KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY, KUMASI

INSTITUTE OF DISTANCE LEARNING


A MATHEMATICAL MODEL OF PROJECT SELECTION UNDER MULTI-PERIOD

CAPITAL RATIONING: CASE STUDY OF OFFINSO MUNICIPAL ASSEMBLY

PROJECTS IN GHANA

KNUST

BY

ASIEDU STEPHEN KWAKYE BEDIAKO (B. ED MATHEMATICS)


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# DECLARATION

I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

ASIEDU STEPHEN K. BEDIAKO
    (PG6318211)

----------------------------------------      -------------------------      --------------------
Student Name & ID            Signature            Date

Certify by:

PROF. S. K. AMPONSAH

----------------------------------------      -------------------------      --------------------
Supervisor(s) Name            Signature            Date

Certified by:
PROF. S. K. AMPONSAH

----------------------------------------      -------------------------      --------------------
Head of Dept. Name            Signature            Date

Certified by:
PROF. I. K. DONTWI

----------------------------------------      -------------------------      --------------------
Dean, IDL            Signature            Date

# ABSTRACT

Project selection is the process of choosing a project or set of projects to be implemented by an organization. Since projects in general require a substantial investment in terms of money and resources, both of which are limited, it is of vital importance that the projects that an organization selects provide good returns on the resources and capital invested. This requirement must be balanced with the need for an organization to move forward and develop. This research work is concerned with the use of Mathematical models, specifically, Integer Programming (IP) and Linear Programming (LP) formulations, for the purpose of selecting feasible projects and maximizing the returns from the batch of projects of some District Assemblies (DAs) and Municipal Assemblies (MAs) in Ghana. The research work was a case study of Offinso Municipal Assembly's projects in Ghana. In these models, we maximize the Net Present Value (NPV) and maintain the net discount cash inflow for each project in each period. Sensitivity analysis on the project parameters were also carried out to test the extent to which project selection is sensitive to the changes in the parameters of the models. These facts are supposed to give the Offinso Municipal Assembly (OMA) management some guidance in their considerations of various alternatives with regards to the limited investment capital.

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Finally, l wish to express special thanks to my family, Nana Osei Bosompem III Adako-Jachie hene who has been the backbone and inspiration to my tertiary education and a friend in need Mr. Stephen Opoku and all who in diverse ways saw this work through.

I am hopeful that this study would be beneficial to anybody who reads it and have interest in project selection under Multi-Period Capital Rationing.

## DEDICATION

This research work is dedicated to the almighty God for his abundant grace, guidance and protection that has brought me this far.

I would like to dedicate this research work to my parents, Mr. Yaw Asiedu and Madam Akosua Achiaa of blessed memories, my siblings whose spiritual and moral support have sustained me through.

Finally, to my beloved wife, Mrs. Beverly Serwaa Bediako, my lovely children Christiana Osei Bediako, Josephine Gyankama Bediako, Nana Achiaa Bediako and Akosua Ampomah Bediako without whose moral and spiritual support in addition to their encouragement l could not have come this far.
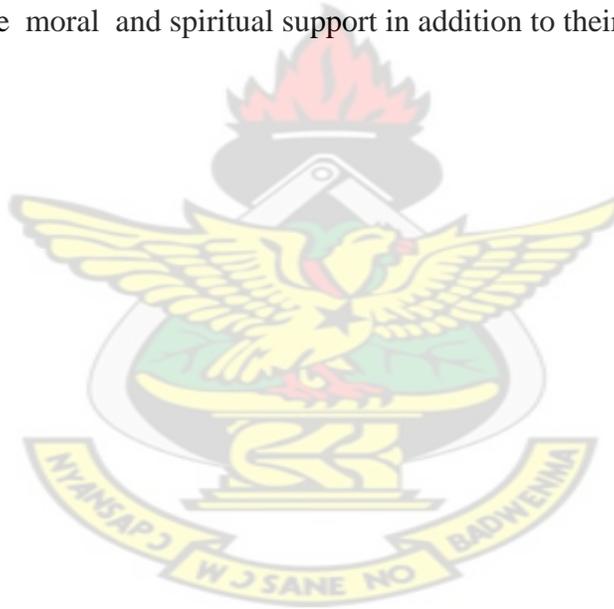
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# CHAPTER ONE

## 1.0 INTRODUCTION

A recent survey of the capital budgeting practices of large companies showed that over seventy-five percent (75%) of companies used payback as an appraisal method, often in conjunction with other techniques. The same survey showed that only seventeen percent (17%) of the companies used Net Present Value (NPV) as their primary evaluation technique in spite of generally acknowledged Technical Superiority of NPV over payback (Pike, 1995). This would seem to suggest that much of the academic preoccupation with refining measurement techniques may be distorted. Nevertheless, investment opportunities are of far greater importance to the companies than the particular appraisal method used, since successful investment appraisal is entirely dependent on the accuracy of the cost and revenue estimates.

The investment appraisal employed in this study is project selection under multi-period capital rationing. Project selection is the process of choosing a project or set of projects to be implemented by an organization. Since projects in general require a substantial investment in terms of money and resources, both of which are limited, it is of vital importance that the projects that an organization selects provide good returns on the resources and capital invested. This requirement must be balanced with the need for an organization to move forward and develop.

The high level of uncertainty in the modern business environment has made this area of project management crucial to the continued success of an organization as exceptional. Single period

capital rationing with divisible projects is between choosing good projects and poor projects literally representing the difference between operational life and death.

In a perfect world, every organization would find time to approach project selection in an objective, methodical way, using the right criteria and tools. In the real world, project selection is often carried out in a more-or-less intuitive way even though the benefits of systematized, criteria based project selection can make the difference between organization's success or failure. A strategic approach to project selection yields better results for organizations by minimizing risk and maximizing the potential upside. With finite resources and infinite project possibilities, project selection could be the most important step in the project life-cycle. A new project is a response to threats or opportunities, and choosing the best possible response, from a complex web of possibilities, is often beyond the capacity of the human brain. (Seeber, 2011).

Because a successful model must capture every critical aspect of the decision, more complex decisions typically require more sophisticated models. "There is a simple solution to every designer". Project decisions are often high-stakes, dynamic decisions with complex technical issues, precisely the kinds of decisions that are most difficult to model.

The Projects Municipal Assemblies (MAs) and District Assemblies (DAs) choose can define what they want, and the benefits the Assemblies will derive. Thus, project decisions can impact every stakeholder, including, employees, partners, regulators, and shareholders. A sophisticated model may be needed to capture strategic implications.

Project decisions are dynamic because a project may be conducted over several budgeting cycles, with repeated opportunities to slow, accelerate, re-scale, or terminate the project. Also, a

successful project may produce new assets or products that create time-varying financial returns and other impacts over many years. A more sophisticated model is needed to address dynamic impacts.

Project decisions typically produce many different types of impacts on the organization. For example, a project might increase revenue or reduce future costs. It might impact how customers or investors perceive the organization. It might provide new capability or learning, important to future success. Making good choices requires not just estimating the financial returns on investment; it requires understanding of all the ways that projects add value.

Project decisions often entail risk and uncertainty. The significance of a project risk depends on the nature of that risk and on the other risks that the organization is taking. In all these a more sophisticated model is needed to correctly deal with risk and uncertainty.

Project selection is the process of evaluating individual projects or groups of projects, and then choosing to implement some set of them so that the objectives of the parent organization will be achieved. This same systematic process can be applied to any area of the organization's activities in which choices must be made between competing alternatives. For example:

- A manufacturing firm can use evaluation/selection techniques to choose which machine to adopt in a part-fabrication process.

- A television station can select which of several syndicated comedy shows to rerun in its 7:30 p.m. weekday time-slot.

- A construction firm can select the best subset of a large group of potential projects on which to bid.

- A hospital can find the best mix of psychiatric, orthopedic, obstetric, and other beds for a new wing.

Each project will have different costs, benefits, and risks. Rarely are these known with certainty.

In the face of such differences, the selection of a multi-period capital rationing project out of a set is a difficult task. Choosing a number of different projects, a portfolio, is even more complex. Project selection is only one of many decisions associated with project management.

To deal with all of these problems, we use decision aiding models. We need such models because they abstract the relevant issues about a problem from the plethora of detail in which the problem is embedded. Reality is far too complex to deal with in its entirety. An "idealist" is needed to strip away almost all the reality from a problem, leaving only the aspects of the "real" situation with which he or she wishes to deal. This process of carving away the unwanted reality from the bones of a problem is called modeling the problem. The idealized version of the problem that results is called a model.

The portfolio management literature mainstream, the Project Selection Paradigm, regards projects as closed packages ready for choice. However, to generate a portfolio, such packages must be opened to reveal the inside sources of interdependencies among them. Then, the project elements so found may be recombined into new alternatives that better capture the synergies among projects and avoid negative interactions. Thus project selection can be superseded by a Project Portfolio Generation based on projects' and portfolios' reformulation process. (Nascimento, 2013).

Capital rationing is manifested in the situation where the firm or company is unable to initiate all projects, which are apparently profitable because sufficient funds are not available. At a point where investment funds are expected to be limited over several periods of time is called multi-period capital rationing. The effects of capital rationing may develop for internal purposes. For example, it may be decided that investment should be limited to the amount that can be financed solely from retained earnings or kept within a given capital budget. The external and internal factors, which impose quantitative limits, lead to two opposing view-points developing known as the "hard" and "soft" view of capital rationing. The "hard" view is that there is an absolute on the amount of money a company may borrow or raise externally. The "soft" view on the other hand is that rationing by a quantitative limit such as an arbitrary capital expenditure budget should only be seen as a temporary administrative expedient because such a limit is not determined by the market and such a limit would not be imposed by a profit maximizing establishment. Whatever the causes of the limited capital supply available for investment purposes means that, not only must each project cover the cost of the capital but that the project or batch of projects selected must maximize returns from the limited funds available that is, some form of ranking becomes necessary. Ways of achieving this objective include the following rationing possibilities:

(i) Single period capital rationing with indivisible projects.

(ii) Multi-period capital rationing with divisible projects.

(iii) Multi-period capital rationing with indivisible projects.

These entire investment appraisals would be explained in the subsequent chapters.

## 1.1.0  BACKGROUND OF THE STUDY

### 1.1.1 Missions and Objectives of District and Municipal Assemblies

Within the period of colonial rule and post independent era of Ghana, local government concepts have undergone some scientific reorientations to improve its performance. The current structure of Municipal Assemblies (MA) and District Assemblies (DA), for instance, was introduced during the era of the Provisional National Defense Council (PNDC) to make it more responsive and relevant to the modern day demands of democratization, industrialization and development. Among the main objectives and aspirations of the MA's and DA's are as follows: (i) to promote good governance by way of enhancing grass root participation of democracy. (ii) to allow the local people a certain degree of local autonomy to identify and solve local problems better. This is because the local people know what they want and the problems facing them.
(iii) to undertake appropriate activities and harnessing resources that would promote the well-being of the people by sponsoring economic and developmental projects.

In pursuance of the above stated missions, the MA's perform several functions such as;

(i)     Serving as tools to bring government to the people without making the government too distant and alien.

(ii)    Making mobilization of human and material resources for development much easier. This is because the policies and project works at the local level are seen by the people as their own creation and hence they become very proud to nurture them to fruition.

(iii)   Offering the people who take part in their activities greater opportunities for the acquisition of training, skills, experience and exposure, which prepares them for trade

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and leadership positions at the higher level of governance. Thus the MA's more or less serves as training and business organizations for the local people.

(iv)    Providing social and economic infrastructural facilities to complement the efforts of the central government.

## 1.1.2  MA Projects and Sources of Revenue

The Offinso Municipal Assembly (OMA) undertakes variety of economic and developmental projects. Among these projects are:

Construction of markets square, lorry parks, school buildings, water boreholes, clinics, recreational centres, community libraries, latrines (KVIP), roads and putting up of housing facilities, toll booths, sanitation facilities, agricultural projects and electrification. These projects are the main sources of income for the OMA. Apart from these projects, the OMA like any other MA, sources of income are derived from the fourth, fifth and sixth schedules of the Act 462 of 1993 constitution.

Under schedule four, the MA's or DA's has a right to issue vehicle licenses to all vehicles within the MA or DA.

Under schedule five, they derive some of their sources of income from entertainment. Examples of these are concert parties, musical or theatrical performances, video shows, cinemas, dancing discotheques and others to which admission is to be obtained on the payment of money or reward except where the proceeds are being devoted to charity.

The 6$^{th}$ schedule provides the MA the avenue for taxes base on gambling and betting (Act 268, 1965). Gambling (Act NRC Decree 174, 1973), as well as Casino Revenue under the Casino tax decree of 1973 (NRCD, 200).

## 1.2 STATEMENT OF THE PROBLEM

It is evident from the above that the cash inflow for projects and running of the functions of the OMA comes largely from monies accrued from projects and common fund. Usually, the limitation of these funds extends over a long period of time. The OMA operates under multi-period capital rationing, that is, it is unable to initiate all its projects, which are apparently profitable because funds available are insufficient.

The vast increasing number of the youth unemployment problem has hit the Offnso municipality for some time now. This has compelled the OMA to undertake projects creating employment opportunities for the youth. Simultaneously, it wishes to invest in the projects that will maximize returns from the batch of projects it would select with regard to capital limitation. The dilemma of the OMA is that, which of the numerous projects they should invest under the limited funds in order to maximize its profits and also solve the youth unemployment problem.

This study intends to formulate mathematical programming models for solving the OMA project selection under the multi-period capital limitation problem.

## 1.3 OBJECTIVES OF STUDY

The main objectives of this study are:

(i) to formulate Linear Programming (LP) and an Integer Programming (IP) models for solving OMA project selection under multi-period capital rationing problems.

(ii) to maximize the returns from the batch of the projects selected with regards to the capital limitation.

(iii) to carry out sensitivity analysis on the parameters of the models in order to assist the project management to decide on which project(s) worth undertaking.

## 1.4 SIGNIFICANCE OF THE STUDY

The study is to throw more light on the inherent difficulties and problems facing firms, MAs and companies in selection of projects with regards to the capital limitations. The models will serve as a tool for solving various problems of multi-period capital rationing in firms and other establishments.

The study can also be a guide for policy and decision makers in the various firms and organizations to maximize profit from the batch of projects they would undertake.

## 1.5 SCOPE OF THE STUDY

As indicated in the background, the OMA was introduced during the PNDC era that is in 1979. However, the study will cover the project works over the period 2002-2007. It is hoped that the models will be of interest to management and planners of the various MAs and firms in Ghana.

## 1.6  METHODOLOGY

Data for the study is purely a secondary data which shall be obtained from sources such as financial statements, annual reports, monthly reports, daily newspapers and other relevant documents of the OMA.

The data shall critically be scrutinized and classify into tables. Thus projects with different lives will be compared, and financial ratios and Continuous Probabilistic Analysis (CPA) will be used to do the analysis of the data. The LP and IP models for solving project selection problems from optimization theory perspective will be given.

## 1.7  LIMITATIONS

All things being equal, there are no doubts that, the LP and the IP cannot be a useful method of dealing with multi-period capital rationing problems. However, there are few assumptions and limitations, which are worth mentioning. These include the assumption that;

(i)  all functions are linear, may not be realistic.

(ii) the projects and constraints are all being independent of one another.

(iii) the cash flows, resources and constraints are all known with certainty may also not be realistic.

The researchers are also aware that there are other techniques like Pay Back Period (PBP) and Internal Rate of Return (IRR) for selecting optimal projects. However, due to the number of serious limitations they present, only NPV will be used in the formulation of the objective function of the models.

## 1.8 ORGANIZATION OF THE STUDY

In this chapter we considered the background, statement of the problem and objective of the study. The justification, scope, methodology and limitations of the study were also put forward.

Chapter two provides the literature review for the project selection under multi-period capital limitation problem addressed in this thesis. Chapter three examines the procedure and formulation of linear programming and integer programming models for solving programming problems from optimization theory perspective. In this chapter, the sources, the method of collection of data and comparison of data using financial ratios and Continuous Probabilistic Analysis (CPA) will be given. The results from chapter three will be used in chapter four to formulate a suitable optimization problem to compute the objective function or decision rule of the programming problems. MATLAB Software for solving the LP and IP problems formulated in chapter three will also be given. This will be followed by sensitivity analysis of the results. Chapter five which is the final of the study presents the conclusions and recommendations of the study.

## 1.9 SUMMARY

In this chapter, we presented the background, statement of the problem and objectives of the study. The justification, methodology, scope and limitations of the problem were also put forward. In the next chapter, we shall put forward pertinent literature on Project Selection Under Multi-Period Capital Rationing

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# CHAPTER TWO

# LITERATURE REVIEW

## 2.0  INTRODUCTION

In this chapter, the necessary literature on project selection under multi-period capital rationing problems is addressed. It is upon this background information that the programming models will be formulated.

## 2.1  MULTI-PERIOD CAPITAL RATIONING

The multi-period capital rationing has been defined previously to be where investment funds are expected to be limited over several periods. In such circumstances, it becomes difficult to choose the batch of projects, that is, some starting immediately, first or second or others couple of periods hence etc., which yields the maximum returns and yet which remain within the capital limits.

Furthermore, according to InvestorWords.com, capital rationing means limiting a company's new investments, either by setting a cap on parts of the capital budget or by using a higher cost of capital when weighing  the merits of potential investments. This might happen when a company has not enjoyed good returns from investments in the recent past. Capital rationing could take place if a company has excess production capacity on hand.

The problem becomes one of optimizing a factor, (e.g. NPV) where resources are limited, that is, the funds available over the periods being considered. This is recognized as a situation where linear programming and integer programming could be used. Both IP and LP have been used successfully in solving multi-period capital rationing problems. Specifically, the LP method is usually used to solve divisible projects, that is, where a fractional part of a project can be undertaken. Where the projects are not divisible, the only feasible solution tool is IP model. (Beasely, 2003, Powell, 1998).

Gregory et al, (1984) conducted a study on the effects of several production/management, price and risk factors upon channel catfish profitability and analyzed with a multi-period mixed-integer linear programming model. Their analysis included pond size and optimal stocking rates, alternate levels and trends in catfish prices, pond production losses and level of family consumption withdrawals. Their model results indicate that channel catfish offer the potential to significantly increase farm rates of return while providing an avenue of intensive farm growth, without expanding the land base of the farm. The authors however, concluded that the long range financial success of the firm was very sensitive to several of the management and risk factors they examined.

## 2.2  PROJECT SELECTION

Fox and Baker (1985) conducted a research to find out whether different markets conditions affect project selection decisions. The authors developed a simulation model that attempted to include simplified market and production characteristics. They tried to capture the essence of Abernatty and Utterback's model of innovation process so that it may be applied to more specific

setting. Their observed project selection patterns over time indicated an apparent relationship between market time increasing and production cost decreasing projects. The appropriate choice of investment projects depends primarily on the nature of cash flows generated by the projects, the risk level associated with the cash flows and the budgetary, limitations of the corporation over time. For the past four decades, researchers have attempted to present a working investment choice model that considers the various aspects of the budgetary process.

In 1963, the first mathematical programming formulation of the multi-period capital rationing problem was provided (Weingartner, 1963). In his formulation, the net discount cash inflows for projects are maximized while cash outflows and availability of resources are maintained in each period. This formulation has withstood many criticisms over the past three decades. The majority of these criticisms are based on three main features of the models. These being the appropriate selection of an objective function, the determination of a suitable discount rate to account for project returns and the inability of the model to deal with uncertain budgetary constraints.

 In the seminar work by Weingartner, the author provided a framework using a deterministic linear programming approach. His model uses Net Present Value (NPV) as its objective function. The value associated with the timing of a particular cash flow is adjusted by an appropriate discount rate (Kira, 2000).

It is evident from the above surveys that the LP model for project evaluation under capital rationing made use of project cash flows, NPV, IRR and other investment evaluation techniques.

It is imperative that these methods are examined to see how they could be incorporated into the formulation of LP and IP in the proposed study.

According to Omer (2002), project selection is a major problem in managerial decision making. In his study, a deterministic model that schedules project starts was formulated as a binary integer program. In his view, the model is applicable in various settings such as selection of engineering projects in corporate planning or in other planning environments in which the candidate projects are interdependent. His results show that the binary integer model can provide the required solutions in a very reasonable amount of time.

Matheson et al, (1997) developed general fairness measures that may be used as criteria for sustainable project selection. They discussed sustainable development, fair allocation objectives and empirical distance-based measures of fairness, and their evaluation. They developed and extended generalized fairness measures for both intratemporal and intertemporal fairness comparisons. A preliminary application of the extended distance based fairness measures is then performed for a case study of the selection of an electricity supply project. The applied fairness measures indicate that intratemporal fairness, in terms of the distribution of user unit costs, may be increased by choosing the land line alternative and that there is no significant difference among alternatives with respect to intertemporal fairness. According to them, their results provide limited insight into the energy supply problem, however, and it is suggested that further analyses should be conducted when information regarding the environmental impacts and reliability of power supply for each of the alternatives becomes available.

Amponsah et al, (2011) were concerned with the use of mathematical models, specifically linear programming (LP) and integer programming (IP) formulations, for the purpose of selecting potential projects and maximizing the returns from the batch of projects of some Municipal Assemblies (MAs) in Ghana. In these models, they maximize the Net Present Value (NPV) and maintain the net discount cash inflow for each project in each period. Sensitivity analysis on the project parameters was also carried out to test the extent to which project selection is sensitive to the changes in the parameters of the models. According to them, these facts are expected to give the MA management some guidance in their considerations of the various alternatives with regard to the limited investment capital. The authors concluded that the IP model solved a Municipal Assembly's large scale project selection problem and this produced the optimal solution quantities. They added that selection of the small scale project problem is solved efficiently using an IP model.

Yong-tao et al (2010) introduced a quantitative method for assisting contractors to select appropriate projects for bidding by considering multiple attributes and integrating decision group member opinions. The fuzzy technique for order preference by similarity to ideal solution (TOPSIS) method was used to help contractors make decision on project selection. The numerical example demonstrates that the fuzzy TOPSIS approach can be used to simulate the decision process in project selection, and the results provide contractors with valuable insight into the project selection problem.

Cheng (2007) examined the funding mechanism problem underlying the rail transit project selection bias and hypothesizes this problem to be a principal–agent problem. They introduced incentive theory as an analytical tool to model the problem. By applying the incentive theory to analyze the project development process of the US New Starts program, they suggested that the

funding mechanism problem associated with the bias toward capital-intensive rail investments can be viewed as a principal–agent problem between the Federal Transit Administration  (the principal) and local project sponsors (the agent).

Padberg and Wilczak (1999) employed Mathematical programming to obtain an optimal decision rule for project approval in capital budgeting in a non-perfect capital market. They used the framework due to Martin Weingartner to formulate the decision problem in a deterministic setting and derive the optimal rules for the acceptance/rejection of a single project explicitly in the two cases where the borrowing/lending rates for capital are constant and time-dependent, respectively.

Matson (1999) undertook a study which dealt with the current state of the theory and methods of capital budgeting under conditions of capital rationing. He focused on the need for adequate decision support for management and planners dealing with capital budgeting problems. He used a general optimization model in presenting and discussing some major theoretical contributions and proposed methods of calculation in this field.

Beraldi et al (2012) proposed a stochastic version of the capital rationing problem which explicitly accounts for uncertainty. They particularly provided a mathematical formulation framework of stochastic programming with joint probabilistic constraints and a novel solution approach. They concluded that their model could be extended to include specific risk measures.

Capital rationing is a real decision problem in government, yet it has never been seriously addressed in the literature on public budgeting. Conventional methods such as NPV or IRR that frequently appear in the discussion on the subject are limited in their scope. Alternative methods such as mathematical programming, which can substantially overcome some of the limitations of

the conventional methods, have been extensively used in the private sector, but their applications have been few and far between in government. (Khan, 2008).

In their study, Bas and Kahraman (2009) used fuzzification of Weingartner's pure capital rationing model and its analysis to develop a primal-dual pair based on t-norm/t-conorm relation for the constraints and objective function for a fully fuzzified pure capital rationing problem except project selection variables. They performed sensitivity analysis for a change in a budget level or in a cash flow level of a non-basic as well as a basic variable. They analyzed the problem based on duality and complementary slackness results. Their computational analysis and interpretation showed positive results.

Milton and Artur (1996) wanted to reconcile investment theory with actual practice. In their view, the essential features of the environment that give rise to a type of capital budgeting processes observed in practice are managerial incentive problem and asymmetric information. They constructed a simple model that incorporates these two features. They concluded that this procedure obviously deviates from the NPV rule and can result in underinvestment for high productivity projects and overinvestment for low productivity projects.

Andrew and Martin (2012) analyzed the optimal capital budgeting mechanism when divisional managers are privately informed about the arrival of future investment projects. They modeled a firm with a headquarters and one division operating over two periods. The model derived a number of empirical predictions regarding capital budgeting and the investment of financially constrained firms. Their findings showed that consistent with field study evidence, an optimal allocation mechanism can include a stipulation that a capital request for discretionary investment

will be declined with positive probability in the period after a significant investment was made even though this is ex-post suboptimal.

Sunil and Qintao (2009) examined the optimal choice of hurdle rates in an agency-theoretic capital budgeting setting. They modeled a one-period principal-agent relationship between a firm's risk- neutral owner (principal) and its risk-neutral manager (agent). They concluded that the optimal hurdle rate is always lower than what it would be if the manager's project development effort were not subject to moral hazard.

## 2.3.0  PROJECTS AND CASH FLOWS

Every decision the company makes is a capital budgeting decision whenever it changes the company's cash flows. Consider launching a new project involves a phase where the new product is advertised and distributed. Hence the company will have cash outflows for paying advertising agencies, distributors, transportation services etc. Then, for the period of time the company has cash inflows from the sale of the products in the future. Thus, two types of cash flows are identified; cash inflow and cash outflow from the project. Cash flow items include:

**(a) Cash inflows**

The project revenues

Government grants

Resale or scrap value of assets

Tax receipts

Any other cash inflows caused by accepting the project.

**(b) Cash outflows**

Initial investment in acquiring the assets

Project costs, labour, materials etc.

Tax payments

Any other cash outflow caused by accepting a project.

The difficulty with making these decisions are that typically many cash flows are affected and they usually extend over a long period of time. Investment appraisal criteria could be employed in analyzing capital budgeting decisions by aggregating the multiple of the cash flows into one number. Thus, all cash flows have to be included in the analysis whenever they are affected by the decision. (Smith and Whaley, 2004)

**2.3.1  DETERMINATION OF CASH FLOWS IN THE NPV ANAYLSIS**

The incremental net cash flow of an investment proposal is defined as the difference between the company's cash flows if the investment project is undertaken and the company's cash flows if the investment project is not undertaken.

We now show how cash flows are related to accounting numbers and taxes. Define the net cash flow generated by certain assets as:

Net Cash Flow = Cash Inflow – Cash Outflow

$$= \text{Revenue} - \text{Expenses} - \text{Capital Expenditure} - \text{Taxes} \qquad (2.1)$$

The income tax paid is determined by:

$$\text{Taxes} = t \,(\text{Revenue} - \text{Expenses} - \text{Depreciation}) \qquad (2.2)$$

Where t is the corporate tax rate. We note that depreciation is not a cash expenses and only affects cash flows through its effect on taxes. Substituting equation (2.1) into equation (2.2) yields an expression for the company's cash flow:

$$\text{Cash Flow} = (1\text{-} t)\,(\text{Revenue} - \text{Expenses}) + t\,(\text{Depreciation}) - \text{Capital expenditure} \qquad (2.3)$$

The term t (Depreciation) is sometimes known as the depreciation tax shield.

## 2.3.2 Investment Appraisal Techniques

The investment evaluation techniques under consideration in this study include:

Net Present Value (NPV)

Discount Rate (DR)

Payback Period (PBP)

Internal Rate of Return (IRR)

Profitability Index (PI)

## 2.3.3 NET PRESENT VALUE (NPV)

The investment appraisal measure, the researcher wishes to propose here is the Net Present Value (NPV). The NPV of a project is defined as the present value of all future cash flows by an investment, less the cost of the initial cost of investment.

 Let each cash inflow/outflow be discounted back to its PV. Then they are summed. Therefore

$$NPV = \sum_{t=1}^{n} \frac{C_t}{(1+r)^t} - C_0$$ KNUST

(2.4)

Where,

$t$ - the time of the cash flow

$n$ - the total time of the project

$r$ - the discount rate

$C_t$ - the net cash flow (the amount of cash) at time t.

$C_0$ - the capital outlay at the beginning of the investment time (t = 0)

NPV is an indicator of how much value an investment or project adds to the value of the firm. With a particular project, if $C_t$ is a positive value, the project is in the status of discounted cash inflow in the time of $t$. If $C_t$ is a negative value, the project is in the status of discounted cash outflow in the time of $t$. appropriately risked projects with a positive NPV may be accepted. This does not necessarily mean that they should be undertaken since NPV at the cost of capital may

not account for opportunity cost, i.e. comparison with other available investments. In financial theory, if there is a choice between two mutually exclusive alternatives, the one yielding the higher NPV should be selected. The following sums up the NPVs in various situations. Table 1.

| If | It means | Then |
|---|---|---|
| NPV > 0 | the investment would add value to the firm | The project may be accepted |
| NPV < 0 | the investment would subtract value from the firm | The project should be rejected |
| NPV = 0 | the investment would neither gain nor lose value for the firm | We should be indifferent in the decision whether to accept or reject the project. This project adds no monetary value. Decision should be based on other criteria, e.g. strategic positioning or other factors not explicitly included in the calculation. |

However, NPV = 0 does not mean that a project is only expected to break even, in the sense of undiscounted profit or loss (earnings). It will show net total positive cash flow and earnings over its life.

In sum, it is optimal to make a decision that generates positive NPV of their incremental cash values. If there are more than two alternatives, it is optimal to choose the alternative that generates the highest NPV. (Baker, 2007).

### 2.3.4 DISCOUNT RATE

The rate used to discount future cash flows to their present values is a key input of this process. Most firms have a well defined policy regarding their capital structure, so the weighted average cost of capital (after tax) is used with all projects. Some people believe that it is appropriate to use higher discount rates to adjust for risk for riskier projects. Another method is to use a variable discount rate with higher rates applied to cash flows occurring further along the time span, (reflecting the yield curve premium for long-term debt).

Another approach to choosing the discount rate is to decide the rate which the capital needed for the project could return if invested in an alternative venture. If, for example, the capital required for Project A can earn five percent elsewhere, use this discount rate in the NPV calculation to allow a direct comparison to be made between Project A and the alternative. Obviously, NPV value obtained using variable discount rates with the years of the investment duration better reflects the real situation than that calculated from a constant discount rate for the entire investment duration. (Baker, 2007).

For some professional investors, their investment funds are committed to target a specified rate of return. In such cases, that rate of return should be selected as the discount rate for the NPV calculation. In this way, a direct comparison can be made between the profitability of the project and the desired rate of return.

To some extent, the selection of the discount rate is dependent on the use to which it will be put. If the intent is simply to determine whether a project will add value to the company, using the firm's weighted average cost of capital may be appropriate. If trying to decide between alternative investments in order to maximize the value of the firm, the corporate investment rate would probably be a better choice. Using variable rates over time or discounting "guaranteed" cash flows different from "at risk" cash flows may be a superior methodology, but is seldom used in practice. Using the discount rate to adjust for risk is often difficult to do in practice (especially internationally), and is really difficult to do well. An alternative to using discount factor to adjust for risk is to explicitly correct the cash flows for the risk elements, then discount at the firm's rate (Philip, 1997)

## 2.3.5 PAYBACK PERIOD

Numerous surveys have shown that payback is a popular technique for appraising projects either on it own or in conjunction with other methods. Payback can be defined as the period, usually expressed in years, which it takes for a project's net cash inflows to recoup the original investment. The usual decision rule is to accept the project with the shortest payback period. The payback has several advantages and disadvantages. Among these are:

**Advantages**

(i) Uses project cash flows rather than accounting profits and hence is more objectively based.

(ii) Favors quick return projects which may produce faster growth for company and hence liquidity

**Disadvantages**

(i) Payback does not measure overall project worth because it does not consider cash flows after payback period.

(ii) It provides only a crude measure of timing of project cash flows

(iii) In spite of any theoretical disadvantages payback is undoubtedly the most popular appraisal criterion in practice (Pike, 1995).

### 2.3.6 INTERNAL RATE RETURN (IRR)

The IRR of a project is the rate which equates the NPV of the project's cash flows to zero; or equivalently the rate of return which equates the PV of inflows to the PV of the outflows. Internal rate of return rule

IRR is return that equates initial investment with PV of cash flows

$$O = -C_o + \sum_{t=1}^{T} C_t \left[ \frac{1}{(1 + IRR)^t} \right] \qquad (2.5)$$

The decision rules include:

• Accept projects with IRR>r

• Reject projects with IRR<r

The problems with IRR are:

(i) Ignores Value Creation (Scale).

(ii) Assumes cash flows are reinvested at IRR.

(iii) Can have Multiple IRRs if later cash flows are negative.

## 2.3.7 PROFITABILITY INDEX (PI)

Another investment appraisal technique, the PI, is used when the companies or firms have only a limited supply of capital with which to invest is positive NPV projects. This type of problem is referred to as a capital rationing problem. Given that the objective is to maximize shareholder wealth, the objective in the capital rationing problem is to identify that subset of projects that collectively have the height aggregate NPV. To assist in that evaluation, this method requires that each project's PI is computed using:

Profitability Index (PI) $= \dfrac{NPV}{I}$ where $I$ = Initial investment.

The project's PI is then ranked from highest to lowest and then select from the top of the list until the capital budget is exhausted. The idea behind the PI method is that this will provide the subset of projects that maximizes the aggregate NPV.

In general the PI is of limited usefulness and the use of NPV is considered safer.

## 2.4 PROJECT VALUATION

Many authors and researchers consider the possibility of information upgrading in project valuation, but do not present a specific model delineating how learning can be formally embedded into the multi-period capital rationing problems. This study is a step in bridging this gap.

The Stochastic Capital Rationing (SCR) model developed by Kira and Kusy (1990) does not directly consider the issue of uncertain project cash flows in its analysis. Rather, they developed a procedure for the capital budgeting problem wherein both uncertainty in budgetary constraints and returns can be addressed simultaneously. This is realized by utilizing the SCR model and by considering varying standard deviations of project returns in generating the optimal composition of projects.

The purpose of this study is to develop or formulate LP and IP models for solving multi-period capital rationing problems. Specifically, the LP model will be designated to solve multi-period capital rationing (MCR) with divisible project problems while IP will be used to solve MCR with indivisible project problems. The models seek to produce optimal solution quantities (i.e. the projects to be initiated), the value of the objective function (i.e. the total NPV) and the shadow costs (i.e. opportunity costs of the binding constraints).

## 2.5 SUMMARY

In this chapter we presented related literature on multi-period capital rationing, project selection, projects and cash flows, and project evaluation. The next chapter presents the research methodology of the study.

# CHAPTER THREE

## METHODOLOGY

### 3.0    Introduction

This chapter presents the research methodology of the study. We shall put forward some information regarding Offinso Municipality and Offinso Municipal Assembly.

### 3.1    Brief History of Offinso Municipal Assembly

The Asante name *ɔfenso* has been anglicised to Offinso. Offinso Municipal located in the extreme north-western part of the Ashanti Region.  Offinso Municipal lies between longitude 1' 65W and 1' 45E and latitudes 6' 45N and 7' 25 S. The district covers an area of 1255km$^2$. One of the 27 districts in the region and about half of its boundary in the north and west is shared with the Brong Ahafo region. It is also bordered in the east by Ejura-Sekyedumase district and in the south by Kwabre, Sekyere South, Ahafo-Ano South and Atwima-Nwbiagya districts.

The Municipal capital is Offinso New Town. The Municipality is dissected by the main trunk road between Accra and Kumasi to the north. This is part of the Trans-African Highway, which serves as the main gateway to the Ashanti Region from the Northern and Brong-Ahafo Regions.

There are a number of educational institutions in the municipality including kindergarten, Primary, Junior High, Senior High, Polytechnic, Teacher Training and Midwifery Training Schools.

Most culture, heritage, and customs in the district originated with the Akan people. However, migrant settlers from the north have brought their own culture to the area. The Municipality has one major festival, the Mmoaninko Afahye, which is celebrated every four years. It is an

occasion which brings Offinso citizens, home and abroad, together to discuss the development of the Municipality. About 85% of the population are Asante, while the remaining 15% comprises other ethnic groups, the majority of whom hail from the Northern, Upper West, and Upper East Regions.

Christianity is the main religion practiced in the district, although there is a significant Muslim population in the town of Offinso. The Municipality has one paramount chief, the Offinsohene, and several sub-chiefs.

**Municipal Vision**

Offinso Municipal Assembly's vision is to champion decentralization in the Municipality through the effective local Governance Administration for the empowerment of its citizen's public and private sectors and vibrant Civil Society organizations capable of delivering efficient and sustainable services to the populace in the Municipality.

**Municipal Mission Statement**

The Municipal Assembly is the highest political and administrative body of the Municipality and therefore exercises deliberate legislative and executive functions.  It was established by Legislative Instrument (L.I. 1906, 2007).  It has a mission and responsibility to improve the quality of the people through sustainable development.  To achieve this mission, the Assembly has set itself certain objectives:

**Objectives of the Municipality**

To fulfill its mission, the Municipal Assembly has set itself the following objectives:

1. To facilitate the effective functioning of local government administration in the Municipality.

2. To ensure efficiency and effectiveness in the use of resources of the

3. Assembly and Decentralized Department in the Municipality.

4. To monitor, co-ordinate and harmonize the implementation of development

5. Plans and activities in the Municipality.

6. To facilitate the provision of basic social and economic infrastructure and services in the Municipality.

7. To facilitate community based and Private Sector Development in the Municipality.

**Functions**

The functions of the Municipal Assembly, like all other Assemblies are basically derived from the legislative Instruments as mandated by the Local Government Act 462 of 1993.

These functions which are broadly aimed at attaining its objectives and fulfilling its mission of improving the quality of life of its people are to:

- Be responsible for the overall development of the Municipal and ensures the preparation and submission of development plans and budget to the relevant Central Government Agency/Ministry through the Regional Co-coordinating Council (RCC).

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- Formulate and executive plans, programmes and strategies for the effective mobilization of the resources necessary of the overall development of the Municipality.

- Promote and support productive activity and social development in the Municipal and remove any obstacles to initiative and development.

- Initiate programmes for the development of basic infrastructure and provide Municipal works and services in the Municipality.

- Be responsible for the development, improvement and management of human settlements and environment in the Municipality.

- Co-operate with the appropriate national and local security agencies and be responsible for the maintenance of security and public safety in the Municipality.

- Ensure ready access to courts and public tribunals in the Municipality, for the promotion of justice.

- Initiate, sponsor or carry out such studies as may be necessary for the discharge of any of the functions conferred by Act 462 or any other enactment.

- Perform such other functions as may be provided under any other enactment.

- Effectively and efficiently perform these functions, the Assembly requires variety of skills and professional to man its various post created to operate its organization structure.

**Organizational Structure**

The Municipality is made up of the Municipal Chief Executive, 43 Assembly members of which 33 are elected by universal adult suffrage and 10 are appointed by the President in consultation with chiefs and interest groups in the Municipality. The one Member of Parliament (M.P.) in the Municipality is an ex-officio member. The Assembly sitting is chaired by a Presiding Member (PM) elected from among the members of the Assembly.

The Assembly performs its functions through the Executive Committee Chaired by the Municipal Chief Executive and a network of sub-committees. The Executive Committee exercises and co-coordinates functions of the Assembly while the six (6) sub-committees collate and deliberate on issues relevant to their functional areas. The sub-committees are:

- Development planning

- Social Services

- Works

- Finance and Administration

- Justice and Security

- Education

For administrative effectiveness, the Municipal Chief Executive (MCE) is supported by a secretariat or central administration referred to as the office of the Municipal Assembly, which is headed by a Municipal Co-ordinating Director (MCD) who reports to the Municipal Chief Executive and is in charge of the day to day administration of the Assembly. The Assembly

currently    has    staff    strength    of    about    75    employees.

The Municipal Co-ordinating Director is also in charge of all Heads of Decentralized Departments of the Assembly.

### 3.2.0    Linear Programming (LP)

A Linear Programming (LP) is one of the most widely used optimization techniques and perhaps the most effective. The term Linear Programming was coined by George Dantzig in 1947 to refer to problems in which both the objective function and constraints are linear. (Dantzig, 1998).

A Linear Programming is the problem of optimizing linear objective in the decision variables $X_1, X_2, \dots \dots X_n$ subject to linear equality or inequality constraints.

### 3.2.1    Standard Form of Linear Programs

Standard form of Linear Programming is;

$$\text{Maximize F} = \sum_{j=1}^{n} C_j X_j \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{ (3.1)}$$

$$\text{Subject to} \quad \sum_{j=1}^{n} a(i, j) X_j = b_i, \qquad i = 1, 2, \dots, m \qquad \dots\dots\dots\dots\dots \text{ (3.2)}$$

$$l_j \le X_j \le u_j, \quad j = 1, 2\dots, n \qquad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots \text{(3.3)}$$

where $C_j$ is the n objective function. Coefficients, $a(i, j)$ and $b_j$ are parameters in the m linear inequality constraints and $l_j$ and $u_j$ are lower and upper bounds with $l_j \le u_j$. Both $l_j$ and $u_j$ may be positive or negative.

A linear programming can be expressed more conveniently using matrices;

$$\text{Minimize} \quad F = C^1 X \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{(3.4)}$$

$$\text{Subject to:} \quad \mathbf{A}X = b \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{(3.5)}$$

$$l \leq X \leq u \quad \dots\dots\dots\dots\dots\dots\dots\dots \text{(3.6)}$$

$\mathbf{A}$ is an $m \times n$ matrix whose $(i, j)$ element is the constraint coefficient $a(i, j)$ and $C$, $b$, $l$, $u$ are

vectors whose complements are $C_j$, $b_j$, $u_j$ respectively. If any of the equations (3.1- 3.5) were

redundant, that is, linear combinations of the others, they could be detected without changing any

solutions of the system. If there is no solution or if there is one solution for equation (3.5), there

can be no optimization. Thus the case of greatest interest is where the system of equation (3.5)

has more than unknown equations and has at least two and potentially and infinite number of

solutions. This occurs if and only if

$n > m$, and Rank $(\mathbf{A}) = m$

We assume these conditions are true in what follows. The problem of linear programming is to

first detect whether solutions exist, and if so, to find one yielding the minimum $F$.

### 3.2.2 Basic Terminology and Definitions

(i)   A feasible solution to LP problem is a vector $X = (X_1, X_2, \dots\dots X_n)$ that satisfies the equation

   $\mathbf{A}X = b$ and the bounds $l \leq X \leq u$.

(ii)  A linear programming (LP) is feasible if there exists a feasible solution otherwise it is said

   to be infeasible.

(iii) A basic feasible solution is a basic solution in which variables satisfy their bounds

   $l_j \leq X_j \leq u_j$.

(iv)  A non degenerate basic feasible solution in which all basic variables Xj are strictly between

   their bounds, that is, $l_j \leq X_j \leq u_j$.

(v) An optimal solution $X = (X_1, X_2, \ldots \ldots X_n)$ is a feasible solution subject to $C^T X$, $\mathbf{A}X = b$

and $X \leq 0$. Lasdon (2002).

### 3.2.3    Equivalent Forms of LP

A linear programming can take on several forms. We might be maximizing instead of

minimizing. We might have a combination of equality and inequality constraints. Some variables

may be restricted to be non-positive instead of non-negative, or be unrestricted in sign. Two

forms are said to be equivalent if they have the same set of optimal solutions or are either

infeasible or unbounded.

(i)  A maximization problem can be expressed as a minimization problem;

   i.e. maximize $C^T X \equiv$ minimize $-C^T X$

(ii) An equality can be represented as a pair of inequalities;

$$a_i^T x = b_i \equiv \begin{cases} a_i^T \leq b_i \\ a_i^T \geq b_i \end{cases} \quad \text{OR}$$

$$\begin{cases} a_i^T \geq b_i \\ -a_i^T \leq -b_i^T \end{cases}$$

(iii) By adding a slack variable, an inequality can be represented as a combination of equality

   and non-negativity constraints. For example,

$$\sum_{j=1}^{n} a(i, j) x_j \leq b_j$$

   Then we define a slack variable $s_i \geq o$ such that

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$$\sum_{j=i}^{n} a(i, j)x_i + s_i = b_i \,,$$

and the inequality becomes equality.

Similarly, if the inequality is $\sum_{j=1}^{n} a(i, j)_j \, j \geq b_i$ , we write;

$$\sum_{j=1}^{n} a(i, j)X_j - s_j = b_i$$

(iv) Non-positive constraint can be expressed as non-negative constraints. To express $X_j$ we replace $X_j$ everywhere with $- y_j$ and impose the $y_j \geq 0$ .

(v) $X$ may be unrestricted in sign. In such a case we replace $X_j$ everywhere by $\overset{+}{X}_j - \overset{-}{X}_j$ , adding the constraints $\overset{+}{X}_k$, $\overset{-}{X}_j \geq 0$.

In general, an inequality can be represented using a combination of equality and non-negative constraints, and vice versa. Using these rules;

Minimize $\{C^T X, \text{ st} \mathbf{A}X = b\}$ can be transformed into Minimize $\{\mathbf{A}\overset{+}{X} - \mathbf{A}\overset{-}{X} - s = b$ , $\overset{+}{X}, \overset{-}{X}, s \geq 0\}$. The former LP is said to be in canonical form, the latter in standard form.

### 3.2.4    Duality

For any given Linear programming problem called Primal, there is an associated Linear Programming called Dual problem. Duality is an important concept in Linear Programming problem since it is algorithmic and allows us to provide a proof of optimality.

### 3.2.5       Rules for Taking Dual Problems

If PRIMAL problem (P) is minimal problem, then the DUAL (D) problem is maximum problem and vice versa.

In general, using the rules of transforming a Linear Programming in Standard Canonical form, we have that Dual      of Primal (P);

$$\text{Minimize} \quad Z = C^T X$$

$$\text{Subject to} \quad \mathbf{A}\, X \leq b,$$

$Xj \geq 0$

Where $X = (X_1, X_2, ...., X_n)^T$ is any $n$-vector

$C = (C_1, C_2, ...., C_n)^T$ is any $n$-vector

$\mathbf{A} = a(i, j)$ is $m \times n$ matrix and

$b = (b_1, b_2, ...., b_n)^T$ is any m-vector is DUAL (D).

$$\text{Maximize} \quad W = b^T y$$

$$\text{Subject to} \quad \mathbf{A}^T y \geq C, \; y \geq 0.$$

The variables in the P are called Primal variables and the variables in the Dual problem are called Dual variables.

The general rules for converting primal problem of any form into dual problem can be summarized as shown in table 3.1

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**Table 3.1 PRIMAL AND DUAL PROBLEMS**

| PRIMAL PROBLEM | DUAL PROBLEM |
|---|---|
| *Maximization* | *Minimization* |
| <ul><li>Coefficients of objective function</li><li>Coefficients of $i^{th}$ constraints</li><li>$i^{th}$ constraint is an inequality of the form $\leq$</li><li>$i^{th}$ constraint is an equality (=)</li><li>$i^{th}$ variable is unrestricted</li><li>$i^{th}$ variable satisfies $\geq 0$</li><li>Number of variables</li><li>Number of constraints</li><li>If inequality of type $\geq$ occurs in maximization problem convert to type $\leq$ by multiplying through by -1</li></ul> | <ul><li>Right hand sides of constraints</li><li>Coefficients of $i^{th}$ variable</li><li>$i^{th}$ variable satisfies $\geq 0$</li><li>$i^{th}$ variable is unrestricted</li><li>$i^{th}$ constraint is an equality (=)</li><li>$i^{th}$ constraint is an inequality of the type $\geq$</li><li>Number of constraints</li><li>Number of variables</li><li>If inequality of type $\leq$ occurs in minimization problem convert to type $\geq$ by multiplying through by -1</li></ul> |

### 3.2.6 Solution Techniques for Linear Programming

There are several approaches for solving the linear programming problems. Among these techniques are:

(i)     Graphical Approach

(ii)    Simplex Algorithms

(iii)   LINDO Software

(iv)    QSB Package

(v)     MATLAB

(vi)    YE's Interior Point Algorithms

(vii)   Microsoft Excel

The most convenient and effective technique in use now is the Simplex Algorithms.

Essentially, the Simplex Algorithms starts at one vertex of the feasible region and moves (at each iteration) to another (adjacent) vertex, improving (or leaving unchanged), the objective function as it does so, until it reaches the vertex corresponding to the optimal linear programming solution.

The Simplex Algorithm for solving LP's was developed by Dantzig in 1940. A number of different versions of the algorithms have now been developed. One of these later versions, called the Reversed Simplex Algorithms forms basis of the most modern computer packages for solving LP's. (Arsham, 1999).

### 3.2.7    Sensitivity Analysis

Sensitivity analysis (or post-optimal analysis) allows us to observe the effect of changes in the parameters of linear programming problems in the optimal solution.

Given the LP package, it is easier to change the data to see how the solution changes (if at all) as certain key data items change.

As a by-product of using the Simplex Algorithm, we automatically get sensitivity information;

(a) for the variables, the Reduced Cost (also known as Opportunity Cost) column which gives us

for each variable, an estimate of how the objective function will change if we make that

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variable non-zero. This is called "Reduced Cost" for the variables. We can also interpret the Reduced Cost as the amount by which the objective function coefficient for a variable needs to change before that variable becomes non-zero.

(b) for each constraint column headed shadow price indicates by how much the objective function will change if we change the right hand side of the corresponding constraint. This is known as the "Marginal Value" or "Dual Value" for the constraint.

### 3.2.8   LP Formulation and Applications

The conditions for a Mathematical model to be a Linear Programming;

(i)   all variables must be continuous (i.e. can take fractional values)

(ii)  a single objective (minimum or maximum)

(iii) the objective and constraints are linear (i.e. any term is either a constant or constant multiples of an unknown).

The LP's are important in the everyday life because, many practical problems can be formulated as LP's and also there is an algorithm (called the Simplex Algorithm) which enables us to solve LP's numerically relatively easy.

Some of the major application areas of which LP can be applied are;

(i)   Blending

(ii)  Production Planning

(iii) Oil Refinery Management

(iv)  Distribution

(v) Financial and Economic Planning

(vi) Manpower Planning

(vii) Farm Planning

### 3.3.0    An LP Model for Project Selection under Multi-Period Capital Rationing Problem

Considering the information above about the formulation of LP, we formulate formally our LP model for solving OMA Project Selection under multi-period capital rationing problem.

Essentially, we are interested in the OMA's limited capital Qj, for the projects' investment. Which project(s) or portion(s) of a project to be initiated is the main concern of the OMA?

Below are the steps of the LP formulation.

(i) We determine the project NPV's, using

$$\beta_j(NPV) = \sum_{t=1}^{n}\left[\frac{C_t}{(1+r)^t}\right]$$

Where t = 0, 1

$j = 1, 2\dots 5$ and $C$ is the cash flows.

Assuming the results, NPV of the projects to be; Market Circle (A) = $\beta_1$, School Building

(B) = $\beta_2$, Health Post (C) = $\beta_3$, Electrification (D) = $\beta_4$ and Agriculture (E) = $\beta_5$.

(ii) We formulate the problem in LP which means defining the objective function, decision variables and constraints. The objective function of OMA is to maximize NPV. That is;

Maximize Z = $\beta_1 X_A + \beta_2 X_B + \beta_3 X_C + \beta_4 X_D + \beta_5 X_E$

Where the decision variables ( $X_j$ );

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$X_A$ is proportion of project A to be initiated (j=1),

$X_B$ is proportion of project B to be initiated (j=2),

$X_C$ is proportion of project C to be initiated (j=3),

$X_D$ is proportion of project D to be initiated (j=4),

$X_E$ is proportion of project E to be initiated (j=5).

(iii) Subject to the constraints in the OMA problem are the budgetary limitations in periods 0 and 1 (table 3.1). Taking the periods separately, we have;

Capital at time (t) = 0,

$$a_{(1,1)}X_A + a_{(1,2)}X_B + a_{(1,3)}X_C + a_{(1,4)}X_D + a_{(1,5)}X_E \le Q_1$$

Capital at time (t) = 1,

$$a_{(2,1)}X_A + a_{(2,2)}X_B + a_{(2,3)}X_C + a_{(2,4)}X_D + a_{(2,5)}X_E \le Q_2.$$

In addition to this, we deem it necessary to specify the formal constraints regarding the proportions of the projects accepted to ensure that a project cannot be accepted more than once or that NEGATIVE projects are accepted. That is;

$$X_A, X_B, X_C, X_D, X_E \le 1$$
$$X_A, X_B, X_C, X_D, X_E \ge 0.$$

Where $a_{(i,j)}$ = cash flow for each period for each project.

The whole formulation in a compact form is thus;

Maximize Z = $\beta_1 X_1 + \beta_2 X_2 + \beta_3 X_3 + \beta_4 X_4 + \beta_5 X_5$

Subject to

$$a_{(1,1)}X_1 + a_{(1,,2)}X_2 + a_{(1,3)}X_3 + a_{(1,4)}X_4 + a_{(1,5)}X_5 \leq Q_1$$

$$a_{(2,1)}X_1 + a_{(2,2)}X_2 + a_{(2,3)}X_3 + a_{(2,4)}X_4 + a_{(2,5)}X_5 \leq Q_2$$

$$X_1 \leq 1$$

$$X_2 \leq 1$$

$$X_3 \leq 1$$

$$X_5 \leq 1$$

$$X_4 \leq 1$$

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

$$X_4 \geq 0$$

$$X_5 \geq 0$$
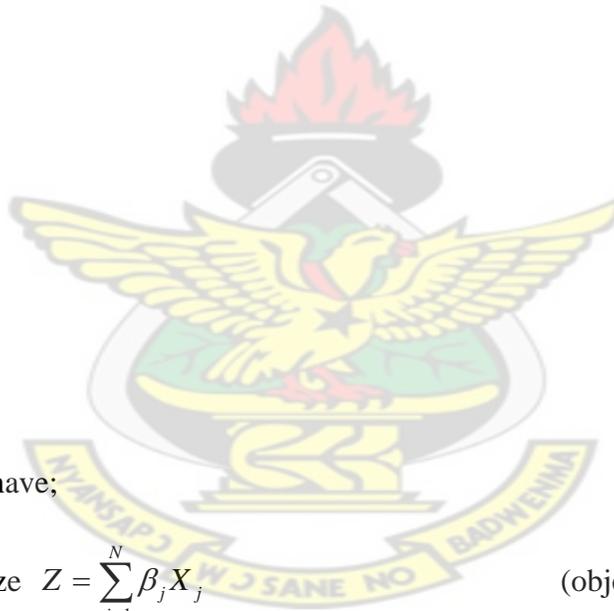
More generally we have;

$$\text{Maximize} \quad Z = \sum_{j=1}^{N} \beta_j X_j \qquad \text{(objective function)}$$

$$\text{Subject to} \quad \sum_{j=1}^{N} a_{(i,j)}X_j \leq Q_j \qquad \text{(constraints)}$$

$$i = 1, 2, \ldots, m$$

$$0 \leq X_j \leq 1 \qquad \text{(Non-negative constraints)}$$

$$j = 1, 2, \ldots, N$$

Where $\{a_{(i,j)}, Q_j \text{ and } \beta_j\}$ are given and $N$ is the number of projects to be invested.

### 3.3.1    Assumptions

The assumptions made in formulating the LP model include:

(i)     All the projects and constraints are independent on one another.

(ii)    Equal investment opportunities are assumed for the projects for each period.

(iii)   The cash flows, resources and constraints are known with certainty.

### 3.4.0   Integer Linear Programming (IP)

In many of the LP problems, certain variables should have been regarded as Integer values. But for the sake of convenience, we let them take fractional values reasoning that the variables were likely to be so large that any fractional part could be neglected. Whilst this is acceptable in some situations, in many cases we must find a numeric solution in which the variables take integer values. The problems in which this is the case are called Integer Programs (IP's) and the subject of solving such programs is called Integer Programming.

### 3.4.1   General Forms of Integer Programming (IP)

(i) An IP in which all the variables are required to be integers is called Pure IP problem,

i.e. Maximize $Z = \sum_{j=1}^{n} a_j X_j$          (objective function)

Subject to $\sum_{j=1}^{n} a_{(i,j)} X_j \leq b_j$          (constraints)

$X_j \leq 0, X_j$   Integer

$j = 1, 2 \dots n$An

(i) IP in which only some of the variables are required to be integers is called Mixed Integer Programming,

i.e. Maximize $Z = \sum Q_j X_j$

Subject to $\sum_{j=1}^{n} a_{(i,j)} X_j \leq P_j$

$$\sum_{j=1}^{n} X_j + \sum y_j \geq W$$

$X_j \geq 0, \quad y = 0, \text{ or } 1$

$j = 1, 2, \ldots, n$

(ii) An IP problem in which all the variables must be equal to 0 or 1 is called a Zero-one Integer Programming,

i.e. Maximize $Z = \sum Q_j X_j$

Subject to $\sum_{j=1}^{n} d_j X_j$ less than $D_j$

$X_j = 0 \text{ or } 1$

(iii) The LP obtained by omitting all integers or 0-1 constraints as variables is called the LP Relaxation of IP.

Any IP may be viewed as LP relaxation plus some additional constraints, the constraints that state which variables must be integers or be 0 or 1. Hence the LP relaxation is less constrained or more relaxed version of the LP. This means that the feasible region for an IP must be contained in the feasible region for the corresponding LP relaxation. LP relaxation for any IP is a maximize problem. This implies that; Optimal value for LP relaxation $\geq$ Optimal Z value for LP.

**3.4.2      Solution Techniques for IP**

We have general purpose (independent of LP being solved) and computationally effective (able to solve large LP's) algorithms (Simplex or interior point) for solving LP's. However, there is no similar general purpose and computationally effective algorithm exists for solving IP's. This means that IP's are harder to solve than LP's.

There are, at least three different approaches for solving IP problems, although they are frequently combined into "hybrid" solution procedures in computational practice. They are;

- Enumerative techniques

- Relaxation and decomposition techniques

- Cutting planes approaches based on polyhedral combinatory.

The most effective and simplest approach to solving a pure IP problem is to numerate all finitely many possibilities. However, due to the "combinatorial explosion" resulting from the parameter "size" only the smallest instances could be solved by such an approach. Sometimes one can implicitly eliminate many possibilities by domination or feasibility arguments. Besides straight-forward or implicit enumeration, the most commonly used enumerative approach is called BRANCH and BOUND, where the "branching" refers to the enumeration part of the solution technique and "bounding" refers to the fathoming of possible solutions by comparison to a known upper or lower bound on the solution value. (Doig, 1960).

To obtain an upper bound on the problem (we presume a maximization problem), the problem is relaxed in a way which makes the solution to the relaxed problem, relatively easy to solve.

All commercial branch-and-bound codes relax the problem by dropping the integrality conditions and solve the resultant continuous LP problem over the set P. If the solution to the relaxed linear programming problem satisfies the integrality restrictions, the solution obtained is optimal. If the LP is infeasible, then so is the integer program. Otherwise, at least one of the integer variables is fractional in the LP solution. One chooses one or such fractional variables and "branches" to create two or more sub problems which exclude the prior solution but do not eliminate any feasible integer solutions. These new problems constitute "nodes" on a branching tree, and an LP problem is solved for each node created. Nodes can be fathomed if the solution to the sub problem is infeasible, satisfies all of the in the integrality restrictions, or has an objective function value worse than a known integer solution. (Powell, 1998).

What makes this problem difficult is the fact that the variables are restricted to integers (zero or one). If the variables are allowed to be fractional (takes all values between zero and one for example) then we would have an LP which we can easily solve.

Suppose that we are to solve this LP relaxation of the problem (replace $x_j = 0$ or 1, j=1, 2, 3, 4 by $0 \leq x_j \leq 1$,

j =1, 2, 3, 4). Then using MATLAB package we get $x_2 = 0.5$, $x_3 = 1$, $x_1 = x_4 = 0$ of value 0.65 (i.e. the objective function value of the optimal linear programming solution is 0.65).

As a result of this, we now know something about the optimal integer solution, namely that it is $\leq 0.65$, i.e. this value of 0.65 is a (upper) bound on the optimal integer solution. This is because when we relax the integrality constraints, we (as we are maximizing) end up with a solution value at least that of the optimal integer solution (and may be better).

Consider this LP relaxation solution. We have a variable $x_2$ which is fractional when we need it to be integer. How can we rid ourselves of this troublesome fractional value? To remove this troublesome fractional value we can generate two new problems:

- original LP relaxation plus $x_2 = 0$
- original LP relaxation plus $x_2 = 1$

Then, it will be claimed that the optimal integer solution to the original problem is contained in one of these two new problems. This process of taking a fractional variable (a variable which takes a fractional value in the LP relaxation) and explicitly constraining it to each of its integer values is known as branching. It can be represented diagrammatically as below (in a tree diagram that is how the name-tree search arises).

Now two new LP relaxations are to be solved to get:

- P1 - original LP relaxation plus $x_2 = 0$, solution $x_1 = 0.5$, $x_3 = 1$, $x_2 = x_4 = 0$ of value 0.6
- P2 - original LP relaxation plus $x_2 = 1$, solution $x_2 = 1$, $x_3 = 0.67$, $x_1 = x_4 = 0$ of value 0.63

This can be represented diagrammatically as below.
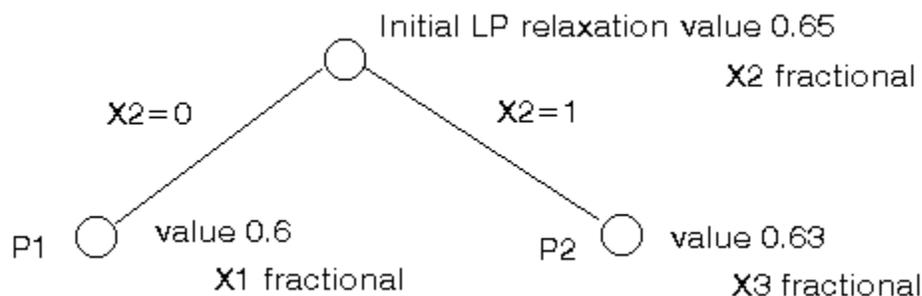
To find the optimal integer solution, just repeat the process, choosing one of these two problems, choosing one fractional variable and generating two new problems to solve.

Choosing problem P1 we branch on $x_1$ to get our list of LP relaxations as:

- P3 - original LP relaxation plus $x_2= 0$ (P1) plus $x_1= 0$, solution $x_3= x_4= 1$, $x_1= x_2= 0$ of value 0.6

- P4 - original LP relaxation plus $x_2= 0$ (P1) plus $x_1 = 1$, solution $x_1 = 1$, $x_3= 0.67$, $x_2 = x_4= 0$ of value 0.53

- P2 - original LP relaxation plus $x_2= 1$, solution $x_2= 1$, $x_3= 0.67$, $x_1= x_4= 0$ of value 0.63. This can again be represented diagrammatically as below.
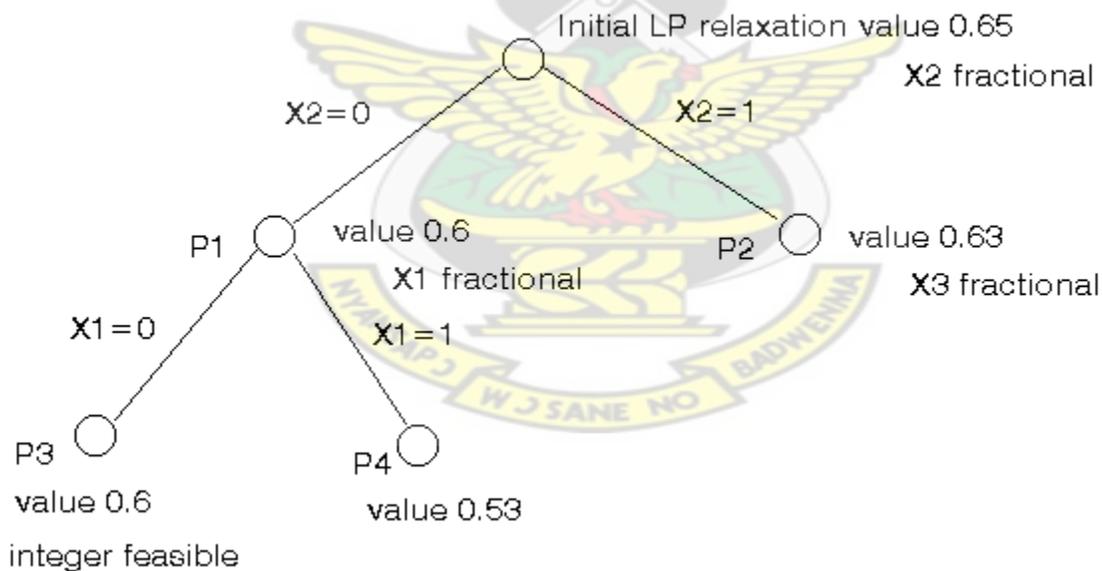


At this stage we have identified an integer feasible solution of value 0.6 at P3. There are no fractional variables so no branching is necessary and P3 can be dropped from our list of LP relaxations.

Hence we now have new information about our optimal (best) integer solution, that, it lies between 0.6 and 0.65 (inclusive).

Consider P4, it has value 0.53 and has a fractional variable ($x_3$). However if we were to branch on $x_3$ any objective function solution values we get after branching can never be better (higher) than 0.53. As we already have an integer feasible solution of value 0.6 P4 can be dropped from our list of LP relaxations since branching from it could never find an improved feasible solution. This is known as *bounding* - using a known feasible solution to identify that some relaxations are not of any interest and can be discarded.

Hence we are just left with:

- P2 - original LP relaxation plus $x_2 = 1$, solution $x_2 = 1$, $x_3 = 0.67$, $x_1 = x_4 = 0$ of value 0.63

  Branching on $x_3$ we get

- P5 - original LP relaxation plus $x_2 = 1$ (P2) plus $x_3 = 0$, solution $x_1 = x_2 = 1$, $x_3 = x_4 = 0$ of value 0.5

- P6 - original LP relaxation plus $x_2 = 1$ (P2) plus $x_3 = 1$, problem infeasible

Neither of P5 or P6 lead to further branching so we are done, we have discovered the optimal integer solution of value 0.6 corresponding to $x_3 = x_4 = 1$, $x_1 = x_2 = 0$.

The entire process we have gone through to discover this optimal solution (and to prove that it is optimal) is shown graphically below.
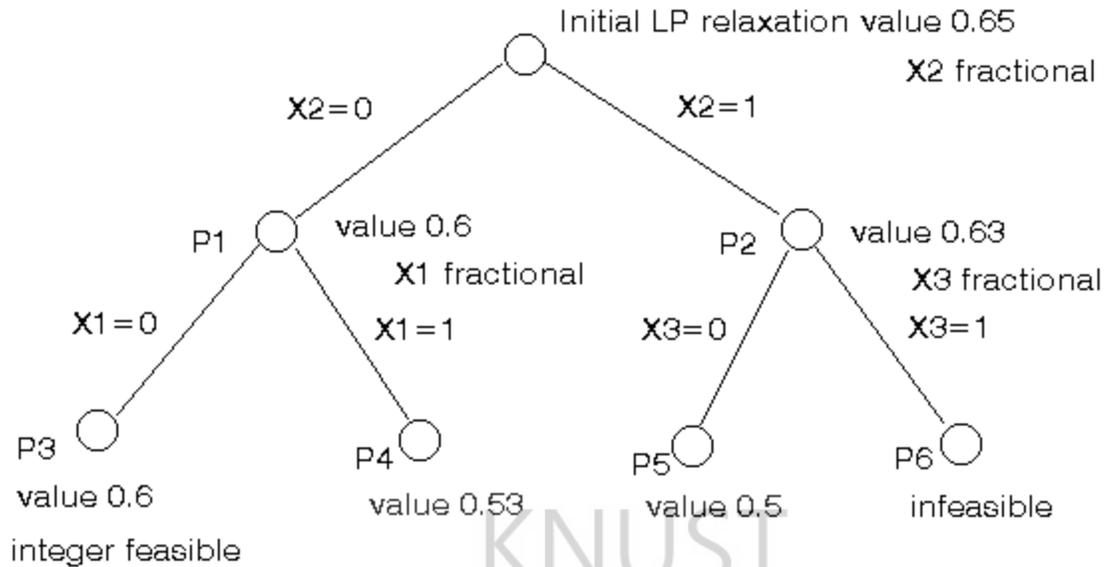
Thus, the optimal integer solution for this problem is $0.6m or $600.

### 3.4.3   IP APPLICATIONS

The major application areas which IP can be applied are;

(i)  a manpower scheduling problems concerned with security personnel.

(ii) a church location problem

(iii) capital budgeting problems

(iv) traveling salesperson problem

(v) the cutting stock problem

(vi) vehicle routing

### 3.5.0   IP Model for Project Selection under Multi-Period Capital Rationing Problem

Based on the background information about IP, we formulate our IP model for solving OMA

project selection under multi capital rationing problem.

**Conditions**

The dilemma of OMA is that it cannot invest in all N-projects suitable for investment which run n-years and have characteristics as shown in Table two because $\sum d_{(i,j)}$ is greater than $R_j$ where $d_{(I,j)}$ is the least capital requirement for $j$ projects and $R_j$ is the capital for investment.

We have a decision problem here: which projects would OMA select in order to maximize the total returns? To formulate IP for OMA problem the steps are;.

**Step 1: Decision Variables**

Define decision variables;

Let $X_j = \begin{cases} 1, & if \quad OMA \quad investes \quad in \quad project \quad j \\ 0, & if \quad OMA \quad does \quad not \quad invest \quad in \quad project \quad j \end{cases}$

$j = 1, \quad 2, \quad ...N$

That is, the $X_j$ are integer variables which must take one of the two possible values (zero or one). This represents Binary decision (e.g. do project = 1 or not to do project = 0)

**Step 2: Constraints**

Define the constraints;

Let $d_{(i,j)}$ = capital requirements for j projects,

$R_j$ = available capital for j projects for each year.

Then the constraints relating to availability of capital funds each year are;

$$\sum_{j=1}^{N} d_{(i,j)} X_j \leq R_j$$

$i = 1, \quad 2,....,m$

$X_j = 0$ OR 1

$j = 1, \quad 2,..., N$

## Step 3: Objective Function

Let the total profit be $\sum P_j X_j$

Then maximize the total Return, i.e. Maximize $Z = \sum_{j=1}^{N} P_j X_j$

This gives a complete IP model for OMA's problem as;

`Maximize $Z = \sum_{j=1}^{N} P_j X_j$ (objective function)

Subject to $\sum_{j=1}^{N} d_{(i,j)} X_j \leq R_j$ (constraints)

$i = 1, \quad 2,..., m$

$X_j = 0$ OR 1 (non-negative constraints)

$j = 1, \quad 2,..., N$

In more compact form we have;
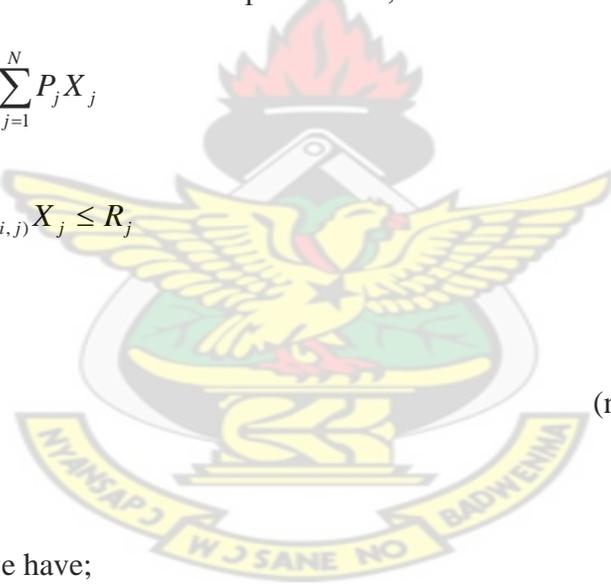
Maximize $Z = P_1 X_1 + P_2 X_2 + ....... + P_N X_N$

Subject to

$d_{(1,1)} X_1 + d_{(1,2)} X_2 + ...... + d_{(1,N)} X_N \leq R_1$

$d_{(2,1)} X_1 + d_{(2,2)} X_2 + ....... + d_{(2,N)} X_N \leq R_2$

$\qquad \qquad - \qquad \qquad -$

$\qquad \qquad - \qquad \qquad -$

$\qquad \qquad - \qquad \qquad -$

$d_{(m,1)} X_1 + d_{(m,2)} X_2 + ....... + d_{(m,N)} X_N \leq R_N$

$X_j = 0 \text{ or } 1$

$j = 1, \quad 2,....., N$

This completes our IP model

.

### 3.6.0 Summary

(i) In writing down the complete IP, we include the information that $x_j = 0$ or 1 ($j = 1,...,N$) as a reminder that the variables are integers

(ii) The usefulness of defining the variables is to take zero-one values - e.g. in the objective the term $p_1 x_1$ is zero if $x_1 = 0$ (as we want since no return from project 1 if we do not do it) and $p_1$ if $x_1 = 1$ (again as we want since get a return of $p_1$ if we do project 1). Hence effectively the zero-one nature of the decision variable means that we always capture in the single term $p_1 x_1$ what happens, when we do the project and when we do not do the project.

(iii) Note that the objective and constraints are linear (i.e. any term in the constraints/objective is either a constant or a constant multiplied by an unknown).Here we deal only with linear integer programs (IP's with a linear objective and linear constraints). It is plain that there do exist non-linear integer programs - these are, however, outside the scope of this thesis.

### 3.6.1   Extensions to the LP and IP Models

The extensions to the models include;

1. projects of different lengths

2. projects with different start/end dates

3. adding capital inflows from completed projects

4. projects with staged returns

5. carrying unused capital forward from year to year

6. mutually exclusive projects (can have one or the other but not both)

7. projects with a time window for the start time.

For the amendment of LP and IP models to deal with these extensions, see **Appendix A**.

### 3.6.2    Importance of IP Model

The IP or quantitative modeling techniques are increasingly being used for financial problems due to its numerous advantages. The advantages of using the models for project selection include;

(i)  enable us to maximize profit, rather than relying on an ad-hoc judgmental approach.

(ii) can be easily extended to deal with a larger number of potential investment opportunities.

(iii) can be used for sensitivity analysis, for example to see how sensitive our project selection decision is to change in the data.

# CHAPTER FOUR

## DATA COLLECTION AND ANALYSIS

### 4.0    Introduction

In this chapter, the secondary data extracted from the published financial statements, monthly and yearly reports of OMA from 2002-2007 and other supporting documentary evidence obtained from the OMA's department, is analyzed with the help of the LP and IP models formulated in the third chapter.

### 4.1.0   Sources and Data Collection

A capital budgeting is the process of considering alternative capital projects and selecting those alternatives that provide the most profitable return on available funds, within the framework of the company's goals and objectives. A capital project is any available alternative, to purchase, build, lease or renovate buildings, equipment, or other long range major items of property or projects. The alternative selected usually involves large sums of money and results in a large increase on fixed assets for several years. Once a company builds a plant or undertakes some other capital expenditure, the company becomes less flexible regarding future plans (Hermanson et al, 1992).

Based on this fact, most of the necessary information about cash inflows and cash outflows of the OMA's small scale and large scale projects were extracted from the OMA's financial statements, monthly and annual reports for the period 2002-2007. The annual net cash flow which is the difference between the cash inflows and cash outflows during each period for the under listed projects were then estimated and recorded (Tables 4.1, 4. 2)

The concepts of capital budgeting can be applied to not-for-profit organizations, such as universities, schools, districts, cities and not-for-profit hospitals. Since these organizations are not subject to as many taxes as profit-making organizations, the cash flows related to taxes are usually zero or near zero. Due to this, we ignored the tax factor in the estimation of the annual net cash flows for the OMA projects. (Equation 3 in Chapter 3). The OMA projects were classified into small scale and large scale projects. We also estimated the discount factors at cost of capital of ten percent (10%) for each cash inflow for each project and the corresponding NPV at ten percent (10%) (Table 4.1). The distribution of cash outlay for OMA small scale projects are shown in Table 4.2 which also shows the capital requirements, for each project, available capital at each period and the corresponding capital returns.

**Table 4.1:** Annual Net Cash Flow for OMA Large Scale Projects for 2002-2007 in GH¢ $\times 10^2$ Million

| Year | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | NPV at 10% | P.I |
|---|---|---|---|---|---|---|---|---|
| Project/ Period | 0 | 1 | 2 | 3 | 4 | 5 | | |
| Market Circle | 100 | -100 | -200 | -400 | 600 | 500 | 264 | 0.82 |
| School Building | 400 | 500 | -1000 | 1200 | -1400 | 1200 | 719 | 0.80 |
| Health Post | 250 | 200 | -360 | 500 | -400 | 0 | 237 | 0.92 |
| Electrification | 30 | 50 | 60 | 60 | 150 | -90 | 217 | 0.33 |
| Agriculture | 10 | 20 | -10 | 0 | 30 | 50 | 72 | 0.72 |
| Discount Factors | 1.00 | 0.909 | 0.826 | 0.751 | 0.683 | 0.621 | | |
| Capital Limitation $Q_j$ | 550 | 500 | | | | | | |

**Table 4.2**    Distribution of Capital Requirement for OMA Small Scale Projects for 2002-2007 in GH¢ $\times 10^2$ Million

| Year | 2002 | 2003 | 2004 | 2005 | 2006 | 2007 | Capital Returns |
|------|------|------|------|------|------|------|-----------------|
| Project/ Period | 0 | 1 | 2 | 3 | 4 | 5 | |
| KVIP ($X_1$) | 20 | 30 | 20 | 60 | 40 | 50 | 50 |
| Refuse Facility ($X_2$) | 50 | 60 | 20 | 60 | 50 | 20 | 40 |
| Water Borehole ($X_3$) | 40 | 60 | 0 | 60 | 45 | 80 | 45 |
| Taxi Circle ($X_4$) | 10 | 0 | 10 | 20 | 10 | 10 | 25 |
| Toll Booth ($X_5$) | 0 | 10 | 10 | 20 | 15 | 10 | 10 |
| Available Capital $R_j$ | 200 | 150 | 300 | 150 | 120 | 400 | |

### 4.1.1   Statistical Analysis

We used Continuous Probabilistic Analysis (CPA) for the analysis of the cash flows and NPV of the various projects. That is, we showed the variability of the project outcomes which results from the variability of the individual project cash flows. This will enable us to make probability assessment of the likelihood of the various project cash flows and variability (risk) of using the project's NPV in our proposed models.

The most useful measure for statistical purposes is the standard deviation. Hence we first, compute the mean (NPV), dispersion and variance of the period's cash flows. The projects' standard deviation is then obtained by combining the discounted standard deviations of the individual cash flows, using what is known as the statistical sum.

Having calculated the means and standard deviations of the various projects, the relative variability of the distribution of the project cash flows were then computed, using the formula;

Coefficient of Variation = $\dfrac{\sigma i}{\chi}$ x 100%, where $\chi$ is the mean of the cash flows. The results were

then compared and recorded (Table 3 & 4).

**Table 4.3:**    Distribution of Large Scale Projects Cash flows (2002-2007)

| Project | Mean $(\bar{x})$ | Standard deviation $(\sigma i)$ | Coefficient of variation/% |
|---|---|---|---|
| Market Circle | 266.67 | 205.48 | 77 |
| School Building | 250.00 | 373.05 | 39 |
| Health Post | 285.00 | 160.59 | 56 |
| Electrification | 73.33 | 38.59 | 53 |
| Agriculture | 20.00 | 16.32 | 82 |

**Table 4.4:**    Distribution of Small Scale Projects Cash flows (2002-2007)

| Project | Mean $(\bar{x})$ | Standard deviation $(\sigma i)$ | Coefficient of variation/% |
|---|---|---|---|
| KVIP $(X_1)$ | 36.67 | 14.91 | 41 |
| Refuse Facility $(X_2)$ | 43.33 | 16.99 | 39 |
| Water Borehole $(X_3)$ | 47.50 | 24.79 | 52 |
| Taxi Circle $(X_4)$ | 10.00 | 5.77 | 58 |
| Toll Booth $(X_5)$ | 10.83 | 6.07 | 56 |

The results in tables three and four will enable us to make probability statements about the projects outcomes which reflect the variability's expected in each period's cash flows and the distribution of the competing projects to be compared favorably

### 4.2.0 Implementation of LP Model

Applying LP model;

Maximize $Z = \beta_j X_j$

Subject to $\sum_{j=1}^{n} a_{(i,j)} X_j \leq b_j$

$i = 1, \quad 2,....,m$

$0 \leq X_j \leq 1$

$j = 1, \quad 2,....,n$

To OMA's capital rationing data (table 1), we formulate the Assembly's project selection as an LP so as to maximize the returns whilst satisfying the OMA's capital limitations.

Essentially, we are interested in the OMA's limited capital $Q_j$ for the project investment and the proportions of a project or a project to be initiated. Hence, we have the LP;

Maximize $Z = 264X_1 + 719X_2 + 237X_3 + 217X_4 + 72X_5$

Subject to $100X_1 + 400X_2 + 250X_3 + 30X_4 + 10X_5 \leq 550$

$\qquad\qquad 100X_1 + 500X_2 + 200X_3 + 50X_4 + 20X_5 \leq 500$

$0 \leq X_j \leq 1, \; j = 1, \quad 2,...,5$

In this LP problem, it is assumed that answers involving fractional variable values will be acceptable since the amount required for each period is reasonably large and this will not cause too many problems.

### 4.2.1 Solution to the LP, using MATLAB Package

There are more than two variables in our LP problem, hence it is considered as large scale LP

problem that can be solved easily by software package like MATLAB.

The LP module in the MATLAB package is called "Linprog". To solve an LP problem,

$\min_{x} f^T X$ such that

$$A X \leq b$$

$$A_{eq} X = b_{eq}$$

$lb \leq X \leq ub$

Where f, x, b, beq, lb, and ub are vectors and A and Aeq are matrices, using linprog,

the Syntax is;

x = linprog (f, A, b, Aeq, beq)

x = linprog (f, A, b, Aeq, beq, lb, ub)

x = linprog (f, A, b, Aeq, beq, lb, ub, x0)

x = linprog (f, A, b, Aeq, beq, lb, ub, x0, options)

[X fval] = linprog (...)

[X, fval, exitflag] = linprog (...)

[X, fval, exitflag, output] = linprog (...)

[X, fval, exitflag, output, lambda] = linprog (...)

Putting the parameters of the LP model into the "linprog" we have

$$f = \begin{bmatrix} -264, -719, -237, -217, -72 \end{bmatrix}$$

$$A = \begin{bmatrix} 100 & 400 & 250 & 30 & 10 \\ 100 & 500 & 200 & 50 & 20 \end{bmatrix}$$

$$b = [550,500]$$

$$lb = zeros(5,1)$$

$$ub = ones(5,1)$$

$$b_{eq} = []$$

$$A_{eq} = []$$

[X, fval, exitflag, output, lambda] = linprog (f, A, b, Aeq, beq, lb, ub, [], []);

Enter

X, fval, lambda.ineqlin, lambda.lower

Entry of the solution to the LP problem is shown below;

| Decision Variables | Solution Variables | Unit Cost or Profit | Total Contribution | Shadow Price | Reduction Cost |
|---|---|---|---|---|---|
| Market ( $X_1$ ) | 1.00 | 264 | 264 | 0 | 0 |
| School ( $X_2$ ) | 0.66 | 719 | 474.54 | 1.438 | 0 |
| Health ( $X_3$ ) | 0 | 237 | 0 | 0 | 50.6 |
| Electrification ( $X_4$ ) | 1.00 | 217 | 217 | 0 | 0 |
| Agriculture ( $X_5$ ) | 1.00 | 72 | 72 | 0 | 0 |

Optimal solution (Max. objective function) = $GH \not\subset 1027.5 \times 10^2 M$

### 4.2.2 Interpretation of Solution

The solution indicates that construction of market place $(X_1 = 1)$, providing Agricultural inputs $(X_3 = 1)$, and electrification projects $(X_4 = 1)$ can be done by the OMA within its capital limitation. At the same time, 0.66 portions of a unit classroom block $(X_2)$ can be initiated while $(X_3 = 0)$ indicates that the Assembly cannot put up health post within the time frame with the limited capital. This investment plan uses all the funds available in year zero and year one.

The shadow prices indicate that the amount by which the NPV of the optimal plan (i.e. $GH \not\subset 1027.5 \times 10^2 M$) could be increased if the budgetary constraints could be increased. For every $GH \not\subset 1M$ relaxation of the constraint in period two, $GH \not\subset 1.438 \times 10^2 M$ extra NPV could be obtained. The shadow prices also indicate that extra funds in period zero is not required. That is, the marginal value or dual value is not needed in the first constraint.

The Reduced Cost of 50.6 in $X_3$ indicates the amount by which the objective coefficient for a variable $X_3$ needs to change before becoming non-zero is $GH \not\subset 50.6 \times 10^2 m$.

The slack or surplus column gives, for a particular constraint, the difference between the Left Hand side of the constraint when evaluated at the LP optimal (i.e. evaluated at $X_j$ ) and the Right Hand side of the constraint.

However, in the LP solution, all the constraints are tight or binding (i.e. have zero surplus or slack). None of the constraints is loose (i.e. have non-zero surplus).

All these facts may give OMA management some guidance in their considerations of the various alternative sources of capital.

**4.2.3.0 Sensitivity Analysis**

As explained earlier on, sensitivity analysis or post optimal analysis permits us to observe the effect of changes in the parameters of the LP problem on the optimal solution.

At this point, we study the impact of changing;

the objective function coefficient (cost coefficient of our LP for multi capital rationing problem)

the Right Hand Side (RHS) coefficient of a constraint of our LP model.

The MATLAB LP package provides us with the sensitive information (the Reduced Cost) and shadow prices. Hence we change the data items concern and resolve the LP to see how the solution changes as certain parameters change.

## 4.2.3.1 Changing the RHS Coefficient of Constraints

The shadow price of the $j^{th}$ constraint gives the amount by which the optimal Z-value is increased in the maximize problem if we increase the RHS coefficient of constraints by one. Hence, we increase the $b_2$ by one (i.e. 500 +1=501) and resolve the LP to get;

| Decision Variables | Solution Variables | Unit Cost or Profit | Total Contribution | Shadow Price | Reduction Cost |
|---|---|---|---|---|---|
| Market ($X_1$) | 1.00 | 264 | 264 | 0 | 0 |
| School ($X_2$) | 0.66 | 719 | 474.54 | 1.438 | 0 |
| Health ($X_3$) | 0 | 237 | 0 | 0 | 50.6 |
| Electrification ($X_4$) | 1.00 | 217 | 217 | 0 | 0 |
| Agriculture ($X_5$) | 1.00 | 72 | 72 | 0 | 0 |

Optimal Solution (maximum objective function) = GH¢ 1029

**Interpretation of Solution**

It is observed that a unit change in the constraint $X_3$ does not affect the solution values and reduced cost or opportunity cost.

Also, a unit increased in the capital funds for the constraint (Health Post ($X_3$)) has created a marginal value of 1.5 in the optimal objective function value (NPV) for a project investment. That is, the optimal objective function value increased from 1027.6 to 1029, with a difference of 1.5 which is approximately equal to the shadow price of 1.438 for the constraint $X_3$.

From this observation, we can deduce that in the maximization problem, if RHS of the $j^{th}$ constraint is increased by an amount $\Delta b_j$, then assuming the current optimal solution, the new optimal Z-value can be found from;

New optimal Z-value = Old optimal Z-value + $\Delta b_j$ (constraint $j$ shadow prices)

This analysis is very useful in planning because it enables the management to identify the most sensitive parameters or elements in the projects. Once the elements of the projects are identified, further analysis and study can take place on these elements trying to establish the likelihood of the variability and range of values that might be expected to make a more reasoned decision whether or not to proceed with the project.

### 4.2.3.2        Changing the Coefficient of the Objective Function

The Reduced Cost, 50.6 in $X_3$ row shows that the amount by which the objective function coefficient for the variable $X_3$ should change to make it non-zero. Hence we alter coefficient of $X_3$ in the objective function by +50.6 and resolve our LP problem.

Solving by the MATLAB software, we have;

| Decision Variables | Solution Variables | Unit Cost or Profit | Total Contribution | Shadow Price | Reduction Cost |
|---|---|---|---|---|---|
| Market ($X_1$) | 1.00000 | 264 | 264 | 0 | 0 |
| School ($X_2$) | 0.38042 | 719 | 273.52 | 1.438 | 0 |
| Health ($X_3$) | 0.69895 | 287.6 | 201.20 | 0 | 0 |
| Electrification ($X_4$) | 1.00000 | 217 | 217 | 0 | 0 |
| Agriculture ($X_5$) | 1.00000 | 72 | 72 | 0 | 0 |

Optimal solution (maximum objective function) $=$GH¢ $1027.5 \times 10^2$ m.

### 4.2.3.3        Explanation

Addition of the reduced cost of the 50.6 0n the row of variable $(X_2)$ to its corresponding coefficient in the objective function effects no changes in the shadow prices, solution values for variables $X_1, X_4, X_5$ and the optimal objective function.

However, there were sharp variations in some the optimal solution values. The coefficient of variable $(X_2)$ decreased from 0.662 to 0.38042 while variable $X_3$ increased from 0 to 0.69895. Thus, increasing the NPV per unit on variable $X_3$ impacts a sharp change on the optimal solution. Given the sensitivity analysis of one or more of the key factors of projects like this, the management's task is to decide whether the project is worthwhile.

### 4.3.0    Implementation of IPModel

IP works reasonably well where there is a hierarchy of decisions to be made. For instance, building a new factory enables various consequential activities to take place. Although the solution depends on the values of all the decision variables, setting the values of the most important ones restricts the values of the decision variables representing the consequential activities. In such a case, the IP code will usually be work out for itself which are the most important decisions and determine those first or we can assist it by specifying the hierarchy of decisions explicitly.

Based on these IP principles, we apply the IP model for multi capital rationing problem;

$$\text{Maximize } \sum_{j=1}^{N} P_j X_j$$

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Subject to $\sum_{j=1}^{N} d_{(i,j)} X_j \leq R_j$, $X_j = 0\,or\,1$ $\qquad j = 1,\quad 2,...,N$

to the OMA's small scale project selection problem data (table 2).

### 4.3.1 Purpose

The OMA's project manager wishes to select from N-potential projects for investments so that by the end of n-years, the project selected will maximize returns from these batch of projects with regards to his capital limitation.

This problem can be formulated, (using the data in table 4.2) as zero-one IP problem (do a project = 1, do not project = 0). That is we fit the data into our IP model as:

$$\text{Maximize } Z = 50X_1 + 40X_2 + 45X_3 + 25X_4 + 10X_5$$

$$\text{Subject to } 20X_1 + 50X_2 + 40X_3 + 10X_4 + 0X_5 \leq 200$$

$$30X_1 + 60X_2 + 60X_3 + 0X_4 + 10X_5 \leq 150$$

$$20X_1 + 20X_2 + 0X_3 + 10X_4 + 10X_5 \leq 300$$

$$60X_1 + 60X_2 + 60X_3 + 20X_4 + 20X_5 \leq 150$$

$$40X_1 + 50X_2 + 45X_3 + 10X_4 + 15X_5 \leq 120$$

$$50X_1 + 20X_2 + 80X_3 + 10X_4 + 10X_5 \leq 400$$

$$X_j = 0\,or\,1, \quad j = 1,\quad 2,...,5$$

$$\text{Where KVIP} = X_1, \text{ Refuse} = X_2, \text{ Water} = X_3, \text{ Taxi} = X_4 \text{ and Toll Booth} = X_5$$

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This is a Binary decision problem and can be solved easily by MATLAB software as shown below:

### 4.3.2    Solution to IP, using MATLAB

The IP module in the MATLAB package is called "Bintprog". To solve an IP problem of the form;

$$\min_{X} f^{T} X \text{ such that}$$

$$AX \leq b$$

$$A_{eq} = b_{eq}$$

All variables are integer;

where f, b, and beq are vectors, A and Aeq are matrices, and the solution x is required to be a binary integer vector -- that is, its entries can only take on the values 0 or 1. Using "bintprog", the syntax is

x = bintprog (f)

x = bintprog (f, A, b)

x = bintprog (f, A, b, Aeq, beq)

x = bintprog (f, A, b, Aeq, beq, x0)

x = bintprog (f, A, b, Aeq, beq, x0, options)

[X, fval, exitflag] = bintprog (...)

[X, fval, exitflag, output] = bintprog (...)

Putting the data into IP model, the we have;

$$f = \begin{bmatrix} -50, -40, -45, -25, -10 \end{bmatrix}$$

$$A = \begin{bmatrix} 20 & 50 & 40 & 10 & 0 \\ 30 & 60 & 60 & 0 & 10 \\ 20 & 20 & 0 & 10 & 10 \\ 60 & 60 & 60 & 20 & 20 \\ 40 & 50 & 45 & 10 & 15 \\ 50 & 20 & 80 & 10 & 10 \end{bmatrix}$$

KNUST

$$b = \begin{bmatrix} 200, 150, 300, 150, 120, 400 \end{bmatrix}$$

[X, fval, exitflag, output] = bintprog (f, A, b)

The solution to the IP problem (output from bintprog) is shown below;

| Decision Variable | Solution Value | Unit Cost or Profit | Total Contribution | Reduced Cost |
|---|---|---|---|---|
| KVIP $(X_1)$ | 1 | 50 | 50 | 0 |
| Refuse Facility $(X_2)$ | 0 | 40 | 0 | 40 |
| Water Borehole $(X_3)$ | 1 | 45 | 45 | 0 |
| Taxi Circle $(X_4)$ | 1 | 25 | 25 | 0 |
| Toll Booth $(X_5)$ | 0 | 10 | 0 | 10 |

| Iterations | = 15 |
|---|---|

| Optimal objective function value | = 120 |
|---|---|

| Nodes | = 11 |
|---|---|

| Algorithm | = 'LP-branch-and-bound' |
|---|---|

### 4.3.3 Interpretation of Solution

It is obvious that the optimal decision is to choose to do projects; KVIP, Water Borehole and Taxi Circle whilst the OMA cannot provide Refuse Facilities and Toll Booths within its capital limitation for the next five years unless the capital investment is reviewed.

The optimal decision achieves maximum returns of $GH \not\subset 120 \times 10^2 m$. It is evident that the model has assisted the project manager to select large number of the viable projects that maximize profit. This is better than relying on an ad-hoc judgmental approach to the project investment.

The model can be used for sensitivity analysis, for instance, to examine how sensitive our project selection decision is to changes in the parameters of the model.

### 4.4 Findings

The LP and IP models revealed quite interesting facts about project investment parameters. The models;

(i) Enabled us to maximize profit rather than depending on an ad-hoc judgmental approach to project investment.

(ii) Enabled the OMA to deal with a larger number of viable investment opportunities.

(iii) Assisted the OMA project manager to see that a unit change in the capital funds constraint $x_j$ could create marginal value in the optimal objective function (NPV). That is, in maximization problem,

New optimal z-value = Old optimal z-value + $\Delta b_j$ (constraint j shadow prices).

(iv) A change in the coefficient of the objective function by the reduced cost can cause a sharp variation in some of the coefficient of constraints $x_j$. A unit increase in the NPV impacts some change in the optimal solution.

(v) Good computer packages (solvers) exist for finding optimal solutions to IP's. Many of the computational advances in IP optimal solution methods (e.g. constraint aggregation, coefficient reduction, problem reduction, automatic generation of valid inequalities) are included in these packages. Often the key to making successful use of such packages for any particular problem is to put effort into a good formulation of the problem in terms of the variables and constraints. By this we mean that for any particular IP there may be a number of valid formulations. Deciding which formulation to adopt in a solution algorithm is often a combination of experience.

# CHAPTER FIVE

## SUMMARY, CONCLUSION AND RECOMMENDATION

### 5.0 Introduction

In this chapter, the summary of the results from the previous chapters, conclusions, discussions and recommendations about our findings are presented.

### 5.1 Summary

The primary objective in this thesis is to design linear programming and integer programming models for solving Offinso Municipal Assembly's projects selection under multi-period capital rationing problems.

A second objective is to maximize the return from the batch of the projects selected with regard to the capital limitation, and also use the models to do sensitivity analysis on the project parameters in order to assist project managements decide effectively which projects worth undertaking.

Data used for this thesis is purely secondary data extracted from financial statements, annual reports, monthly reports, daily newspapers and other relevant documents from the Assembly. The financial ratios such as Net Present Value, Profitability Index, and also LP and IP models were used in the analysis of the data. The discount factors at cost of capital at ten percent (10%) for each cash flow for each project, NPV's at percent (10%) and relative Profitability Index (PI) were computed and the results tabulated (Table 4.1).

The LP model is designed to solve OMA's project selection under multi period capital rationing with large scale project problems and this produces the optimal solution quantities ( i.e. the projects to be initiated), the value of the objective function (i.e. the total NPV) and opportunity costs of the binding constraints. On the other hand, selection of small scale project problems are solved effectively by IP model or quantitative modeling techniques.

Factors which come into play in choosing which IP solution method is appropriate are:

(i)     The size of the IP (variables and constraints)

(ii)    Time available to build the model (formulation plus solution algorithm)

(iii)   Time available for computer solution once the model has been built

(iv)    Experience

The solutions to both LP and IP models can be given by software such as:

MATLAB

QSB Package

Microsoft Excel 2003

The model can be used to do sensitivity analysis, for instance, to examine how sensitive over project selection decision is to change in the parameters of the model. This invariably helps the project management to decide effectively on the projects which worth undertaking.

## 5.2 Discussion and Conclusion

As explained earlier on, the idea behind the profitability index (PI) is that the PI provides the subset of projects that maximizes the aggregate NPV. However, this is not always the case. The projects were ranked with the most highly PI first i.e. Health Post ninety two percent (92%) viable, followed by Market Circle eighty two percent (82%), School Building eighty percent (80%) and Agriculture seventy seven percent (77%). This provided the highest aggregate NPV value of 1392. This value is much higher than the LP model optimal objective function of 1027.5. The total capital available for investment is 1050. We note here that the PI leads to wrong conclusion, a decision that could have reduced the OMA wealth. The larger coefficient of variation of Agriculture eighty two percent (82%) and Market Circle seventy seven percent (77%) means that the probability of selecting these projects are more certain than others.

The LP model assisted the project manager to accept the hundred percent (100%) construction of market place, hundred percent (100%) agriculture inputs, hundred percent (100%) electrification projects and sixty six percent construction of classroom blocks but failed to accept the construction of health post at the total cost of GH¢ 1027.5x $10^2$ million against the investment capital of GH¢1050 x $10^2$ million (Table 4.1).

After post-analysis of the projects' parameters using the LP model, we tend to accept hundred percent (100%) construction of market place, thirty eight percent (38%) construction of classroom block, 69.9% of construction of health post, hundred percent (100%) of both electrification and agriculture inputs at the cost of GH¢ 1027.5 x $10^2$ million. This yielded an acceptable optimal decision.

Much of the information obtainable as by-product of the solution of the LP model can be useful to management in estimating the effect of changes without going to the expense of resolving the LP. Similarly, using the IP model, the optimal decision for maximizing the returns is to do the projects: KVIP, Water Boreholes and Taxi Circle at the cost of GH¢ $120 \times 10^2$ million.

The helpful observation is that, our mathematical models support project management in the following ways. The models will enable the project managers in:

Problem identification, that is, diagnosis of the problem from its symptoms if not obvious, delineation of the sub-problem to study, establishment of objectives, limitation and requirements.

Formulation of project selection problem as LP and IP.

Model validation. This involves running the algorithm for the model on the computer in order to ensure that the input data is free from errors.

Solution of the model, that is, standard computer packages or especially developed algorithms can be used to solve the model. The solution involves many solutions under varying assumptions to establish sensitivity.

Implementation, that is, the implementation of the results of the study or algorithm for solving the models serves as an operational tool.

The advantages of using a software package to solve LP and IP models, rather than a judgmental approach to project selection problem are:

Actually maximize profit, rather than believing that our judgmental solution maximize profit, we may end up having bad judgment.

Makes the project selection decision one that we can solve in a routine operational manner on a computer, rather than having to exercise judgment each and every time we want to solve it.

Problems that can be appropriately formulated as LP are almost always better solved by computers than by people.

Can perform sensitivity analysis very easily using a computer.

**5.3 Recommendation**

To encourage the use of LP and IP models to solve project selection problems:

(i) The decision rule where capital rationing exists is to maximize the return from the project(s) selected rather than simply accept or reject decisions of the projects in isolation

(ii) Studies have shown that in the project selection problems where proportions or fractional parts of a projects and a whole project(s) are desired to be initiated within a period, the best decision tool is the LP model. Whenever only a whole project is desired to be invested, the IP model or quantitative modeling techniques is the suitable decision making tool.

(iii) Due to the large amount of data involved and the complexity of the mathematical techniques involved, project managers are highly implored to use computers to solve capital rationing problems, particularly for risk evaluation (NPV) and sensitivity analysis.

(iv) The models can be amended to deal with the following extensions:

- projects of different lengths

- projects with different start/end dates

- adding capital inflows from completed projects

- projects with staged returns

- carrying unused capital forward from year to year

- mutually exclusive projects (can have one or the other but not both)

- projects with a time window for the start time.

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**APPENDIX A**

EXTENSIONS TO THE MATHEMATICAL MODELS OF PROJECT SELECTION

Consider the IP model for SMA's project selection problem:

$$\text{`Maximize } Z = \sum_{j=1}^{N} P_j X_j \qquad \text{(objective function)}$$

$$\text{Subject to } \sum_{j=1}^{N} d_{(i,j)} X_j \leq R_j \qquad \text{(constraints)}$$

$$i = 1, \quad 2,...,m$$

$$X_j = 0 \text{ OR } 1 \qquad \text{(non-negative constraints)}$$

$$j = 1, \quad 2,...,N$$

This basic model can be amended to deal with extensions or projects with the following characteristics:

(a)     Projects of different lengths

Projects with different lengths are easily dealt with, just set their capital requirement in any year in which the project does not exist (i.e. has not started or has already ended) to zero. For example if project $X_1$ only runs for 2 years (instead of 3 years) and hence finishes in year 2 then the capital requirement constraint for year 3 becomes:

$$0X_1 + d_{(3,2)} X_2 + d_{(3,3)} X_3 + d_{(3,4)} X_4 \leq R_3$$

(b)    Projects with different start/end dates

Projects with different start/end dates are dealt with in a similar manner as projects of different lengths, just set their capital requirements in any year in which they do not exist (i.e. they have not started or have already ended) to zero. For example if project $X_1$ starts and ends in year 2; project $X_2$ starts in year 2 and project $X_4$ ends in year 2 then the capital requirement constraints become:

$$0X_1 + 0X_2 + d_{(1,3)}X_3 + d_{(1,4)}X_4 \leq R_1 \quad \text{(Year 1)}$$

$$d_{(2,1)}X_1 + d_{(2,2)}X_2 + d_{(2,3)}X_3 + d_{(2,4)}X_4 \leq R_2 \quad \text{(Year 2)}$$

$$0X_1 + d_{(3,2)}X_2 + d_{(3,3)}X_3 + 0X_4 \leq R_3 \quad \text{(Year 3)}$$

(c)    Adding capital inflows from completed projects

If project $X_1$ finishes in year 2, and all of the return from project $X_1$ is available as capital in year 3 then this can be formulated by changing the capital requirement constraint for year 3 to:

$$0X_1 + d_{(3,2)}X_2 + d_{(3,3)}X_3 + d_{(3,4)}X_4 \leq R_3 + d_{(3,2)}X_1$$

We note that we have $0X_1$ above as project $X_1$ finishes in year 2 and hence have no capital requirement in year 3.

A question arises here in that if we put the return from a project into the capital for future years then should we be counting the return from the project in the objective function as well? One way to address this for project $X_1$ here is to say that the return is split into two - one part $y_1$ (say)

that is counted as return taken in the objective and one part $y_2$ (say) that is taken as return used for future capital in year 3. Then amending our formulation we have

$$y_1 + y_2 = d_{(3,1)} X_1$$

a balancing equality equation to correctly account for the return (since we may choose not to do project $X_1$)

$$0X_1 + d_{(3,2)}X_2 + d_{(3,3)}X_3 + d_{(3,4)}X_4 \le R_3 + y_2$$

to account for the capital added in year 3

Maximize $Z = y_1 + P_2X_2 + P_3X_3 + P_4X_4$

to account for the return declared as profit.

We note that $y_1$ and $y_2$ (both $\ge 0$) are continuous (fractional) variables so adding them in this way means that we have a mixed-integer program (MIP).

If we were to solve this MIP numerically then we would find the optimal split between taking the return from project $X_1$ as return in the objective ($y_1$) and reinvesting it as available capital in year 3 ($y_2$).

(d)     Projects with staged returns

In this extension we have that a project gives a return at various stages over its lifetime, and this return can (perhaps) be used as capital to fund ongoing (or new) projects. To illustrate how this can be formulated consider project $X_1$ which gives a total return of $P_1$. Suppose now that this project gives a return of $q$ (such that $q + S = P_1$) at the end of year 2, and the remaining return of

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*S* in year 3. Suppose further that all of this "early" return can be used as available capital in year 3. Then the capital requirement constraint in year 3 becomes:

$$d_{(3,1)}X_1 + d_{(3,2)}X_2 + d_{(3,3)}X_3 + d_{(3,4)}X_4 \leq R_3 + q$$

(e)    Carrying unused capital forward from year to year

In the example as currently formulated we have capital available in each year ($R_1$ in year 1, $R_2$ in year 2 and $R_3$ in year 3). In any particular year all of this capital may not be consumed by the projects that we choose to do. Suppose that we are allowed to carry forward (from year to year) *r%* of any capital that is unused. To formulate this introduce linear (fractional) variables $C_1$ and $C_2$ ($\geq 0$) with $C_1$ being the unused capital in year 1 and $C_2$ being the unused capital in year 2. Then the constraints of the problem become:

$$d_{(1,1)}X_1 + d_{(1,2)}X_2 + d_{(1,3)}X_3 + d_{(1,4)}X_4 + C_1 = R_1 \text{ (Year 1)}$$

$$d_{(2,1)}X_1 + d_{(2,2)}X_2 + d_{(2,3)}X_3 + d_{(2,4)}X_4 + C_2 = R_2 + 0.01C_1r \text{ (Year 2)}$$

$$d_{(3,1)}X_1 + d_{(3,2)}X_2 + d_{(3,3)}X_3 + d_{(3,4)}X_4 = R_3 + 0.01C_2r \text{ (Year 3)}$$

Note here that in years 1 and 2 we have made use of the equality relationship:

Capital used + unused capital = capital available

We note also how we have introduced additional variables ($C_1$ and $C_2$) that make our task of formulating the problem easier.

(f)    Mutually exclusive projects (can have one or the other but not both)

Suppose that projects $X_3$ and $X_4$ are mutually exclusive, i.e. we can choose to do one, or other, of these projects but not both. Then this can be formulated by adding to the problem the constraint:

$X_3 + X_4 \leq 1$

This allows us to do neither of the projects $(X_3 = X_4 = 0)$. If we wish to insist that we must do exactly one of these projects we can use $X_3 + X_4 = 1$

(g)    Projects with a time window for the start time

Suppose that project $X_1$ can start either in year 1, when it has the characteristics given above, or in year 2, when it has a different characteristic - still the same return of $P_1$ but a capital requirement of $k_1$ in year 2 and $k_2$ in year 3, where $k_1$, $k_2$ are different capital requirements. Then this can be formulated by introducing a new zero-one variable $y$ with $y = 1$ representing choosing to do project $X_1$ starting in year 2, $y = 0$ representing not choosing to do project $X_1$ starting in year 2. Then the capital requirement constraints for years 2 and 3 become:

$d_{(2,1)}X_1 + d_{(2,2)}X_2 + d_{(2,3)}X_3 + d_{(2,4)}X_4 + k_1 y \leq R_2$ (Year 2)

$d_{(3,1)}X_1 + d_{(3,2)}X_2 + d_{(3,3)}X_3 + d_{(3,4)}X_4 + k_2 y \leq R_3$ (Year 3)

We also need to add a constraint to prevent project $X_1$ being started at more than one start time (i.e. project $X_1$ starting in year 1 and project $X_1$ starting in year 2 are mutually exclusive)

$X_1 + y \leq 1$

And the objective becomes

Maximize $Z = P_1 X_1 + P_2 X_2 + P_3 X_3 + P_4 X_4 + P_1 y$

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SUMMARY OF IMPORTANT FORMULAS

Payback Period (PBP)

This is the cost of investment divided by the cash flow period.

i.e. $PBP = \dfrac{\cos t \ of \ investment}{cash \ flow \ period}$ ……………………………… (1)

Net Present Value (NPV)

The Net Present Value (including the time of money) of initial and future flows is given by the equation

$$NPV = \sum_{t=1}^{N} \left( \frac{C_t}{(1+r_P)^t} \right)$$ ……………………………………….(2)

Internal Rate of Return

This is the interest or discount rate of which the future net cash flows equals the initial cash outlay.

$$O = \sum_{t=1}^{N} \left[ \frac{C_t}{(1+IRR)^t} \right] - C_o$$ …………………………………… (3)

Profitability Index (P.I)

This is the NPV per unit initial investment.

i.e. $Profitability\ Index = \dfrac{NPV}{Initial\ Investment}$ … ………...... (4)

v.        Net Cash Flow = Cash Inflow − Cash Outflow ……. (5)

**ILLUSTRATIONS**

Use of PBP, NPV and IRR to Evaluate Two Potential Projects

Two alternative projects are under consideration. Project A has a project life of ten (10) years and requires an initial investment of *GH¢100 million* with an annual cash flow after taxes of *GH¢ 20 million* per year for each of four (4) years followed by *GH¢ 10 million* for years five through to ten. Project B has a life of ten (10) years and requires the same investment but has cash flows of *GH¢ 15 million* per year. Based on this information, evaluate projects A and B using:

Payback Period

Internal Rate of Return

Net Present Value, assuming an interest rate of ten percent (10%).

**SOLUTION**

The respective payback periods are:

Project A

It requires four years at *GH¢ 20 million* plus two years at *GH¢ 10 million* or a total of six years to recover the investment.

Project B

$$Payback\ Period = \frac{100\ million}{15\ million} = 6.67\ years$$

These payback periods are quite close.

To find the NPV of the two periods we calculate using equation (2).

Project A

$$NPV = \frac{100}{(1+0.10)^0} + \frac{20}{(1+0.10)^1} + \frac{20}{(1+0.10)^2} + \frac{20}{(1+0.10)^3} + \frac{20}{(1+0.10)^4} + \sum_{k=5}^{10} \frac{10}{(1+0.10)^k} = -GH \not\subset 7.13\ m$$

Project B

$$NPV = \frac{100}{(1+0.10)^0} + \sum_{k=1}^{10} \frac{15}{(1+0.10)^k} = -GH \not\subset 7.83\ m$$

Again the values are quite close.

To find the IRR of two projects we calculate r with NPV 5.0using equation (3)

Project A

$$O = \frac{100}{(1+r)^0} + \sum_{k=1}^{4} \frac{20}{(1+r)^k} + \sum_{k=5}^{10} \frac{10}{(1+r)^k}$$ , the solution is $r = 8.06\%$ annually.

Project B

$$O = \frac{100}{(1+r)^0} + \sum_{k=1}^{10} \frac{15}{(1+r)^K}$$ , the solution is $r = 8.14\%$ annually.

Presumably, neither of the projects would be favorable. Calculations such as made in their case engender a high degree of uncertainty because of the long periods involved, so that a decision between projects, if implemented, is a toss-up.

In general, the payback period and accounting measures are illegitimate shortcuts, and we need to develop proper decision making tools for the problems they try to address. The IRR has a number of serious limitations. Since all criteria are optimal only to the degree that they agree with the NPV rule. It is best to use NPV rule rather than anything else.