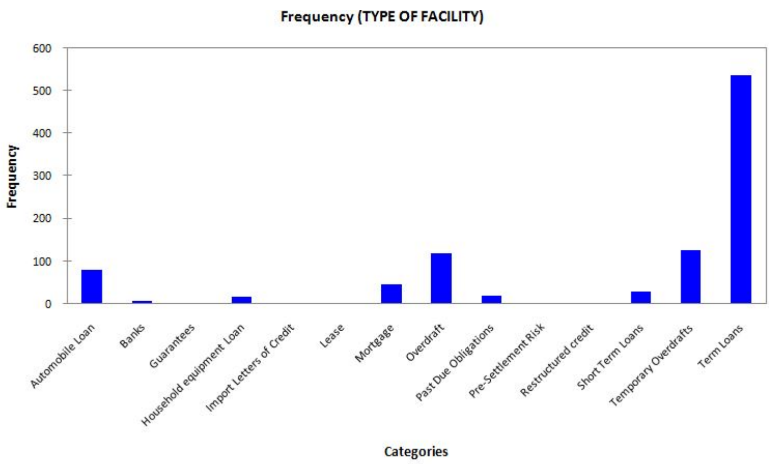

Figure 4.2: Exposure by types of facilities

Table 4.3: Descriptive statistics – Types of facilities

TYPES of FACILITY	FREQUENCE	PERCENTAGE(%)
Automobile Loan	80.00	8.00
Banks	9.00	0.90
Guarantees	3.00	0.30
Household Equipment Loan	17.00	1.70
Import Letter of Credit	4.00	0.40
Lease	2.00	0.20
Mortgage	48.00	4.80
Overdraft	119.00	11.90
Past Due Obligations	21.00	2.10
Pre-Settlement Risk	1.00	0.10
Restructured Credit	1.00	0.10
Short Term Loans	31.00	0.31
Temporary Overdraft	128.00	12.80
Term Loan	536.00	53.60

4.5.1 Definition of Variables

Mortgage: A legal agreement by which a bank, building society,etc lends money at interest in exchange for taking title of the debtors property.

Quarantees: A formal assurance from a lending institution ensuring that a liabilities of a debtor will be met.

Lease: A legal contract or instrument conveying property to another for a period determinable at the will of either the lessor or lessee in considering of rent or either compensation.

Import Letter Of Credit: is an unconditional undertaking, given by a bank (the "Issuing Bank") at the request of their customer (the Applicant or Importer) to pay the Beneficiary (or Supplier) against stipulated documents, provided all the terms and conditions in the Letter of Credit are complied with.

Household Equipment Loan: a facility provided to purchase household equipment such as generating sets, electronics, furniture etc for personal use.

Overdraft: occurs when money is withdrawn from a bank account and the available balance goes below zero. In this situation the account is said to be "overdrawn"

Pre-Settlement Risk: the risk that one party of a contract will fail to meet the terms of the contract and default before the contract's settlement date, prematurely ending the contract.

Restructured Credit:new loan that replaces the outstanding balance on an older loan, and is paid over a longer period, usually with a lower installment amount.

Past Due Obligations: is a loan payment that has not been made as of its due date. A borrower who is past due may be subject to late fees, unless the borrower is still within a grace period.

Automobile Loan: a facility provided to purchase an automobile.

Term Loan:A term loan is a loan from a bank for a specific amount that has a specified repayment schedule and a floating interest rate. Term loans almost always mature between one and 10 years.

4.6 Loss Distribution

Finding the loss distribution was the main goal of the study. The data was analysed using the CreditRisk$^+$ model to generate a loss distribution for the portfolio. Table 4.4 shows some statistical properties of the generated loss distribution. The expected losses in one year for the credit portfolio is GHS4,375.627. Credit Value at Risk at the 99% confidence interval was found to be GHS26,036,909. The corresponding economic capital (This is given by the VaR – Expected loss) is GHS21,661,282. Values at risk at other levels are also shown in the table. The standard deviation of losses was found to be GHS 5,415,395.52

Table 4.4: Loss Distribution

Confidence Level	Credit Loss Amount(VaR)
50	2,158,489
75	5,511,728
95	20,674,794
97.5	22,350,147
99	26,036,909
99.5	27,722,555
99.75	30,956,692
99.9	41,534,685

Figure 4.3 gives a graphical description of the empirical loss distribution. Thus, credit loss is the loss that a business or financial organization records which is caused by customers not paying money they owe. As expected for a loss distribution, it is assumes only non-negative values and is long tailed. The distribution could be used to model both small losses and large losses arising from the credit portfolio. To obtain the PDF for the distribution, the loss data generated by the model was fitted to various theoretical distributions to obtain a distribution that fits best and hence estimate its parameters. Table 4.6 gives a list of the theoretical distributions that were fitted and the p-value obtained for each distribution. The data fitted well to a four parameter Beta distribution

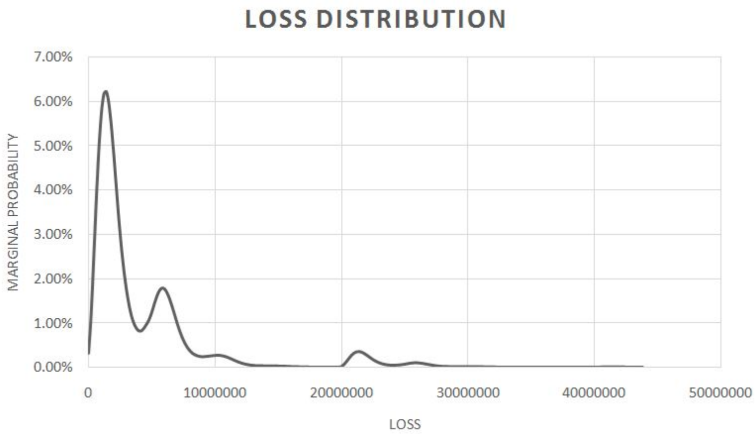

Figure 4.3: Distribution of losses in the portfolio

$$B(x;\alpha,\beta,c,d) = \frac{\sqrt{(\alpha+\beta)}}{\sqrt{(\alpha)}\sqrt{(\beta)}}p\tfrac{(x-c)^{\alpha-1}(d-x)^{\beta-1}}{(b-a)^{\alpha+\beta-1}}, c \leq x \leq d; \alpha, \beta > 0$$

Table 4.5: Maximum likelihood estimate of parameters

PARAMETER	VALUE	STANDARD ERROR
Alpha	1.094	0.097
Beta	1.093	0.097
c	-442,378.990	512377.285
d	44,237,999.990	0.00

Table 4.6: Theoretical distributions fitted using ExcelStats

Distribution	p-value
4 parameter Beta	1
Binomial	
Negative binomial (1)	
Negative binomial (2)	<0.0001
Chi-square	<0.0001
Erlang	<0.0001
Exponential	<0.0001
Fisher-Tippett (1)	<0.0001
Fisher-Tippett (2)	<0.0001
Gamma (1)	<0.0001
Gamma (2)	0.022
GEV	0.288
Gumbel	<0.0001
Log-normal	<0.0001
Logistic	0.386
Normal	0.413
Normal (Standard)	<0.0001
Poisson	
Student	<0.0001
Weibull (1)	<0.0001
Weibull (2)	0.045
Weibull (3)	0.156

CHAPTER 5

Conclusion and Recommendations

5.1 Introduction

This paper has applied an actuarial approach to credit risk modeling using the CreditRisk$^+$ model. An empirical loss distribution was generated using the data obtained from the bank. The generated distribution was compared to known theoretical distributions to determine the distribution that best fits and hence estimate its parameters.

5.2 Conclusion

The result of this study has provided a portfolio based approach to credit risk management. The portfolio consists of several individual loans. Losses could occur on the individual loans which would affect the portfolio quality. This study used techniques in estimating claim severity in the insurance industry to estimate the expected loss on a credit portfolio. The unexpected loss at 99% confidence interval was also estimated. It was found that the distribution generated is a four parameter Beta distribution. This type of model is more practically applied to developing economies as significantly less parameters need to be calibrated. The expected value of the portfolio loss is an important index for management's reserve policy: the higher the expected loss, the higher the reserves to be set aside. More over the expected value of the loss is a basic element to detect if each credit position is conveniently priced or not.

The Economic capital (also known as the credit risk capital) defined as the additional maximum loss, above the expected loss of the portfolio determined by a specified probability level (95%) in this study over a time horizon (1 year) is also an index to measure the losses suffered in an unlikely event. Also, given the distribution of losses, stress testing the portfolio becomes relatively easier.

5.3 Recommendations

Banks and other lenders often have a credit portfolio management team devoted to looking at the big picture containing all of the loans issued by such an institution. It is recommended that these managers assign different risk levels to each of the loans and reach a final assessment about whether or not the lender is too exposed to damage done by potential defaults. It is also recommended that financial institutions in the business of lending should study the model in addition to models they already use. Although credit worthiness is determined on an individual basis, a portfolio based approach would also be instrumental in estimating the expected and unexpected (worst case scenario) losses arising from portfolio of credit exposures. CreditRisk+ model has become the most popular due its tractability. Banks and other lenders can therefore, use this module as the portfolio loss distribution can be calculated analytically such that Monte Carlo simulation can be avoided. Further research is recommended on how model and price an insurance policy or a credit default swap (CDS) for a portfolio of bank loans using the distribution of unexpected losses in the portfolio. A claim event would be said to have occurred when the losses incurred rises beyond an agreed threshold.

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