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Multi Criteria Ranking of Telecommunication Carriers using Promethee Method

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BY

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DECLARATION

I hereby declare that this submission is my own work towards the MSc. and that, to the best of my knowledge; it contains neither material previously published by another person nor material, which has been accepted for the award of any other degree of the University, except where the acknowledgement has been made in the text.

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DEDICATION

What shall I render unto the Lord for all his benefits towards me? I dedicate this thesis to my husband and mother for their undying love and support that has brought me this far in life.

ACKNOWLEDGEMENT

The Lord is my strength and my song, he has also become my salvation, I thank you almighty God for seeing me through it all even when I felt like given up. I am very grateful to my supervisor Mr Kwaku Darkwa for his immense contributions, advice and support he gave me in making this work a success. I say a very big thank you to the entire staff of Airtel Ghana limited especially the engineers at Network operation Center, Network Optimization and Core Network for given me the necessary advice which has given birth to this work. To my husband Rev. George Appiah Asante, I render unlimited appreciation for taken high interest in my academics. Finally to Mr. Emmanuel Gorh and all those who contributed in diverse ways towards this thesis I say God richly bless you.

ABSTRACT

The selection of a carrier for a Mobile Operator is done based on a myriad of criteria for a given number of telecommunication carriers. There is an urgent need for the switch engineer to have an easy approach in ranking carrier to specific destination with high level of accuracy. Promethee methods have taken an important place among the existing outranking multiple criteria methods. In this thesis the promethee method was used to rank multiple carriers in telecommunication based on given criteria data. Conclusions and recommendations were given based on results from outranking of carriers.

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INTRODUCTION

CHAPTER 1

1.1 Background of Study

Decision making is the study of identifying and choosing alternatives based on the values and preferences of the decision maker. Making a decision implies that there are alternative choices to be considered, and in such a case we want not only to identify as many of these alternatives as possible but to choose the one that best fits with our goals, objectives, desires, values, and so on. (Harris,1980). According to Baker et al. (2001), decision making should start with the identification of the decision maker(s) and stakeholder(s) in the decision, reducing the possible disagreement about problem definition, requirements, goals and criteria.

Routing management is a longstanding problem that confuses almost all telecommunication related companies, includes service operator companies, equipment manufacturers, billing system/platform vendors, and so on. Nowadays, routing management is not only equipment dependent, but is also dependent on the company's operational flow. Unlike Internet Service Provider (ISP) service in which the total network topology is mostly controlled by an ISP provider, telecommunication operators may not own the whole network, and thus need to have contracted partners, including service providers or carriers, to provide full-ranged service to its customers. For these operators, customer calls or user calls are sent according to an internal and pre-deployed routing logic to its service providers or carriers depending upon the destination or the service traffic to available routes, and it can thus be a clear guide for the engineering staff on route deploying for the next period (or billing cycle) to come, while achieving multiple pre-defined goals automatically.

(Chang et al, 2011). A telecom carrier is a company that is authorized by regulatory agencies to operate a telecommunications system. (Paul Ruffolo, august 2000)

Promethee methods have taken an important place among the existing outranking multiple criteria methods. The number of practitioners which are applying this method to practical multi criteria decision problems and researchers who are further developing or are interested in sensitivity aspects of these methods increase year by year (Wim De Keyser and Peter Peeters,1994)

Bharti Airtel, is a leading global telecommunications company with operations in 19 countries across Asia and Africa; has its brand across 16 African countries including Ghana. In Ghana Airtel replaced Zain Ghana in June,2008. With the unveiling of the new brand identity, Airtel becomes the master brand for all the group's 19 operations covering over 200 million customers.
Airtel Ghana, bring together all its operations under a single strong and unique brand identity and also offer its customers high quality products and services; consistent quality of service, reliability, innovation and affordability. Airtel achieve efficiencies and savings through centralised purchasing, and is committed to extend its network deeper into communities which do not have access to Airtel to bridge the digital divide on the continent. The red colour on the Airtel logo reflects the warmth and vibrancy of the African continent, the colour of life and the sun at dusk. "These qualities are reflected in Airtel's brand personality of being brave, sensitive and empathetic," (Philip Sowah, 2008)

1.2 Problem Statement

The switch engineer is faced with an appreciable number of subscriber complaints of calls that are unable to terminate to their destination and ninety nine percent of such complaints consist of international calls that are routed using multiple carriers. The selection of a carrier is done based on a myriad of criteria for a given number of telecommunication carriers namely Bharti India, Belgacom, AT&T, Gateway and British Telecom. There is an urgent need for the switch engineer to have an easy approach in ranking carrier to specific destination with high level of accuracy.

1.3 Objectives

- Model carrier linkage as a multi criteria decision problem
- Rank carriers using Promethee method
- Analyse results from Promethee ranking of carriers
- Draw Technical conclusion and recommendation

1.4 Methodology

There is a high level of competition between network operators in Ghana due to recent introduction of Mobile number portability (MNP), with MNP subscribers can move into any network of their choice without changing their subscriber identity module (SIM). Ghana presently have six network operators (Globacom, Airtel, Vodafone, Expresso, Tigo and MTN) all competing for the same market and any network problems that are avoidable such as switch configuration issues cannot be tolerated.

Network Operators have one goal of maximising profit by providing good services that will retain and attract subscribers and also to reduce operational expenses (OPEX) of the network.

The model is purely mathematical and encompasses decision making methods from Factor rating, Promethee method, and Analytic hierarchy process (AHP). There will be a study of the afore mentioned decision making methods and used to solve the multi criteria ranking of telecommunication carriers problem. The data for this thesis is a collection of actual figures used by network operators as well as those obtained using mathematical tools.

Figures such as per unit cost for carriers are reviewed annually whiles issues pertaining to network quality, answer to seizure ratio (ASR), post dialing delay(PDD) etc varies and can objectively or subjectively be determined.

The subject matter needs both theoretical and practical approach thus there will be personal discussions will engineers that work on telecommunication routing issues. I work by reviewing literature on the subject from the University library and from the internet. Finally software developers and lectures will also be contacted for their input on the subject.

1.5 Justification

No matter how brilliant and invaluable your idea, it is worthless unless you can share it with others. The hallmark of effective communication is the coherent verbal projection of your ideas so that your listener receives the message that you intend to send. (Barbara Stennes, 2007)

The benefits of effective communication are many and obvious as they enhance all aspects of our personal and professional lives. Ineffective or misunderstood communication in our personal lives may give rise to problems or embarrassments but in our professional lives the results of misunderstanding may have much more serious results. (Imo Staff, 2002)

The importance of choosing the best carrier for telecommunication routing cannot be over emphasized, every call drop, bad quality, and high call tariffs in the network has the tendency to cause a subscriber to port to another network and causes financial loss to the operator.

Socially and emotionally there is nothing frustrating as picking a handset to make a call only for that important call to be rejected because of avoidable network issues.

The services provided by telecommunication providers in Ghana are under the monitoring of the National Communication Authority (NCA) and providers are charged penalties for providing poor services in the country. Ghana has an amount of money charged on every successful call made thus any call failure dependent on the network operator causes financial loss to the operator and Ghana as a whole.

1.6 Chapter Organisation

The thesis is to be organised in five chapters, the first chapter is the introductory chapter and contains the background of study, the problem statement, the objectives, methodology and justification of the thesis.

Chapter two is on literature review and here key words in topic, objectives and methodology are reviewed. These include the problem, model, methods used as well as solution to the problem and conclusion.

The Chapter three deals with model formulation and variants, method of solution and illustrative examples.

The Fourth chapter will deal with study and analysis of data collected the thesis winds up in chapter five with conclusions and recommendations.

CHAPTER 2

LITERATURE REVIEW

2.1 Decision Making Problem

According to (Figuera et al., 2005), to make a decision one need to know the problem, the need and purpose of the decision, the criteria of the decision, their sub- criteria, stakeholders and groups affected and the alternative actions to take. One then tries to determine the best alternative, or in the case of resource allocation, needed priorities for the alternatives to allocate their appropriate share of the resources. Decision making has become a mathematical science today, it formalises the thinking one uses so that, what one have to do to make better decisions is transparent in all its aspects,

(Anonymous, 2005) suggested that the decision making process must identify root causes, limiting assumptions, system and organizational boundaries and interfaces, and any stakeholder issues. The goal is to express the issue in a clear, one-sentence problem statement that describes both the initial conditions and the desired conditions The problem statement must however be a concise and unambiguous written material agreed by all decision makers and stakeholders.

(Evangelos, 1995) explained that decision making involves many criteria and sub-criteria used to rank the alternatives of a decision. Not only does one need to create priorities for the alternatives with respect to the criteria or sub-criteria in terms of which they need to be evaluated, but also for the criteria in terms of a higher goal, or if they depend on the alternatives, then in terms of the alternatives themselves. The criteria may be intangible, and have no measurements to serve as a guide to rank the alternatives, and creating priorities for the criteria themselves in order to weigh the priorities of the alternatives and add over all the criteria to obtain the desired overall ranks of the alternatives is a challenging task. In many industrial engineering applications the final decision is based on the evaluation of a number of alternatives in terms of a number of criteria,

2.1.1 Decision Making Steps

(Saaty, 2008) suggested that in order to make a decision in an organised way to generate priorities, the decision can be decompose into the following steps

1) Define the problem and determine the kind of knowledge sought.
2) Structure the decision hierarchy from the top with the goal of the decision, then the objectives from a broad perspective, through the intermediate levels (criteria on which subsequent elements depend) to the lowest level (which usually is a set of the alternatives).
3) Construct a set of pair wise comparison matrices. Each element in an upper level is used to compare the elements in the level immediately below with respect to it.
4) Use the priorities obtained from the comparisons to weigh the priorities in the level immediately below. Do this for every element. Then for each element in the level below add its weighed values and obtain its overall or global priority. Continue this process of weighing and adding until the final priorities of the alternatives in the bottom most level are obtained.

(Janos, 1995) also stipulated that the decision making process can be summarized with the steps below

1) Define the problem: This process must identify root causes, limiting assumptions, system and organizational boundaries and interfaces, and any stakeholder issues. The goal is to express the issue in a clear, one-sentence problem statement that describes both the initial conditions and the desired conditions.
2) Determine requirements: requirements are conditions that any acceptable solution to the problem must meet. Requirements spell out what the solution to the problem must do. In mathematical form, these requirements are the constraints describing the set of the feasible (admissible) solutions of the decision problem.

3) Establish goals: Goals are broad statements of intent and desirable programmatic values. In mathematical form, the goals are objectives contrary to the requirements that are constraints. The goals may be conflicting but this is a natural concomitant of practical decision situations.

4) Identify alternatives: Alternatives offer different approaches for changing the initial condition into the desired condition. Be it an existing one or only constructed in mind, any alternative must meet the requirements.

5) Define criteria to discriminate among alternatives: Decision criteria, which will discriminate among alternatives, must be based on the goals. It is necessary to define discriminating criteria as objective measures of the goals to measure how well each alternative achieves the goals. According to Baker et al. (2001), criteria should be able to

- discriminate among the alternatives and to support the comparison of the performance of the alternatives,
- complete to include all goals,
- operational and meaningful,
- non-redundant,
- Few in number.
- In some methods, Keeney and Raiffa (1976), non-redundancy is required in the form of independency.

6) Select a decision making tool: the selection of an appropriate tool is not an easy task and depends on the concrete decision problem, as well as on the objectives of the decision makers.

7) Evaluate alternatives against criteria: Every correct method for decision making needs, as input data, the evaluation of the alternatives against the criteria. Depending on the criterion, the assessment may be objective (factual), with respect to some commonly shared and understood scale of measurement (e.g. money) or can be subjective (judgmental), reflecting the subjective assessment of the evaluator.

2.1.2 Multi Decision Making

Nemhauser et al, (1989) explained that it is very important to make distinction between the cases whether we have a single or multiple criteria. A decision problem may have a single criterion or a single aggregate measure like cost.

Then the decision can be made implicitly by determining the alternative with the best value of the single criterion or aggregate measure. We have then the classic form of an optimization problem: the objective function is the single criterion; the constraints are the requirements on the alternatives. Depending on the form and functional description of the optimization problem, different optimization techniques can be used for the solution, linear programming, nonlinear programming, discrete optimization, etc.

Steuer , (1986) stipulated that the case when we have a finite number of criteria but the number of the feasible alternatives (the ones meeting the requirements) is infinite belongs to the field of multiple criteria optimization. Also, techniques of multiple criteria optimization can be used when the number of feasible alternatives is finite but they are given only in implicit form. Decision making problems when the number of the criteria and alternatives is finite, and the alternatives are given explicitly are called multi attribute decision making problems.

2.2 Analytic Hierarchy Process (AHP)

Saaty, (1977) introduced the Analytic Hierarchy Process (AHP). AHP is a multi-criteria decision-making approach and has attracted the interest of many researchers mainly due to the nice mathematical properties of the method and the fact that the required input data are rather easy to obtain. The AHP is a decision support tool which can be used to solve complex decision problems. It uses a multi-level hierarchical structure of objectives, criteria, subcriteria, and alternatives. The pertinent data are derived by using a set of pair wise comparisons. These comparisons are used to obtain the weights of importance of the decision criteria, and the relative performance measures of the alternatives in terms of each individual decision criterion. If the comparisons are not perfectly consistent, then it provides a mechanism for improving consistency.

Some of the industrial engineering applications of the AHP include its use in integrated manufacturing (Putrus, 1990), in the evaluation of technology investment decisions (Boucher and McStravic, 1991), in flexible manufacturing systems (Wabalickis, 1988), layout design (Cambron and Evans, 1991), and also in other engineering problems (Wang and Raz, 1991).

(UK DTRL, 2000) researched that the Analytic Hierarchy Process (AHP) was proposed by Saaty (1980). The basic idea of the approach is to convert subjective assessments of relative importance to a set of overall scores or weights. AHP is one of the more widely applied multi attribute decision making methods. The methodology of AHP is based on pair wise comparisons of the following type 'How important is criterion C_i relative to criterion C_j?' Questions of this type are used to establish the weights for criteria and similar questions are to be answered to assess the performance scores for alternatives on the subjective (judgmental) criteria.

The foundation of the Analytic Hierarchy Process (AHP) is a set of axioms that carefully delimits the scope of the problem environment

According to Merkin, (1979) AHP is based on the well-defined mathematical structure of consistent matrices and their associated right eigenvector's ability to generate true or approximate weights, The AHP methodology compares criteria, or alternatives with respect to a criterion, in a natural, pair wise mode. The AHP uses a fundamental scale of absolute numbers that has been proven in practice and validated by physical and decision problem experiments. The fundamental scale has been shown to be a scale that captures individual preferences with respect to quantitative and qualitative attributes just as well or better than other scales (Saaty 1980, 1994). It converts individual preferences into ratio scale weights that can be combined into a linear additive weight for each alternative

The resultant can be used to compare and rank the alternatives and, hence, assist the decision maker in making a choice. Given that the three basic steps are reasonable descriptors of how an individual comes naturally to resolving a multi criteria decision problem, then the AHP can be considered to be both a descriptive and prescriptive model of decision making.

The AHP is perhaps, the most widely used decision making approach in the world today. Its validity is based on the many hundreds (now thousands) actual applications in which the AHP results were accepted and used by the cognizant decision makers (DMs), Saaty (1994).

2.2.1 Applications of Analytic Hierarchy Process

Stuart, (1994) conducted a study on Using AHP for Decision Making in Engineering; some challenges. In many industrial engineering applications the final decision is based on the evaluation of a number of alternatives in terms of a number of criteria. This problem may become a very difficult one when the criteria are expressed in different units or the pertinent data are difficult to be quantified. The Analytic Hierarchy Process (AHP) is an effective approach in dealing with this kind of decision problems. The study examined some of the practical and computational issues involved when the AHP method is used in engineering applications.
Some of the industrial engineering applications of the AHP include its use in integrated manufacturing (Putrus, 1990), in the evaluation of technology investment decisions (Boucher and McStravic, 1991), in flexible manufacturing systems (Wabalickis, 1988), layout design (Cambron and Evans, 1991), and also in other engineering problems (Wang and Raz,1991).

As an illustrative application consider the case in which one wishes to upgrade the computer system of a computer integrated manufacturing (CIM) facility. There is a number of different configurations available to choose from. The different systems are the alternatives. A decision should also consider issues such as: cost, performance characteristics CPU speed, memory capacity, RAM, etc.), availability of software, maintenance, expendability, etc. These may be some of the decision criteria for this problem.

In the above problem we are interested in determining the best alternative (i.e., computer system). In some other situations, however, one may be interested in determining the relative importance of all the alternatives under consideration. For instance, if one is interested in funding a set of competing projects (which now are the alternatives), then the relative importance of these projects is required (so the budget can be distributed proportionally to their relative importance). Multi-criteria decision-making (MCDM) plays a critical role in many real life problems. It is not an exaggeration to argue that almost any local or federal government, industry, or business activity involves, in one way or the other, the evaluation of a set of alternatives in terms of a set of decision criteria. Very often these criteria are conflicting with each other. Even more often the pertinent data are very expensive to collect.

Sanjay et al, (2009) probe into the study of the Analytical Hierarchy Process Applied to Vendor Selection Problem: Small Scale, Medium Scale and Large Scale Industries.
The paper endeavored to investigate the problem of vendor selection using Analytical Hierarchy Process (AHP) in Small-scale industries (SS), Medium-scale industries (MS) and Large-scale industries (LS) under different criteria for the same. In India, industries having investment in plant and machinery less than rupees ten million are called small-scale industries (Singh et al, 2003). Similarly, industries having investments between ten million and one thousand million in plant and machinery are considered as medium scale industries (Karandikar, 1999), whereas for large scale industries, investments in plant and machinery is more than one thousand million has been considered as criteria (Singh et al, 2005). Decision criteria used for vendor selection can be different depending on the size of a buyer organisation. Large companies use a different set of criteria and a formal approach when selecting suppliers compared to small and medium sized enterprises (Pearson et. al., 1995).

AHP makes the selection process very transparent. It also reveals the relative merits of alternative solutions for a Multi Criteria Decision Making (MCDM) problem. (Drake, P.R.,1998).

AHP approach is a subjective methodology (Cheng and Li, 2001); information and the priority weights of elements may be obtained from a decision-maker of the company using direct questioning or a questionnaire method.

It is generally agreed in the literature that the following makes the supplier selection decision making process difficult and/or complicated (de Boer, 1998, Murlidharan et.al, 2001):

The present study presents a comparative analysis of different group aggregation methods adopted in AHP by testing them against social choice axioms with a case study of Delhi transport system. The group aggregation (GA) methods and their correctness were tested while prioritizing the alternative options to achieve energy efficient and less polluting transport system in Delhi.

It was observed that among all group aggregation methods, geometric mean method (GMM) - the most widely adopted GA method of AHP - showed poor performance and failed to satisfy the most popular "pareto optimality and non-dictatorship axiom" raising questions on its validity as GA method adopted in AHP. All other group aggregation methods viz. weighted arithmetic mean method with varying weights and equal weights (WAMM, WeAMM) and arithmetic mean of individual priorities (AMM) resulted in concurring results with the individual member priorities.

The study demonstrated that WeAMM resulted in better aggregation of individual priorities compared to WAMM. Comparative analysis between individual and group priorities demonstrates that the arithmetic mean (AMM) of priorities by individual members of the group showed minimum deviation from the group consensus making it the most suitable and simple method to aggregate individual preferences to arrive at a group consensus.

Maggie et al, (2007) adopted the AHP method in; An application of the AHP in vendor selection of a telecommunications system. Vendor selection of a telecommunications system is an important problem to a telecommunications company as the telecommunications system is a long-term investment for the company and the success of telecommunications services is directly affected by the vendor selection decision.

Furthermore, the vendor selection of a telecommunications system is a complex multi-person, multi-criteria decision problem. The group decision-making process can be improved by a systematic and logical approach to assess priorities based on the inputs of several people from different functional areas within the company. The analytic hierarchy process (AHP) can be very useful in involving several decision-makers with different connecting objectives to arrive at a consensus decision. In their paper, an AHP-based model was formulated and applied to a real case study to examine its feasibility in selecting a vendor for a telecommunications system. The use of the proposed model indicated that it can be applied to improve the group decision making in selecting a vendor that satisfies customer specifications. Also, it was found that the decision process is systematic and that using the proposed AHP model reduced the time taken to select a vendor.

Ivan et al, (2008) conducted a study on Application of AHP Method in Traffic Planning. Achieving competitiveness on the market ensures business continuity within terms of globalization. Consequently, competitiveness is determined by various factors which grading and evaluation require corresponding approach. The final result is a set of information used as basis for making the concrete decisions. Traffic of goods and services has a special importance in ensuring the concrete business, not just in logistic sense. Traffic planning and making decisions relevant to that area directly influence the business. Today there are different methodologies and techniques of planning in field of traffic. The choice of technology usually depends upon business management. Application of AHP method is one of the possibilities that can be used within mentioned circumstances. Their paper analysed the possibilities of applying AHP method in making decisions regarding planning and implementation of plans in traffic and ensuring the qualitative business logistics.

Irfan, (2009) determined the The Application of AHP Model to Guide Decision Makers:

A Case Study of E-Banking Security. In the case of banking industries, better management of information security has been realized as an important factor to ensure safety of all financial transactions.

Under IT management umbrella, he observed several terms such as information technology governance, information security management, and information systems audit. In order to fulfill the requirements, banking industries follow several international standards to comply with, such as COBIT and

ISO 27001. The case study was base on Indonesian banks which have implemented information security policy and audit systems based on COBIT or ISO 27001. COBIT or Control Objectives for Information and related Technology is a framework consists of a set of best practices for IT management with a subset of information security and assurance part. Likewise, ISO 27001 is an international standard for information security management with best practice recommendations on information security management, as well as risks and controls within the context of an overall Information Security Management System (ISMS). Deciding appropriate information security policy is not an easy task since there are many aspects should be considered appropriately. Therefore, there is a strong requirement to assist evaluation in this field. He therefore proposed an evaluation method based on Analytic Hierarchy Process (AHP) which considered all relevant aspects of information security as a guidance framework.

2.3 The Promethee Method

Brans, (1982) developed the Promethee I (partial ranking) and Promethee II (complete ranking) and presented for the first time in 1982 at a conference organised by R. Nadeau and M. Landry at the Université Laval, Québec, Canada (L'Ingéniérie de la Décision. Elaboration d'instruments d'Aideà la Décision).

The same year several applications using this methodology were already treated by G. Davignon in the field of Heath care. A few years later J.P. Brans and B. Mareschal developed Promethee III (ranking based on intervals) and Promethee IV (continuous case). The same authors proposed in 1988 the visual interactive module GAIA which is providing a marvellous graphical representation supporting the Promethee methodology.

Brans and Mareschal, (1992) further suggested two nice extensions: Promethee V (including segmentation constraints) and Promethee VI (representation of the human brain).

A considerable number of successful applications has been treated by the Promethee methodology in various fields such as Banking, Industrial Location, Manpower planning, Water resources, Investments, Medicine, Chemistry, Health care, Tourism, Ethics in OR, Dynamic management. The success of the methodology is basically due to its mathematical properties and to its particular friendliness of use.

The decision table is the starting point of the Promethee methodology introduced by Brans and Vincke (1985) and Brans et al. (1986). The scores a_{ij} need not necessarily be normalized or transformed into a common dimensionless scale.

We only assume that, for the sake of simplicity, a higher score value means a better performance. It is also assumed that the weights w_i of the criteria have been determined by an appropriate method (this is not a part of the Promethee methods

Preference function based outranking method is a special type of MCDM tool that can provide a ranking ordering of the decision options. The Promethee (preference ranking organization method for enrichment evaluation) method was developed by Brans and Vincke in 1985. The Promethee I method can provide the partial ordering of the decision alternatives, whereas, Promethee II method can derive the full ranking of the alternatives.

2.3.1 Applications of Promethee method

Ali, (2011) used the Promethee method combined with multi-objective linear programming (MOLP) to develop a model for outsourcing. The model was built based on two phases. First, with using Promethee, we start to rank the priority of our partners. In the second phase, we assign the products to partner with multi-objective linear programming based on the priorities that was earned from the first phase. Minimizing costs and defect products, maximizing on time delivery and referring demands to better suppliers are the major goals in this article.

The model is improved to solve the problem of a company that periodically purchases different products from different suppliers to fulfill its aggregate demand.

Each of suppliers can provide a few products. However, it was expected that these suppliers have different characteristics.

The supplier evaluation and order allocation plan of the company is a strategic issue. In general, these plans are made for a time period of at least 6-12 months, due to economic and market conditions, because of difficulty in determine the environmental coefficient and related parameters, especially in a medium time horizon

In this study, order allocation methodology focuses on developing the MOLP approach to allocate order quantities to satisfy the aggregate demand of a company considering prices, rejected units rate, on time delivery, and suppliers ranks calculated from the PROMETHEE.

Constanta, (2005) adopted the Promethee method together with the AHP to study Water Resources Planning. The projects goal is the rational water resources management of Nestos River in relation to the operation of two recently constructed dams. The management of the water supply system should balance the needs for irrigation, the needs of the Public Electrical Corporation for hydropower generation, as well as environmental requirements given the presence of valuable natural ecosystems in the area. In order to evaluate the projects, the Analytic Hierarchy Process (AHP) and PROMETHEE multi criteria methods are used. The projects evaluation is based on economic, social, environmental and cost criteria. Alternative scenarios on the availability of water resources are also incorporated in the model.

The management of the water supply system should balance the needs for irrigation, the needs of the Public Electrical Corporation for hydropower generation, as well as environmental requirements given the presence of valuable natural ecosystems in the area. Water management covers a wide range of activities, in which technical, economic, environmental and social issues are involved. Since several groups with divergent interests are also concerned in determining the public resources management, human value and judgment systems are parts of the decision problem. Therefore, the elements to be considered in designing an efficient strategy are numerous, and their relationships are extremely complicated and highly nonlinear. Given the complexity of the decision process, much attention has been paid to multiple criteria decision-making (MCDM) approaches in order to enhance the ability to make sound decisions in water resources management.

The evaluation of the irrigation projects in this study was based on a multi criteria analysis carried out via two multi criteria methods: the Analytic Hierarchy Process (AHP) and the PROMETHEE method The application of the methods was supplied by data deriving from a Ministry of Agriculture study as well as from a recent study realized by Democritus University of Thrace in which the economic, social, and environmental characteristics of the region are fully analyzed. The necessary computations were realized with Expert Choice 9.0 and Decision Lab, two software packages developed for AHP and PROMETHEE methods respectively.

Shankar, (2010) adopted the Promethee II method in Facility Location Selection. Selecting a location for a new organization or expansion of an existing facility is of vital importance to a decision maker. The cost associated with acquiring the land and facility construction makes the facility location a long-term investment decision. The best location is that which results in higher economic benefits through increased productivity and good distribution network. Selecting the proper facility location from a given set of alternatives is a difficult task, as many potential qualitative and quantitative criteria are to be considered. This paper solves a real time facility location selection problem using PROMETHEE II (preference ranking organization method for enrichment evaluation) method which is an effective multi-criteria decision-making (MCDM) tool often applied to deal with complex problems in the manufacturing environment.

Slavica et al, (2008) probed into the use of Promethe method in studying the Muti Criteria Decision in the Choice of Advertising tools. The illustration of the practical application of this method to a hypothetical example of item-selection in the branding of a new product is done. Using the calculations obtained by the Decision Lab Program, the analysis of the achieved results is done and the decision about the most effective branding item is made.

Most of the economic, political, financial and industrial problems are multi-criteria ones. The issue about the choice and ranking of alternatives is not easy, at all, to solve. In particular, there is no optimal solution; neither alternative is the best for every criterion.

No one buys a car only because of the price, conformity or quality; the performance and the prestige are also considered. On the other hand, we do not react in the same way while taking into consideration some criterion.

The choice is subjected to the taste of the individuals. Better quality implies a higher price. A compromise should be considered, but it should be the most beneficial compromise.

Recently, some decision aid methods have been developed, that is, decision support systems, whose role is to help the decision maker, in the process of selecting the most optimal alternative.

PROMETHEE-GAIA methodology is known as the most efficient and the easiest to use among the methodologies in the field of multi-criteria decisions.

Also, the newly-developed software, available to the individuals, is called the Decision Lab. The software has been developed together with the Canadian company Visual Decision, as a support in decision-making, and is applicable to all problems.

Masoud et al, (2009) developed a model for Selecting Suitable Semantic Web Service Composition, Using Promethee Method and Non-Functional Parameters. So far many methods are introduced to compose Web services. In order to automate Web service composition, concepts from semantic Web technology are used. By using this technology, we can compose Web services in order to response users' request automatically or semi-automatically. By the growth of Web services amount, the possible composition of Web services which can satisfy users' requests will increase. In this paper a new classification of Web service non-functional parameters is introduced and a new approach is proposed to select more suitable semantic Web service composition among all alternatives. The PROMETHEE method selects the best composition based on non-functional parameters and user's preferences. PROMETHEE is a method for multi criteria decision making method which can be used in our approach with good performance.

The non-functional parameters are gathered from researches in the domain of Web services non-functional parameters, performance and Quality os Service (QoS). In the first level we have the hierarchy root which is called selecting best composition based on non-functional parameters. In the second level, three basic parameters quality of service, provider and costs are introduced. In the third level each of the three above parameters are divided into their sub-parameters. QoS divided into performance, reliability, availability, accessibility, integrity, regularity, scalability and resources.

Provider includes profile, policy and number of service. Profile has some information about provider and number of services means how many service this provider has developed. Costs divided into three main views. Rental costs which includes information about renting services costs, buy costs that has information about purchasing prices and invocation costs. Most of parameters in the level 3 will break into some sub-parameters.

Harry et al, (2009) used the multi criteria decision aid approach to study; A PROMETHEE based uncertainty analysis of UK police force performance rank improvement.

The police forces in the UK are periodically compared with each other on their performance, by government and non-government bodies. The study demonstrated the employment of PROMETHEE in an investigation of the targeted performance rank improvement of individual UK police forces (with their 'most similar forces' groups). The graphical representations presented offer an insight into the implications of such a PROMETHEE based series of perceived improvement analyses. The goals of this study are twofold, firstly to exposit PROMETHEE based uncertainty analysis in rank improvement and secondly, how the subsequent results can form part of the evidence to aid in their performance strategies.

Tien et al, (2004) conducted a research on the; Application of the PROMETHEE technique to determine depression outlet location and flow direction in DEM.

With the fast growing progress of computer technologies, spatial information on watersheds such as flow direction, watershed boundaries and the drainage network can be automatically calculated or extracted from a digital elevation model (DEM).

The stubborn problem that depressions exist in DEMs has been frequently encountered while extracting the spatial information of terrain. Several filling-up methods have been proposed for solving depressions. However, their suitability for large-scale flat areas is inadequate. This study proposes a depression watershed method coupled with the Preference Ranking Organization Method for Enrichment Evaluations (PROMETHEEs) theory to determine the optimal outlet and calculate the flow direction in depressions. Three processing procedures are used to derive the depressionless flow direction: (1) calculating the incipient flow direction; (2) establishing the depression watershed by tracing the upstream drainage area and determining the depression outlet using PROMETHEE theory; (3) calculating the depressionless flow direction.

The developed method was used to delineate the Shihmen Reservoir watershed located in Northern Taiwan. The results show that the depression watershed method can effectively solve the shortcomings such as depression outlet differentiating and looped flow direction between depressions. The suitability of the proposed approach was verified.

2.4 Factor Rating Method

According to Amponsah and Darkwa, (2011) the factor rating method is popular because a wide variety of factors from education to recreation to labour skill can be objectively included. Sources have suggested several factors that have deemed as important enough to be included in the factor rating method. Although they are not the only ones that can be included the equation, they are there to give the prospective firm a starting point. The suggested factors include: labour, cost, labour availabity, proximity to raw materials and suppliers, proximity to markets, state and local governments fiscal policies, environmental regulation, utilities, site cost, transportation, quality of life issues within the community , foreign exchange and quality of government. When using factor rating method, the following steps must be followed strictly and religiously. These are:

1. Develop a list of relevant factors
2. Assign weight to each factor to reflect its relative importance in the company's objectives.
3. Develop a scale for each factor (for example 1 to 10, or 1 to 100 points)
4. Have management or related people score each relevant factor using the scale developed in step three.
5. Multiply the score by the weight assigned to each factor and total the score for each location.
6. Make a recommendation based on the maximum point score, considering the result of qualitative approaches as well.

2.4.1 Applications of Factor Rating Method

(Dan et al, 2010) dealt into a presentation on Capacity Planning and Facility Location Using the Factor Rating Method. In their study they outlined that Capacity is the maximum output rate of a production or service facility. Capacity planning is the process of establishing the output rate that may be needed at a facility: Capacity is usually purchased in "chunks". Strategic issues to be considered as: how much and when to spend capital for additional facility & equipment.

Tactical issues concerned about are workforce & inventory levels, & day-to-day use of equipment. Facility location is the process of identifying the best geographic location for a service or production facility. They stipulated the below as steps in the Factor Rating Method

1. Develop a list of relevant factors.
2. Assign a weight to each factor reflecting its relative importance to the firm.
3. Develop a rating scale for the factors.
4. Score each location on each factor based on the scale.
5. Multiply the scores by the weights for each factor and total the weighted scores for each location.

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Nkumbwa, (2010) probed into Facility Location Strategies using the Factor Rating Method. It was explained that Location contributes Up to 25% of the product's selling cost and once a company commits to a location, many costs are fixed and difficult to change such as energy and Labor. Location depends on the type of business; Manufacturing – minimizing cost, Retail and professional services – maximizing revenue, Warehouse – cost and speed of delivery.

He outlined the following steps as essential in solving any problem using the Factor Rating Method.

1. State relevant factors in terms of "max" or "min"
2. Assign weights to each factor (should add to 100%)
3. Assign rating to each factor (1-5) (1=poor, 5=excellent)
4. Multiply scores by weights for each factor & total
5. Calculate percent of total
6. Compare top 2 alternatives (using percent as a basis of comparison)

2.5 Multi Attribute Utility Theories (MAUT)

According to Torrance, (2011) Multi Attribute Utility Theory (Keeney and Raiffa and Farquhar) is concerned with expressing the utilities of multiple-outcomes or consequences as a function of the utilities of each attribute taken singly. The theory specifies several possible functions (additive, multiplicative and multilinear) and the conditions (independence conditions to be met) under which each would be appropriate. As a practical matter Keeney and Raiffa suggest that for four or more attributes the reasonable models to consider are the additive and the multiplicative. Standard MAULT has been developed for the case of a single decision maker or a single decision making unit.

Josias, (2006) researched that The MAUT approach is an attempt to rigorously apply objective measurement to decision making. The basic hypothesis of MAUT is that in any decision problem, there exists a real valued function or utility (*U*), defined by the set of feasible alternatives that the decision-maker seeks, consciously or not, to maximize (Olson, 1996). Each alternative results in an outcome, which may have a value on a number of different dimensions. MAUT seeks to measure these values, one dimension at a time, followed by an aggregation of these values across the dimensions through a weighting procedure. The simplest and most widely used aggregation rule is to take the weighted linear average. In this case, each weight is used in conjunction with each criterion value to produce the final utilities. it allows the decision-maker to allocate relative weights to the various criteria (Mickelson, 1998).

Dyer et al, (1992) explained that On the MAUT side, the additive value model for multiple objectives of Churchmanand Ackoff (1954) and others was axiomatized by Debreu (1960), Luce and Tukey (1964). Krantz (1964). and Scott (1964).

Later summaries and extensions appear in Krantz, Luce, Suppes and Tversky (1971) and Wakker (1989). The earlier axiom systems were followed by axioms for multi attribute models in expected utility theory by Pollak (1967). Keeney (1968), Fishburn (1970) and others. A good summary is given by Keeney and Raiffa (1976) and a synthesis of these models appears in Dyer and Sarin (1979).

According to Ralph, (2009) the Multi-Attribute Utility Theory is an evaluation scheme which is very popular by consumer organizations for evaluating products. According to MAUT, the overall evaluation v(x) of an object x is defined as a weighted addition of its evaluation with respect to its relevant value dimensions. The common denominator of all these dimensions is the utility for the evaluator. For example, a digital camera can be evaluated on the value dimensions quality of image, flash, viewfinder, operation time, and handling.

Dr. Yan, (2011) discussed that MAUT uses utility functions to convert numerical attribute scales to utility unit scales. Assign weights to these attributes and then calculate the weighted average of each consequence set as an overall utility score and compare alternatives using the overall utility score

Winterfeldt, et al, (1973) researched that Multi-attribute utility theory (MAUT) combines a class of psychological measurement models and scaling procedures that can be applied to the evaluation of alternatives with multiple value relevant attributes. For example, MAUT can be used to analyze preferences between cars described by the attributes cost, comfort, prestige, and performance. MAUT may also be applied as a decision aiding technology for decomposing a complex evaluation task into a set of simpler subtasks. For example, the decision maker might be asked to assess the utility of each alternative with respect to each attribute and to assign importance of weights to each attribute. Then an appropriate combination rule is used to aggregate utility across attributes. Two major theoretical approaches to multi-attribute utility assessment have been developed. Both provide an axiomatic justification for the existence of a utility function over multi-attributed an alternative which decomposes into single attribute utility functions.

The approaches to the representations, however, differ substantially. The theory of conjoint measurement (Krantz, 1964; Luce and Tukey, 1964; Krantz, Luce, Suppes, and Tversky, 1971) simultaneously constructs the overall and single attribute utility functions. The conjoint measurement representation preserves the decision maker's preference ordering for riskless decisions, but it cannot necessarily be applied to decision under risk, where alternatives are not only multi-attributed but also uncertain.

Multi-attribute expected utility theory (Fishburn, 1965, 1970; Keediey, 1969, 1971, 1973; Raiffa, 1969), on the other hand, was explicitly designed for decisions under risk. The utility function U obtained with this approach not only preserves the decision maker's riskless preference order, but also may be used in expected utility computations to select among risky alternatives.

2.51 Applications of Multi Attribute Utility Theory (MAUT)

Edwards, (1971) typically worked with simple additive models. The argument for MAUT as a decision technology goes as follows: Since the evaluation of multi-attributed alternatives is often difficult, leading to inconsistent judgments and simplistic strategies, the choice problem is first structured by determining the basic dimensions of importance. Then the evaluation task is decomposed into the evaluation of each alternative with respect to each attribute, and the estimation of importance weights for the different attributes. Weights and single attribute utility functions are aggregated using a weighted additive model to generate an overall evaluation.

Torrance, (1982) used MAUT to study Application of Multi-Attribute Utility Theory to Measure Social Preferences for Health States.The health states in this study are defined according to the four attribute health state classification system. The system was developed to classify and to follow for life, the health outcomes of randomly selected infants in an evaluation of neonatal intensive care (Boyle et al, 1982) All selected children (age rang 2-15 years) have had their current health status classified, their past health pattern reconstructed and their future health pattern forecast using the health state classification system. The children represent a wide variety of disabilities, mostly chronic. Each attribute in the Health State Classification System is subdivided into a number of levels such that each person can be classified at every point in time into one level on each attribute. A social preference function defined over the health states is required. Since each feasible combination of attribute levels defines a a unique health state, the system implicitly includes a large number of different states; too many to measure preference explicitly using holistic utility assessment thus MAUT was used.

Josias, (2006) researched on Transportation corridor decision-making with multi-attribute utility theory. In his research he provided a description of how decisions concerning transportation programmes and projects can be made in the context of sustainable transportation. He provided information on identifying appropriate performance measures for sustainable transportation and then quantifying these measures with a traffic simulation model as well as transportation environmental models. The quantified performance measures were then used with three decision making methodologies. The test bed used for this study comprised a transportation corridor in Tshwane, South Africa and one in Houston, Texas. A method based on the multi-attribute utility theory (MAUT) techniques was found to be the best because a broad range of quantitative and qualitative sustainability issues can be included in the decision-making process.

Ralph, (2009) studied Rules for Using Multi-Attribute Utility Theory for Estimating a User's Interests. In his study, he showed that Multi-Attribute Utility Theory (MAUT), a prescription for evaluating objects, can be ascribed as evaluation process to a user when estimating the user's interests and proposed some rules for the application of MAUT.

Kabassi et al, (2006) researched on Multi-Attribute Utility Theory and Adaptive Techniques for Intelligent Web-Based Educational Software. The paper described how the Multi-Attribute Utility Theory can be combined with adaptive techniques to improve individualized teaching in an Intelligent Learning Environment (ILE). The ILE is called Web F-SMILE; it operates over the Web and is meant to help novice users learn basic skills of computer use. Tutoring is dynamically adapted to the individual learner based on the learner modeling component of the system and the Multi-Attribute Utility Theory (MAUT) that is employed to process the information about the user. As a result, MAUT provides a way for the system to select on the fly the best possible advice to be presented to users. Advice is dynamically formed based on adaptive presentation techniques, where adaptation is performed at the content level and adaptive navigation support, which is performed at the link level of the hyperspace of the tutoring system. The adaptivity of learning depends on factors such as the learner's habits, prior knowledge and skills, which are used as criteria for the application of MAUT in the educational software.

In this way, a novel combination of MAUT with adaptive techniques is used for intelligent web-based tutoring

CHAPTER THREE

METHODOLOGY

3.1 The Multi decision Making Problem

There are a lot of methods used in solving multi decision making problems, this chapter will probe into these methods and provide sample problems and their solutions.

Consider a multi-attribute decision making problem with m criteria and n alternatives. Let $C_1,..,C_m$ and $A_1,..,A_n$ denote the criteria and alternatives, respectively. A standard feature of multi-attribute decision making methodology is the decision table as shown in Figure 3.1 below. Each row belongs to a criterion and each column describes the performance of an alternative. The score a_{ij} describes the performance of alternative A_j against criterion C_i. For the sake of simplicity the assumption that a higher score value means a better performance is made since any goal of minimization can be easily transformed into a goal of maximization.

As shown in decision table Figure 3.1, weights $w_1,..., w_m$ are assigned to the criteria. Weight w_i reflects the relative importance of criteria C_i to the decision, and is assumed to be positive. The weights of the criteria are usually determined on subjective basis. They represent the opinion of a single decision maker or synthesize the opinions of a group of experts using a group decision technique. The values $a_{ij},..., a_{in}$ associated with the alternatives in the decision table are used in the Multi Attribute Utility Theory (MAUT) methods (see Figure 3.1) and are the final ranking values of the alternatives. Usually, higher ranking value means a better performance of the alternative, so the alternative with the highest ranking value is the best of the alternatives.

		x_1	.	.	x_n
		$\mathbf{A}_1$	.	.	$\mathbf{A}_n$
w_1	$\mathbf{C}_1$	a_{11}	.	.	a_{m1}
.	.	.	.	.	.
.	.	.	.	.	.
w_m	$\mathbf{C}_m$	a_{m1}	.	.	a_{mn}

Figure 3.1: Multi decision Making model

Multi-attribute decision making techniques can partially or completely rank the alternatives: a single most preferred alternative can be identified or a short list of a limited number of alternatives can be selected for subsequent detailed appraisal. Besides some monetary based and elementary methods, the two main families in the multi-attribute decision making methods are those based on the Multi Attribute Utility Theory (MAUT) and Outranking methods.

The family of Multi Attribute Utility Theory (MAUT)) methods consist of aggregating the different criteria into a function, which has to be maximized. Thereby the mathematical conditions of aggregations are examined. This theory allows complete compensation between criteria, i.e. the gain on one criterion can compensate the loss on another (Keeney and Raiffa, 1976).

The concept of outranking was proposed by Roy (1968). The basic idea is as follows. Alternative A_i outranks A_j if on a great part of the criteria A_i performs at least as good as A_j (concordance condition), while its worse performance is still acceptable on the other criteria (non-discordance condition). After having determined for each pair of alternatives whether one alternative outranks another, these pair wise outranking assessments can be combined into a partial or complete ranking. Contrary to the Multi Attribute Utility Theory (MAUT) methods, where the alternative with the best value of the aggregated function can be obtained and considered as the best one, a partial ranking of an outranking method may not render the best alternative directly. A subset of alternatives can be determined such that any alternative not in the subset be outranked by at least one member of the subset.

The aim is to make this subset as small as possible. This subset of alternatives can be considered as a shortlist, within which a good compromise alternative should be found by further considerations or methods.

Triantaphyllou and Mann (1989) modelled the structure of the typical decision problem to consists of a number, say *M*, of alternatives and a number, say *N*, of decision criteria. Each alternative can be evaluated in terms of the decision criteria and the relative importance (or weight) of each criterion can be estimated as well.

Let a_{ij} (i=1,2,3,...,*M*, and *N*=1,2,3,...,*N*) denote the performance value of the *ith* alternative (i.e., A_i) in terms of the *j-th* criterion (i.e., C_j). Also denote as W_j the weight of the criterion C_j. Then, the core of the typical Multi Criteria Decision Making (MCDM) problem can be represented by the following decision matrix:

		Criterion				
		C_1	C_2	C_3	…	C_N
		W_1	W_2	W_3	…	W_N
Alt	A_1	a_{11}	a_{12}	a_{13}	…	a_{1N}
	A_2	a_{21}	a_{22}	a_{23}	…	a_{2N}
	A_3	a_{31}	a_{32}	a_{33}	…	a_{3N}
	.	.	.	.	.	.
	.	.	.	.	.	.
	A_M	a_{M1}	a_{M2}	a_{M3}	…	a_{MN}

Figure 3.2: Triantaphyllou and Mann MCDM

Given the above decision matrix, the decision problem considered in this study is how to determine which is the best alternative. A slightly different problem is to determine the relative significance of the *M* alternatives when they are examined in terms of the *N* decision criteria combined. In a simple Multi Criteria Decision Making (MCDM) situation, all the criteria are expressed in terms of the same unit (example, dollars).

However, in much real life Multi Criteria Decision Making (MCDM) problems different criteria may be expressed in different dimensions. Examples of such dimensions include dollar figures, weight, time, political impact, environmental impact, etc. It is this issue of multiple dimensions which makes the typical Multi Criteria Decision Making (MCDM) problem to be a complex one.

3.2 Analytic Hierarchy Process (AHP) Theory

According to Geoff, (2004) Considering n elements to be compared, $C_1 \dots C_n$ and denote the relative 'weight' (or priority or significance) of C_i with respect to C_j by a_{ij} and form a square matrix $A=(a_{ij})$ of order n with the constraints that $a_{ij} = 1/a_{ji}$, for $i \neq j$, and $a_{ii} = 1$, all i.

Such a matrix is said to be a reciprocal matrix. The weights are consistent if they are transitive, that is $a_{ik} = a_{ij}a_{jk}$ for all *i*, *j*, and *k*. Such a matrix might exist if the a_{ij} are calculated from exactly measured data.

Then find a vector ω of order *n* such that $A\omega = \lambda\omega$. For such a matrix, ω is said to be an eigen vector (of order n) and λ is an eigen value. For a consistent matrix, $\lambda = n$.

For matrices involving human judgement, the condition $a_{ik} = a_{ij}a_{jk}$ does not hold as human judgements are inconsistent to a greater or lesser degree. In such a case the ω vector satisfies the equation $A\omega = \lambda_{max}\omega$ and $\lambda_{max} \geq n$. The difference, if any, between λ_{max} and n is an indication of the inconsistency of the judgements. If $\lambda_{max} = n$ then the judgements have turned out to be consistent. Finally, a Consistency Index can be calculated from $(\lambda_{max}-n)/(n-1)$. That needs to be assessed against judgments made completely at random and (Saaty, 1980) has calculated large samples of random matrices of increasing order and the Consistency Indices of those matrices. A true Consistency Ratio (CR) is calculated by dividing the Consistency Index for the set of judgments by the Index for the corresponding random matrix. (Saaty, 1980) suggests that if that ratio exceeds 0.1 the set of judgments may be too inconsistent to be reliable. In practice, CRs of more than 0.1 sometimes have to be accepted. A CR value of 0 means that the judgements are perfectly consistent.

(Anagnostopolous, 2005) determined that hierarchy evaluation is based on pair wise comparisons. The decision-maker compares two alternatives A_i and A_j using a criterion and assigns a numerical value to their relative weight. The result of the comparison is expressed in a fundamental scale of values ranging from 1 (A_i, A_j contribute equally to the objective) to 9 (the evidence favoring A_i over A_j) the highest possible order of affirmation. Given that the n elements of a level are evaluated in pairs using an element of the immediately higher level, an nxn comparison matrix is obtained.

A comparison matrix is consistent if and only if $a_{ij}*a_{jk} = a_{ik}$ for all i, j, k. AHP measures the inconsistency of judgments by calculating the consistency index CI of the matrix, as

$$CI = (n_{max} - n)/(n-1) \tag{3.1}$$

The consistency index CI is in turn divided by the average random consistency index RI to obtain the consistency ratio defined as

$$CR = CI / RI \quad (3.2)$$

The RI index is a constant value for an *n*x*n* matrix, which has resulted from a computer simulation of *n*x*n* matrices with random values from the 1-9 scale and for which $a_{ij} = 1/a_{ji}$. If CR is less than 5% for a 3x3 matrix, 9% for a 4x4 matrix, and 10% for larger matrices, then the matrix is consistent.

$$RI = \frac{1.98(n-2)}{n} \quad (3.3)$$

Once its values are defined, a comparison matrix is normalized and the local priority (the relative dominance) of the matrix elements with respect to the higher level criterion is calculated.

The overall priority of the current level elements is calculated by adding the products of their local priorities by the priority of the corresponding criterion of the immediately higher level. Next, the overall priority of a current level element is used to calculate the local priorities of the immediately lower level which use it as a criterion, and so on, till the lowest level of the hierarchy is reached. The priorities of the lowest level elements (alternatives) provide the relative contribution of the elements in achieving the overall goal.

To make comparisons, we need a scale of numbers that indicates how many times more important or dominant one element is over another element with respect to the criterion or property with respect to which they are compared.

Table 3.2 exhibits the scale. Table 3.1 exhibits an example in which the scale is used to compare the relative consumption of drinks. One compares a drink indicated on the left with another indicated at the top and answers the question: How many times more, or how strongly more is that drink consumed than the one at the top? One then enters the number from the scale in Table 3.2 that is appropriate for the judgment: for example enter 9 in the (coffee, wine) position meaning that coffee consumption is 9 times wine consumption. It is automatic that 1/9 is what one needs to use in the (wine, coffee) position. Note that water is consumed more than coffee, so one enters 2 in the (water, coffee) position, and ½ in the (coffee, water) position. One always enters the whole number in its appropriate position and automatically enters its reciprocal in the transpose position.

Table 3.1: Judgments on relative Consumption of Drinks

	Coffee	*Wine*	*Tea*	*Beer*	*Sodas*	*Milk*	*Water*
Coffee	1	9	5	2	1	1	1/2
Wine	1/9	1	1/3	1/9	1/9	1/9	1/9
Tea	1/5	2	1	1/3	1/4	1/3	1/9
Beer	1/2	9	3	1	1/2	1	1/3
Soda	1	9	4	2	1	2	1/2
Milk	1	9	3	1	1/2	1	1/3
Water	2	9	9	3	2	3	1

Table 3.2: Fundamental scale of absolute numbers by Saaty, 1980

Intensity of Importance	*Definition*	*Explanation*
1	Equal Importance	Two activities contribute equally to the objective
2	Weak or slight	
3	Moderate importance	Experience and judgement slightly favour one activity over another
4	Moderate plus	
5	Strong importance	Experience and judgement strongly favour one activity over another
6	Strong plus	
7	Very strong or demonstrated importance	An activity is favoured very strongly over another; its dominance demonstrated in practice
8	Very, very strong	
9	Extreme importance	The evidence favouring one activity over another is of the highest possible order of affirmation
Reciprocals of above	If activity *i* has one of the above non-zero numbers assigned to it when compared with activity *j*, then *j* has the reciprocal value when compared with *i*	A reasonable assumption
1.1–1.9	If the activities are very close	May be difficult to assign the best value but when compared with other contrasting activities the size of the small numbers would not be too noticeable, yet they can still indicate the relative importance of the activities.

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3.2.1 Illustrative example of AHP Method

Considering making a choice between three universities based on some selected criteria as shown in the Table 3.3 below:

Tab 3.3: Choosing a University Problem

UNIV/CRITERIA	A	B	C	Weight
Location	12.9	27.7	59.4	17%
Reputation	54.5	27.3	18.2	83%

The ranking of each University is based on computing the composite weights as follows:

$$U(A) = (12.9 \times 0.17) + (54.5 \times 0.83) = 47.428$$

U(B) = (27.7 x 0.17) + (27.3 x 0.83) = 27.368

U(C) = (59.4 x 0.17) + (18.2 x 0.83) = 25.204

3.2.2 Calculating Consistency in University Selection Problem

Considering n criteria, this establishes n x n matrix A. The criteria in row (i=1,2,…n) is ranked relative to every other criterion. AHP uses a discrete scale 1-9 in which a_{ij}=1 signifies i and j are of equal importance, a_{ij}=5 signifies i is strongly more important than j and a_{ij}=9 indicates that i is extra ordinary important than j. All other intermediate values between 1 and 9 are interpreted correspondingly. Consistency in judgment requires that $a_{ij=}k$ automatically implies that $a_{ji=}1/k$, also all diagonal elements in a_{ii} of A must be equal to 1 because they rank a criterion against itself. Using the scale 1-9 in Table 3.2, the pair wise comparison matrix A can be derived.

$$A_L = \begin{pmatrix} 1 & 1/2 & 1/5 \\ 2 & 1 & 1/2 \\ 5 & 2 & 1 \end{pmatrix}$$

Summing all the column elements C1=8, C2=3.5 and C3=1.7, we thus divide all column elements by their respective column sum value to form the matrix:

$$N_L = \begin{pmatrix} 1/8 & 0.5/3.5 & 0.2/1.7 \\ 2/8 & 1/3.5 & 0.5/1.7 \\ 5/8 & 2/3.5 & 1/1.7 \end{pmatrix}$$

$$N_L = \begin{pmatrix} 0.125 & 0.143 & 0.118 \\ 0.25 & 0.286 & 0.294 \\ 0.625 & 0.571 & 0.588 \end{pmatrix}$$

Since N_L does not have identical columns the matrix is inconsistent. Thus compute the following:

$\sum$R1/3= (0.125+0.143+0.118)/3=0.129

$\sum R2/3 = (0.25+0.286+0.294)/3=0.277$

$\sum R3/3 = (0.625+0.571+0.588)/3=0.594$

$$A_{L1} = \begin{pmatrix} 0.125 & 0.143 & 0.118 \\ 0.25 & 0.286 & 0.294 \\ 0.625 & 0.571 & 0.588 \end{pmatrix} \times \begin{pmatrix} 0.129 \\ 0.277 \\ 0.594 \end{pmatrix}$$

$$A_{L1} = \begin{pmatrix} 0.3863 \\ 0.832 \\ 1.793 \end{pmatrix}$$

$$n_{max} = (0.3863 + 0.832 + 1.793) = 3.0113$$

Consistency index CI is given as:

$$\mathrm{CI} = (\mathrm{n_{max}} - \mathrm{n})/(\mathrm{n} - 1)$$

$$\mathrm{CI} = \frac{(3.0113 - 3)}{3 - 1} = 0.0565$$

The random consistency index is expressed below:

$$\mathrm{RI} = \frac{1.98(\mathrm{n} - 2)}{\mathrm{n}}$$

$$\mathrm{RI} = \frac{1.98(3 - 2)}{3} = 0.66$$

Finally, compute the consistency ratio as below:

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{RI}} = \frac{0.0565}{0.66} = 0.0856$$

Since CR<0.1 the level of inconsistency in A_L acceptable.

In conclusion, based on the computations university A has the highest composite weight (47.428) and thus is the number one university, followed by university B (27.368) and university C (25.204) respectively.

3.3 Illustrative example of Factor Rating Method

Tema Oil refinery, headquarters in Tema, must decide among three sites for the construction of a new oil processing center. The firm has selected seven factors listed below as a basis for evaluation and has assigned rating weights on each factor.

Table 3.4a: Factor Rating Method for sites selection Problem

Factor	Factor Name	Weight	Loc. A	Loc. B	Loc. C
1	Prox. To port	5	100	80	80
2	Power source	3	80	70	100
3	Work force	4	30	60	70
4	Dist from Tema	2	10	80	60
5	Comm. Desire	2	90	60	80
6	Equip.	3	50	60	90

	Supplier				
7	Econ activities	1	90	60	60

Table 3.4b: Calculating ratio of rate

Factor	Factor Name	Weight	Ratio of rate	Loc. A	Loc. B	Loc. C
1	Prox. To port	5	5/20=0.25	100	80	80
2	Power source	3	3/20=0.15	80	70	100
3	Work force	4	4/20=0.2	30	60	70
4	Dist from Tema	2	2/20=0.1	10	80	60
5	Comm. Desire	2	2/20=0.1	90	60	80
6	Equip. Supplier	3	3/20=0.15	50	60	90
7	Econ activities	1	1/20=0.05	90	60	60
		∑=20				

Table 3.4c: Calculating aggregate scores in Factor rating

Ratio of rate	Loc. A	Loc. B	Loc. C
0.25	100 x 0.25	80 x 0.25	80 x 0.25
0.15	80 x 0.15	70 x 0.15	100 x 0.15
0.2	30 x 0.2	60 x 0.2	70 x 0.2
0.1	10 x 0.1	80 x 0.1	60 x 0.1
0.1	90 x 0.1	60 x 0.1	80 x 0.1
0.15	50 x 0.15	60 x 0.15	90 x 0.15
0.05	90 x 0.05	60 x 0.05	60 x 0.05

Table 3.4d: Selecting criteria in factor rating

Loc. A	Loc. B	Loc. C
25	20	20
12	10.5	15
6	12	14
1	8	6
9	6	8
7.5	9	13.5
4.5	3	3
∑=65	∑=68.5	∑=79.5

From the aggregate score Location C is the best followed by Location B and Location A respectively.

3.4 The Promethee Theory

Following Brans and Mareschal, (1994), in order to take the deviations and the scales of the criteria into account, a preference function is associated to each criterion. For this purpose, a preference function $P_i(A_j,A_k)$ is defined, representing the degree of the preference of alternative A_j over A_k for criterion C_i. Consider a degree in normalized form, so that

$0 \leq P_i(A_j,A_k) \leq 1$ and

$P_i(A_j,A_k) = 0$ means no preference or indifference,

$p_i(A_j,A_k) \approx 0$ means weak preference,

$p_i(A_j,A_k) \approx 1$ means strong preference, and

$p_i(A_j,A_k) = 1$ means strict preference.

In most practical cases $p_i(A_j,A_k)$ is function of the deviation $d=a_{ij}-a_{ik}$, i.e. $p_i(A_j,A_k) = p_i(a_{ij}-a_{ik})$, where p_i is a non decreasing function, $p_i(d)=0$ for $d \leq 0$, and $0 \leq p_i(d) \leq 1$ for $d > 0$. A set of six typical preference functions was proposed by Brans and Vincke (1985) and Brans et al. (1986). The simplicity is the main advantage of these preferences functions: no more than two parameters in each case, each having a clear economic significance.

A multi criteria preference index $\pi(A_j,A_k)$ of A_j over A_k can then be defined considering all the criteria:

$$\pi(a,b) = \sum_{j=1}^{k} \left(\mathrm{W_j} P_j(a,b) \right) \tag{3.4}$$

Where k=1,2,3…,6

This index also takes values between 0 and 1, and represents the global intensity of preference between the pair of alternatives. In order to rank the alternatives, the following precedence flows are defined:

Positive Outranking flow:

$$\emptyset^{+}(A_j) = \frac{1}{n-1} \sum_{k=1}^{n} \pi(\mathrm{A_j}, \mathrm{A_k}) \tag{3.5}$$

Negative Outranking flow:

$$\emptyset^{-}(A_j) = \frac{1}{n-1}\sum_{k=1}^{n} \pi(A_k, A_j) \quad (3.6)$$

The positive outranking flow expresses how much each alternative is outranking all the others. The higher $\phi^{+}(A_j)$, the better the alternative. $\phi^{+}(A_j)$, represents the power of A_j, its outranking character. The greater $\phi^{-}(A_j)$, the greater the weakness of A_j. The negative outranking flow expresses how much each alternative is outranked by all the others. The smaller $\phi^{-}(A_j)$, the better the alternative. $\phi^{-}(A_j)$, represents the weakness of A_j, its outranked character.

3.4.1 The Promethee I Partial Ranking

A_j is preferred to A_k when $\phi^{+}(A_j) \geq \phi^{+}(A_k)$, $\phi^{-}(A_j) \leq \phi^{-}(A_k)$, and at least one of the inequalities holds as a strict inequality. A_j and A_k are indifferent when $\phi^{+}(A_j) = \phi^{+}(A_k)$, $\phi^{-}(A_j) = \phi^{-}(A_k)$,

A_j and A_k are incomparable otherwise. In this partial ranking some couples of alternatives are comparable, some others are not. This information can be useful in concrete applications for decision making.

3.4.2 The Promethee II Complete Partial Ranking

If a complete ranking of the alternatives is requested by the decision maker, avoiding any incomparability, the net outranking flow can be considered. This is defined to be

$$\phi(A_j) = \phi^{+}(A_j) - \phi^{-}(A_j). \quad (3.7)$$

3.4.3 Preference Functions and their Features

The preference function p (d) is the function of deviation or difference (d) between values of two evaluated alternatives on the same criterion (perhaps over a set of criterion).

Mathematically, written as $p_i(A_k,A_l)= p_i(d_i(A_k,A_l))$, i=1,…n

The main features of preference functions are

a) Values of the preference functions: these values are within the interval zero to one such that $0 \leq p_i(A_k,A_l) \leq 1$

b) Preference functions are functions that maximize criteria through normalized values such that the higher the value of the function p (d), the preference of A_k to A_l.

c) Most preference functions have one or more of the following parameters p, d, σ. Values of these parameters are always determined by the decision maker and thereby aid in determining the intensity of preference of one alternative over the other on a criterion. The parameter q, indicated along the deviation axis, is the greatest point of deviation (d) between two evaluations, below which the decision maker regards the corresponding alternatives (A_k,A_l)= as indifferent. p which is fixed to the right of the parameter q on the deviation axis measures the lowest point of deviation (d) between two alternatives above which the decision maker expresses strict preference $p_i(A_k,A_l)$ for the first alternative A_k over the second alternative A_l When the deviation d between two evaluations falls between q and p, preference for the alternative A_k over alternative A_l ranges between 0 and 1

The value of a preference function p (d) equals zero when the deviation or difference (d) is below the lower boundary q, in other words, when the value of deviation is less than the value of q: $p(d) = 0$ if $d \leq q$ (in case however, the value of q is not specified it is regarded as zero, $q = 0$)

So long as the deviation value remains a value in between the threshold q and p, the following conclusions are worth noting:

(i) $p_i(A_k,A_l) = 0$ (3.8a)

implies indifference between A_k and A_l or no preference of A_k over A_l

(ii) $p_i(A_k,A_l) \approx 0$ (3.8b)

implies there is a weak preference of A_k over A_l where the symbol " ≈ " denotes a value of $p_i(A_k,A_l)$ closed to zero (0)

(iii) $p_i(A_k,A_l) \approx 1$ (3.8c)

implies a strong preference of A_k over A_l where the symbol " ≈ " denotes a value of $p_i(A_k,A_l)$ closed to 1

(iv) $p_i(A_k,A_l) = 1$ (3.8d)

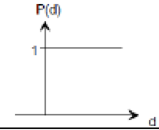
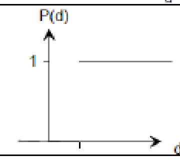
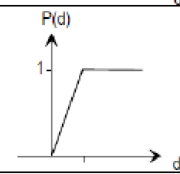
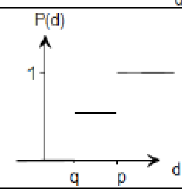
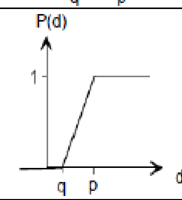
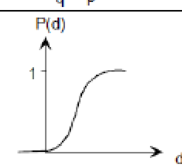
implies a strict preference of A_k over A_l (Brans et al., 1986)

There is also a parameter σ which is regarded as an intermediate value between q and p, therefore the choice of generalized criterion is preceded by the selection of the appropriate parameters.

d) If the upper boundary of deviation p is defined then $p(d) = 1$ if and only if $d \geq p$ also, there are times the value of p is not explicitly stated and in such cases $\lim_{d \to \infty} p(d) = 1$ (Podvezko and Podviezko, 2010)

There exist basically six preference functions in the Promethee method. The 'usual function' is an easy to use preference function and is generally used with quantitative criteria. 'U-shape function' uses a single indifference threshold and is generally used with qualitative criteria. 'V-shape function' uses a single preference threshold and is often used with quantitative criteria. 'Level function' is similar to 'U-shape', but with an additional preference threshold and it is mostly used with qualitative criteria. 'Linear function' is similar to 'V-shape', but with an additional indifference threshold and is often used with quantitative criteria. 'Gaussian function' is rarely used and is best suited for quantitative criteria, (Bertrand, 2009).The graph of each of the preference function is detailed in Table 3.4.

Table 3.5: Different Preference Function available in Promethee

Type of generalized criteria	Analytical definition	Parameters to define	Shape
Type I. Usual criterion	$H(d) = \begin{cases} 0, & d = 0; \\ 1, & \lvert d \rvert > 0. \end{cases}$	--	
Type II. Quasi-criterion	$H(d) = \begin{cases} 0, & \lvert d \rvert \le q; \\ 1, & otherwise. \end{cases}$	p	
Type III. Criterion with linear preference	$H(d) = \begin{cases} \frac{\lvert d \rvert}{p}, & \lvert d \rvert \le p; \\ 1, & \lvert d \rvert > 0. \end{cases}$	p	
Type IV. Level-criterion	$H(d) = \begin{cases} 1, & \lvert d \rvert \le q; \\ 1/2, & q < \lvert d \rvert \le p; \\ 1, & otherwise. \end{cases}$	q, p	
Type V. Criterion with linear preference and indifference area	$H(d) = \begin{cases} 1, & \lvert d \rvert \le q; \\ \frac{\lvert d \rvert - q}{p - q}, & q < \lvert d \rvert \le p \\ 1, & otherwise \end{cases}$	q, p	
Type VI. Gaussian criterion	$H(d) = 1 - \exp\{-\frac{d^2}{2\sigma^2}\}$	σ	

1. Usual Criterion or Preference Function

This function is applicable to cases when the decision maker is only interested in the difference between criteria values. Here there is no allocation of importance for the differences between criteria values. The decision maker only has strict preferences for an alternative with the greatest criteria values. In short, their preference judgement is based on the principle that the "more the better". This type of function is boundary free (neither q nor p is defined).

The decision maker’s focus is only on the evaluation difference and so *p* (*d*) = 1 and if and only if $d_i(A_k,A_l) = C_i(A_k) - C_i(A_k)$ is positive and *p* (*d*) = 0 if $d_i(A_k,A_l) = C_i(A_k) - C_i(A_k)$ is negative and the value of the difference does not matter (Podvezko and Podviezko, 2010).

For example, one job offer is preferred over another if offered salary is higher without assigning any importance to the difference; it is important if distance to the office is higher or smaller; if one candidate for a job knows more languages than another etc. the usual preference function is defined

$$p(d) = \begin{cases} 0, & x \leq 0 \\ 1, & x > 0 \end{cases} \qquad (3.9a)$$

The graph of the preference function is presented in Table 3.4: in the graph the horizontal axis is the deviation axis, d which is the difference between values of two evaluated alternatives on a criterion. The vertical axis labeled *p* (*d*) measures the degree of preference. The meeting point of the two axes is labeled 0 as the point of origin. The upper horizontal line that originates from point 1 on the *p* (*d*) axis and runs parallel to the deviation (*d*) axis marks the maximum value the degree of preference can take.

2. U-shape preference function or the quasi criterion

This differs from the usual preference function by the establishment of the indifference threshold *q*, this indifference threshold marks the lower boundary of the evaluation difference such that when the difference (*d*) between the evaluation of two alternatives is below q the decision maker considers the two alternatives indifferent and the preference function *p* (*d*) = 0 since $d \leq q$. On the other hand, if the evaluated difference between the two alternatives is above *q* then there is a strict preference of one alternative over the other and the preference function *p* (*d*) = 1 since d > *q*, though the function is u-shape our focus is on the right side of it. Hence, to use the u-shape criterion the decision maker has to determine only the value of *q* and this has economic signification- the greatest value of deviation between two alternative actions below which the decision maker declares the affected alternatives indifferent. For example a new job will have strict preference *p* (*d*) = 1 over another if only the salary difference exceeds 500 Ghana cedis (*q*= 500) otherwise the difference will be of no value to the employee and *p* (*d*) = 0. The same way, a candidate becomes preferable to another if the work experience of that candidate is more than another four years (*q*= 4) or that candidate correctly answered at least 4 questions more than another and so on. The algebraic definition of the function is:

$$p(d) = \begin{cases} 0, & x \leq q \\ 1, & x > q \end{cases} \qquad (3.9b)$$

The graph of the u-shape preference function is shown in table 3.4, the graph the horizontal axis is the deviation (*d*) axis which is the difference between the values of two alternatives evaluated on the same criterion. The vertical axis, *p* (*d*), measures the intensity of preference for one alternative over the other. The least value on this axis is zero and the highest is 1.

3. **Level preference function**

This function makes use of the indifference and preference threshold, *q* and *s* respectively which must therefore be defined simultaneously by the decision maker.

As usual, if the value difference between two evaluated alternatives is below indifference threshold *q* then the two alternatives concerned are regarded as indifferent and [*p* (*d*) =0] by the decision maker. If the difference (*d*) is above the preference threshold s, the decision maker expresses a strong preference [*p* (*d*) =1] of one alternative over another. And if the difference d is between q and s then there is a weak preference of one alternative over another denoted by [*p* (*d*) =1/2] as the value of the preference function. The analytical expression is as shown below:

$$\begin{cases} 0, when\ d \le q \\ 0.5, when\ q < d \le s \\ 1 when\ d > s \end{cases} \quad (3.9c)$$

4. **Criterion with linear preference function**

This has a boundary parameter such that if the evaluation difference *d* is below *s* then the preference of the decision maker increases linearly with the difference *d*, if d is above s then the decision maker will have a strict (constant) preference for one option over another. This function is therefore different from the u-shape function in the interval 0 to s where the link between the point of indifference *p* (*d*) = 0 and the point of strict preference of one alternative over another [*p*(*d*) = 1] is linear but not a shift. This preference function has only an upper boundary *s* , a preference threshold above which there is a strict preference for one alternative over another. In effect, the preference threshold *s* is the lowest value of difference (*d*) above which the decision maker has strict preference for one of the corresponding alternatives. The analytical expression for the v-shape preference function is as follows:

$$\begin{cases} 0, when\ d \leq 0 \\ d/s, when\ 0 < d \leq s \\ 1\ when\ d > s \end{cases} \quad (3.9d)$$

5. Criterion with linear preference and indifference area preference function

This function too has the parameters *q* and *s* as defined before and the decision maker has to determine their values. In this case the preference of the decision maker increases but linearly from the point of indifference threshold (*q*) to the point of strict preference threshold (*s*), in other words, the preference function increases steadily and linearly from zero to one based on the formula (*d*-*q*)/(*s*-*q*). The value of this formula suggests the degree of preference of one alternative over another. In view of this, when $q = 0$ the function turns to v-shape preference function. For example, a job seeker already into another job will be indifferent over the job he is engaged in and a new one if the salary difference of these two jobs is less than 500 cedis ($p(d) = 0$). ON the other hand, the seeker expresses strict preference for the new job if the salary of the new job over his current job if the salary offer of the new job offer exceeds 1000 cedis ($p(d) =1$) and there will be a preference of some sort for the new job over his current job if the salary offer of the new one falls within 500 and 1000 cedis.

The preference level is calculated by the formula $p(d)= (d-500)/(1000-500)= (d-500)/500$ The algebraic definition of this function is given as below :

$$\begin{cases} 0, when\ d \le q \\ d - \frac{q}{s} - q, when\ q < d \le s \\ 1\ when\ d > s \end{cases} \tag{3.9e}$$

6. Gaussian Preference Function

This makes use of statistical data involving random values with normal distribution. The decision maker requires only determining the parameter σ of standard deviation of the given random values. The function increase most considerably at values of difference close to parameter σ Preference increases gradually from point zero along with the gradual increase (*d*). As the difference (*d*) in criteria values becomes considerably large so does the preference increases towards the preference threshold 1 but never hit on the exact mark. The algebraic definition is presented below:

$$p(d) = \begin{cases} 0, when\ \ d \le 0 \\ 1 - \exp\left(\frac{-d^2}{2\sigma^2}\right), when\ \ d > 0 \end{cases} \tag{3.9f}$$

3.5 Ranking of Alternatives in Promethee

The two indices π(A_k,A_l) and π(A_l,A_k) connect every pair of alternatives say A_k, A_l to each other . Such a connection or relation is known as outranking relation. Graphically, the relation is often represented by two nodes denoting the two alternatives linked to each other by a corresponding two arcs each for a preference index as presented in Figure 3.1 below:

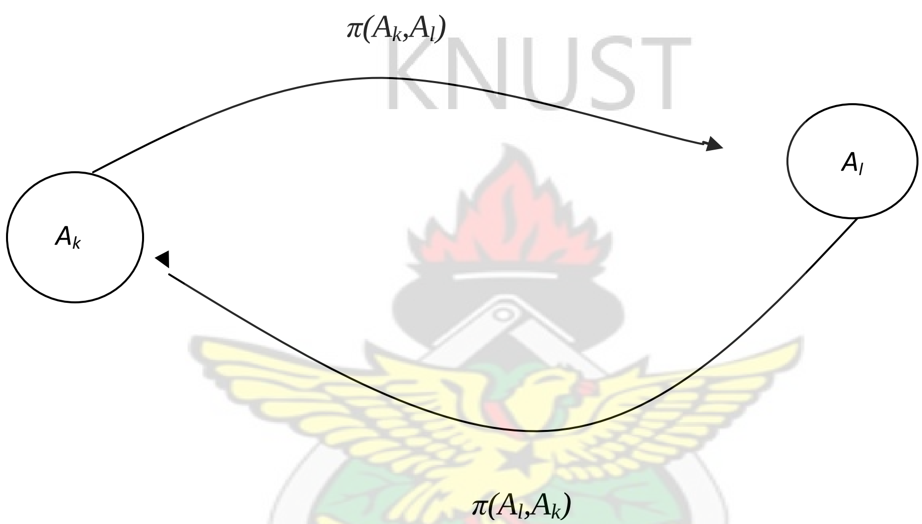


Figure 3.3: Outranking flow relation

From Figure 3.3, the alternatives A_k and A_l are the nodes. The preference index π(A_k,A_l) which links node A_k and A_l as indicated by the arrow of the upper arc of Figure 3.3 shows the magnitude of the preference of the alternative A_k over A_l. The preference index π(A_l,A_k) on the other hand connects node A_l to A_k and is indicated by the arrow of the lower arc of Figure 3.3.

Now given the set of possible alternatives in A, each alternative $A_k \in A$ faces (n-1) other alternatives in A, where n connotes the number of alternatives in A. The PROMETHEE method sums up all preference indices that are in favour of the alternative A_k , to get what is referred to as positive outranking flow:

$$\emptyset^{+}(A_j) = \frac{1}{n-1}\sum_{k=1}^{n} \pi(A_j, A_k) \quad (3.10)$$

It sums up all indices which are not in favour of A_k to be the negative outranking flow:

$$\emptyset^{-}(A_j) = \frac{1}{n-1}\sum_{k=1}^{n} \pi(A_k, A_j) \quad (3.11)$$

So the positive outranking flow shows how the alternative A_k is outranking all else in A over all criteria. It is called the power of A_k or the strength of the outranking character A_k. On the other hand, the negative outranking flow indicates how an alternative A_k is being outranked by all other alternatives in *A*. This measure represents the weakness of the outranked character A_k. The higher the positive outranking flow and the lower the negative flow the better the alternative A_k.

In graphical representation, the positive outranking flow is represented by Figure 3.4

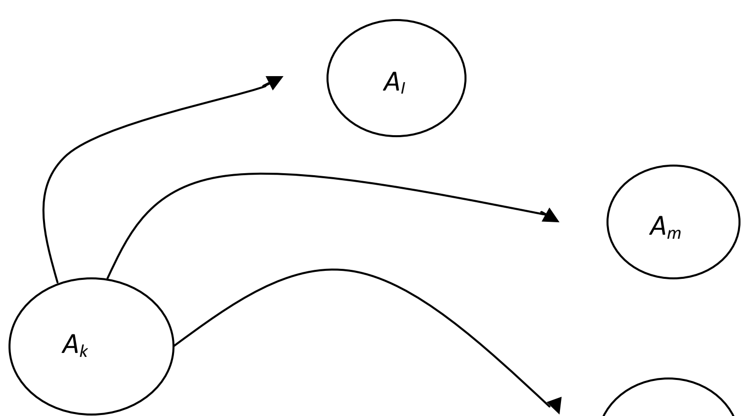


Figure 3.4: Positive outranking flow

From Figure 3.4 the arrows directed at nodes $A_{l,}$ A_m, A_n from node A_k show how the alternative A_k outranks all other alternatives. These directed arrows from A_k are called the positive outranking flow (leaving flows) denoted by $\varnothing^{+}(A_k)$. The negative outranking flow $\varnothing^{-}(A_k)$ is graphically represented by Figure 3.5. In Figure 3.5, the arrows from nodes $A_{l,}$ A_m, A_n directed at node A_k are called the negative outranking (entering flows) and they show how the alternative A_k is outranked by the other alternatives.

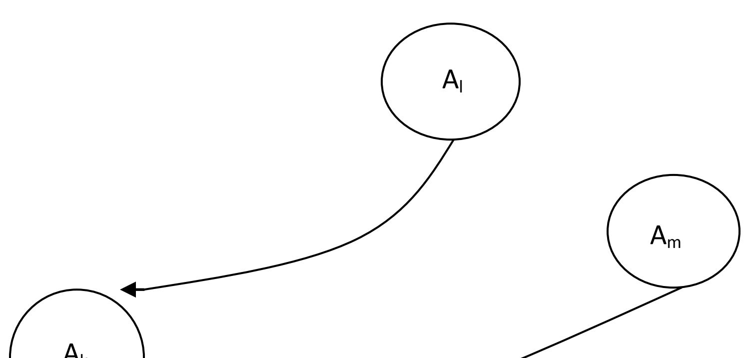


Figure 3.5: Negative outranking flow

The net flow, denoted by $\phi\,(A_k)$, is the difference between the positive flow and the negative flow. Essentially, the net flow is used for PROMETHEE II (complete ranking). The ranking of a finite set of alternatives under PROMETHEE methodology may involve two ranking processes which are namely:

(i) The partial ranking process

(ii) The complete ranking process

3.5.1 Promethee I- The Partial Ranking Method

The partial ranking (PROMETHEE I) establishes the outranking relation existing between various alternatives via the leaving $\emptyset^{+}(A_k)$ and the entering $\emptyset^{-}(A_k)$ flows on node A_k.

The possible outcomes may be denoted by P, I and R, where P, often placed between two alternatives as, A_kPA_l, signifies the preference of the alternative A_k over A_l. A_kIA_l signifies the indifference between alternatives A_k and A_l and A_kRA_l signifies the incomparability of the two alternatives A_k and A_l over all criteria.

The first column in Table 3.5 represents the preference relation which indicates the three possible outcomes when alternatives are compared pair wise. The possible outcomes are

(i) A_kPA_l means A_k is preferred to A_l

(ii) A_kIA_l means A_k is indifferent to A_l

(iii) A_kRA_l means A_k is incomparable A_l

The second column of Table 3.5 labelled cases, give the condition under which a given pair wise comparison of alternatives can be regarded as preference (P), indifference (I) or incomparable (R). The third column is the graphical representation column which shows how one alternative A_k is preferred to A_l by means of a directed arrow from A_k to A_l ($A_k \rightarrow A_l$). However, indifference or incomparable relations are shown by means of a dash (-).These three cases are identified using the following preorders as shown in Table 3.5 below

Table 3.6: Outranking relations for Partial Promethee Method

Preference Relation	Cases	Graphical Representation

A_kPA_l	$\emptyset^+(A_k) > \emptyset^+(A_l)\ and\ \emptyset^-(A_k) < \emptyset^-(A_l)$ $\emptyset^+(A_k) > \emptyset^+(A_l)\ and\ \emptyset^-(A_k) = \emptyset^-(A_l)$ $\emptyset^+(A_k) = \emptyset^+(A_l)\ and\ \emptyset^-(A_k) < \emptyset^-(A_l)$	$A_k \rightarrow A_l$
A_kIA_l	$\emptyset^+(A_k) = \emptyset^+(A_l)\ and\ \emptyset^-(A_k) = \emptyset^-(A_l)$	-
A_kRA_l	$\emptyset^+(A_k) > \emptyset^+(A_l)\ and\ \emptyset^-(A_k) > \emptyset^-(A_l)$ $\emptyset^+(A_l) > \emptyset^+(A_k)\ and\ \emptyset^-(A_l) > \emptyset^-(A_k)$	-

It can be concluded from above that:

(i) A_kPA_l implies a higher power of alternative A_k is matched to a lower weakness of A_k, in relation to A_l. In such a consistency the alternative A_k is automatically preferred to A_l.

(ii) A_kIA_l implies the respective leaving flows and entering flows are the same.

(iii) A_kRA_l implies a higher power of the alternative A_k is associated to a lower weakness of A_l. This type of situation arises when out of a set of criteria, alternative A_k is better than A_l on some, and conversely, the alternative A_l is better than A_k on other criteria. When the flows experience such an inconsistency the alternatives therein are declared incomparable. Over here, PROMETHEE I does not decide which alternative is better than the other. The choice is left to the decision maker to make, based on his or her perception, priorities, knowledge, experience etc. This is the reason why PROMETHEE I is regarded a partial preorder ranking method. It only compares alternatives that are comparable (i.e only those under *P* and *I*) and thus makes the whole ranking incomplete.

The partial ranking can be represented graphically using the leaving and the entering flows. Decision to be made according to this ranking is done by considering the alternative with the highest number of leaving flows. This indicates the alternative most preferred in the comparison to other alternatives.

3.5.2 Promethee II- Complete Ranking

At this stage it is the PROMETHEE II (preorder complete ranking) method which completes the whole ranking process, establishing a relation that links all alternatives be they comparable or incomparable and placing them in their right perspective in a hierarchy from best to worst. If after partial ranking of PROMETHEE I some alternatives are found to be incomparable then we apply PROMETHEE II (the complete ranking) method to finish the ranking process for an optimal decision to be made.

It makes use of only the parameter P and I (preference and indifference respectively). This approach makes use of what is called the net outranking flow, the higher the net flow, the better the alternative.

The alternative A_k is preferable to A_l if and only if $\emptyset(A_k) > \emptyset(A_l)$

(i) The alternative A_k is indifferent to the alternative A_l if and only if $\emptyset(A_k) = \emptyset(A_l)$

Tab 3.7: Two existing relations between alternatives in complete ranking

Preference Relation	Cases	Graphical Representation
A_kPA_l	$\emptyset(A_k) > \emptyset(A_l)$	$A_k \longrightarrow A_l$
A_kIA_l	$\emptyset(A_k) = \emptyset(A_l)$	

3.6 Illustrative example of Promethee Method

In the selection of an appropriate car the table below gives the cost and performance of each car.

The weight assign to Cost is 0.75 and that of performance is 0.25

Table 3.8a: Cost and Performance of three cars

	Car A	Car B	Car C
Cost	50	15	20
Performance	80	60	70

Table 3.8b: Deviations d_1 (A_k, A_l) on the minimizing criterion C_1

Min cost (c_1)	A=50	B=15	C=20
A=50	0	-35	-30
B=15	35	0	5
C=20	30	-5	0

Table 3.8c: Deviations d_2 (A_k, A_l) on the maximization criterion C_2

Max cost (c_2)	A=80	B=60	C=70
A=80	0	20	10
B=60	-20	0	-10
C=70	-10	10	0

3.6.1 Preference Evaluation

In this example the Quasi-criterion preference function was used and is expressed mathematically as below:

$$p(d) = \begin{cases} 0, & d \leq 0 \\ 1, & d > 0 \end{cases} \quad (3.9b)$$

Table 3.8d: Values of $p_1 (A_k, A_l)$ on the minimizing criterion C_1

C_1	i=1	i=2	i=3
k=1	0	0	0
k=2	1	0	1
k=3	1	0	0

Table 3.8e: Values of p_2 (A_k, A_l) on the maximization criterion C_2

C_2	i=1	i=2	i=3
k=1	0	1	1
k=2	0	0	0
k=3	0	1	0

3.6.2 Preference Index

As already shown in equation (3.2) a multi criteria preference index $\pi(A_j, A_k)$ of A_j over A_k can be defined considering all the criteria as expressed below:

$$\pi(a,b) = \sum_{j=1}^{k} \left(W_j P_j(a,b) \right) \quad (3.2)$$

This index also takes values between 0 and 1, and represents the global intensity of preference between the couples of alternatives. In order to rank the alternatives, the following precedence flows are defined:

Table 3.8f: Values of $\pi_1 (A_k, A_l)$ on the minimizing criterion C_1

C_1	A	B	C
A	0	0	0
B	0.75	0	0.75
C	0.75	0	0

Table 3.8g: Values of $\pi_2 (A_k, A_l)$ on the maximization criterion C_2

C_2	A	B	C
A	0	0.25	0.25
B	0	0	0

C	0	0.25	0

3.6.3 Aggregated Preference Index

The aggregated preference index is derived by summing all the individual preference index and the results is summarised in the table below:

Table 3.8h: Aggregated Preference Index

$\pi(A_k, A_l)$	Car A	Car B	Car C
Car A	0	0.25	0.25
Car B	0.75	0	0.75
Car C	0.75	0.25	0

3.6.4 Computation Positive (Leaving) and Negative (Entering) Flow Values

Using the preference functions and weights of the criteria, every action can automatically be compared to each other. From this pair wise comparison, table information can be extracted in order to rank all actions. This is done by computing three different flows.

1. The positive (leaving) flow measures the average degree to which an action is preferred to the other ones. Actions with larger leaving positive flow values should be ranked first.

2. The negative (entering) flow measures the average degree to which the other actions are preferred to that action. Actions with smaller negative flow values should be rank first.

Usually both preference flows lead to somewhat different ranking as in multi criteria context there is usually no ranking completely consistent with all pair wise comparison results. The preference flow formulae are stated below:

Positive Outranking flow:

$$\emptyset^{+}(A_j) = \frac{1}{n-1}\sum_{k=1}^{n} \pi(A_j, A_k) \qquad (3.3)$$

Negative Outranking flow:

$$\emptyset^{-}(A_j) = \frac{1}{n-1}\sum_{k=1}^{n} \pi(A_k, A_j) \qquad (3.4)$$

Net flow:

$$\emptyset = \emptyset^{+} - \emptyset^{-} \qquad (3.5)$$

Table 3.8i: Calculation of Preference flow

$\pi(A_k, A_l)$	Car A	Car B	Car C	$\emptyset^{+}$
Car A	0	0.25	0.25	0.25
Car B	0.75	0	0.75	0.75
Car C	0.75	0.25	0	0.5
$\emptyset^{-}$	0.75	0.25	0.5	
$\emptyset$	-0.5	0.5	0	

3.6.5 Promethee1 Partial ranking of Cars

Promethee I uses partial ranking of the actions includes only preferences that are confirmed by both entering and leaving flows. Incomparability arises when both flows give opposite information because the actions have quiet different profiles and are thus difficult to compare.

Table 3.9: Partial Promethee Ranking

Ranking	Promethee
1	Car B
2	Car C
3	Car A

3.6.6 Promethee II Complete ranking of Cars

Promethee II uses the net flow to rank completely all the actions from the best to the worst. In this case no incomparabilities are possible. In this example both the partial and complete promethee ranking yielded the same results as shown in table 3.8. Car *B* has the highest net flow followed by Car *C* and Car *A*, that is *B*>*C*>*A*.

3.7 The Multi Attribute Utility Theory (MAUT) Approach

Josias, (2006) explained that the MAUT approach can be summarized into the following steps:

Step 1: Identify the various criteria and sub-criteria to be used in the evaluation process.

Step 2: Rank the different criteria and sub-criteria in order of importance.

Step 3: Rate the different criteria and sub-criteria on a scale from zero to one, while reflecting the ratio of relative importance of one criterion over the next.

Step 4: Normalize these weights on a scale from zero to one.

Step 5: Determine criteria values for each alternative by using single-attribute utility functions on linear normalized scales.

Step 6: Calculate the utilities for the alternatives by obtaining the weighted linear sum for the criteria.

Dr. Yan, (2011) explained that MAUT consists of the following steps below:

Step1: Use utility functions to convert numerical attribute scales to utility unit scales

Step2: Assign weights to these attributes and then calculate the weighted average of each consequence set as an overall utility score

Step3: Compare alternatives using the overall utility score

3.7.1 MAUT Utility Functions

There are three fundamental utility functions and discuss below:

1) Additive Utility Function has the following properties:

- A Simplified Utility Model
- Ignores interactions among attributes

Mathematically, for a consequence set that has values $x_1, x_2, \ldots, x_m$ on the attributes of *m* objectives, its overall utility is computed as:

$$U(x_1, x_2, \ldots, x_m) = k_1U_1(x_1) + k_2U_2(x_2) + k_mU_m(x_m) = \sum_{i=1}^{m} k_i U_i(x_i) \quad (3.12)$$

Where $U_i(x_i)$ is the utility function of the ith attribute $0 \leq U_i(x_i) \leq 1$

K_i is the weight of the ith attribute $(k_i + k_2 + \ldots + k_m = 1)$

$0 \leq U(x_1, x_2, .., x_m) \leq 1$

2) Multilinear utility function captures a limited form of interaction, mathematically it is expressed below:

$$U(x,y)= k_xU_x(x) + k_yU_y(y) + (1-k_x-k_y)\ U_x(x)\ U_y(y) \qquad (3.13)$$

$U_x(x)$ is the utility function of x scaled so that $U_x(x^-)=0$ and $U_x(x^+)=1$

$U_y(y)$ is the utility function of y scaled $U_y(y^-)=0$ and $U_y(y^+)=1$

$k_x = U(x^+, y^-)$ is Not relative weight of U_x

$k_y = U(x^-, y^+)$ is Not relative weight of U_y

$k_x + k_y \neq 1$

$U(x^+, y^-) = k_xU_x(x^+) + k_yU_y(y^-) + (1-k_x-k_y)\ U_x(x^+)\ U_y(y^-) = k_x(1) + k_y(0) + (1-k_x-k_y)\ (1)(0) = k_x$

$U(x^-, y^+) = k_xU_x(x^-) + k_yU_y(y^+) + (1-k_x-k_y)\ U_x(x^-)\ U_y(y^+) = k_x(0) + k_y(1) + (1-k_x-k_y)\ (1)(0) = k_y$

3) The Multiplicative utility function

Let x and y be two attributes with values $x_{1\leq \ldots \leq}\ x_n$, $n \geq 2$ and $y_{1\leq \ldots \leq}\ x_m$, $m \geq 2$.

$$U(x,y) = (\ k.u(x,y_1) + 1)\ .\ (k.u(x_1,y) +1) \qquad (3.14)$$

Where

$u(x_1,y_1) = 1$, $u(x_n,y_1) > 1$ and $u(x_1,y_m) > 1$

$$k = \frac{|u(x_n, y_m) - u(x_n, y_1) - u(x_1, y_m)|}{u(x_n, y_1).\, u(x_1, y_m)} > 0, \text{is a scaling constant}$$

3.7.2 Illustrative example of The Multi Attribute Utility Theory (MAUT)

A buyer wants to buy a car with a long expected life span and a low price. The three alternatives under consideration are: the Portalo (a relatively expensive sedan with a reputation for longevity) , the Norushi (renowned for its reliability), and the Standard Motor car (a relatively inexpensive domestic automobile). These three cars are evaluated on both attributes, as in Table 3.10 below:

Table 3.10a: Cost and Criteria of three cars

	Alternatives		
Attributes	Portalo	Norushi	Standard Motors
Price ($)	17000(worst)	10000	8000(best)
Life span (Years)	12(best)	9	6(worst)

The graph below is compares the three cars using a line graph, the horizontal axis gives the price of cars in dollars whiles the vertical axis gives the life span of the cars in years .

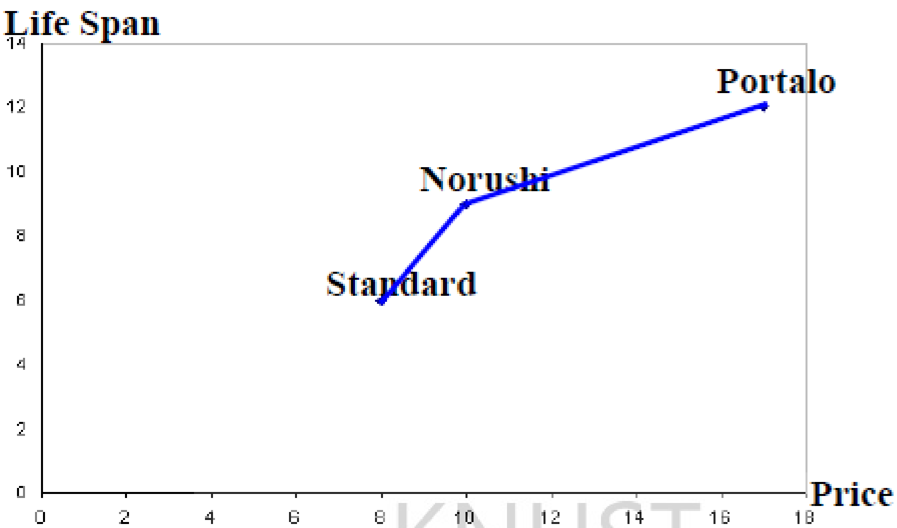


Figure 3.6: Graph of Comparison of three cars.

3.7.3 Additive Utility Function

For a consequence set that has values $x_1, x_2, \ldots, x_m$ on the attributes of m objectives, its overall utility is computed as:

$$U(x_1, x_2, \ldots, x_m = k_1 u_1(x_1) + k_2 u_2(x_2) + \cdots + k_m u_m(x_m) = \sum_{i=1}^{m} k_i u_i(x_i) \tag{3.13}$$

$U_i(x_i)$- the utility function of the ith attribute $0 \leq U_i(x_i) \leq 1$

k_i- the weight of the ith attribute ($k_1 + k_2 + \ldots + k_m = 1$)

$0 \leq U_i(x_1, x_2, \ldots, x_m) \leq 1$

Set U_{Price}(Standard) $= U_{Price}(8000) = 1$, U_{Price}(Portalo) $= U_{Price}(17000) = 0$

U_{Life}(Portalo) $= U_{Life}(12) = 1$, U_{Life}(Standard) $= U_{Life}(6) = 0$

$$U(x) = \frac{x - x_1^-}{x_1^+ - x_1^-} \tag{3.14}$$

x_i^-: is the worst value of attribute x_i and x_i^+: is the best value of x_i

$U_{Price}(\text{Norushi}) = U_{Price}(10000) = (10000 – 17000) / (8000 – 17000) = 0.78$

$U_{Life}(\text{Norushi}) = U_{Life}(9) = (9 – 6) / (12 – 6) = 0.5$

Table 3.10b: Utility values of cars

	Alternatives		
Attributes	Portalo	Norushi	Standard Motors
U_{Price}	0	0.78	1
U_{Life}	1	0.5	0

3.7.4 Weight Assessment (Pricing Out)

Directly specify the ratio of the weights

e.g. $k_{Price}= 2k_{Life}$

Because $k_{Price}+ k_{Life} =1$, then $k_{Price}=2/3$ and $k_{Life} = 1/3$

$U(\text{Portalo}) = 2/3•U_{Price}(\text{Portalo}) + 1/3•U_{Life}(\text{Portalo})$

$= 2/3(0) + 1/3(1) =1/3$

$U(\text{Norushi}) = 2/3•U_{Price}(\text{Norushi}) + 1/3•U_{Life}(\text{Norushi})$

$= 2/3(0.78) + 1/3(0.5) =0.69$

$U(\text{Standard}) = 2/3•U_{Price}(\text{Standard}) + 1/3•U_{Life}(\text{Standard})$

$= 2/3(1) + 1/3(0) = 2/3$

Suppose taking the Standard Motors as the base case. You are indifferent between paying $8000 for 6 years of life span and paying $8,600 for 7 years of life span

U($8,000, 6 Years) = U($8,600, 7 Years)

$k_{Price} \bullet U_{Price}(8000) + k_{Life} \bullet U_{Life}(6) = k_{Price} \bullet U_{Price}(8600) + K_{Life} \bullet U_{Life}(7)$

$U_{Price}(8600) = (8600-17000)/(8000-17000) = 0.933$, $U_{Life}(7) = (7-6)/(12-6) = 0.167$

$k_{Price} \bullet 1 + k_{Life} \bullet 0 = k_{Price} \bullet 0.933 + k_{Life} \bullet 0.167 \Rightarrow 0.067k_{Price} = 0.167k_{Life}$ (Equation 1)

$k_{Price} + k_{Life} = 1$ (Equation 2)

Solve Equations (1) and (2) $\Rightarrow k_{Price} = 0.714$, $k_{Life} = 0.286$

$U(Portalo) = 0.714 \bullet U_{Price}(Portalo) + 0.286 \bullet U_{Life}(Portalo) = 0.286$

$U(Norushi) = 0.714 \bullet U_{Price}(Norushi) + 0.286 \bullet U_{Life}(Norushi) = 0.7$

$U(Standard) = 0.714 \bullet U_{Price}(Standard) + 0.286 \bullet U_{Life}(Standard) = 0.714$

From the calculations of the utility functions, U(Standard) > U(Norushi) > U(Portalo), thus the best alternative is standard motor, followed by Norushi and finally the Portalo.

CHAPTER 4

DATA COLLECTION, ANALYSIS AND RESULTS

4.1 Data Collection

The data used in this study is taken over a twenty four hour surveillance of carrier performance by the network performance team of a mobile operator in Ghana. This data is generated with software that is run on the Mobile Switching Controller (MSC) and thus captures live performance of each of carriers in the network. There are six criteria which are used as performance indicators of the routes. All data are objectively measured by the switch and thus free from any subjective interferences by engineers. This measurement was obtained in January, 2012 from the network performance team of a telecommunication service provider in Ghana.

The data collected is displayed in Table 4.1 below:

The first column, labelled carriers is the column for the six carriers used by the telecommunication provider. The second column, route availability indicates the percentage of carrier trunks available to carry traffic. The third column busy hour traffic gives the total number of traffic for carriers during busy hour. The fourth column, utilization describes the percentage of carrier trunks used in the carrying of traffic. The fifth column, congestion is the percentage of failed calls. The six column answer to seizure ratio (ASR), is the percentage of answered calls.

Table 4.1: Carrier Data from Mobile Provider

CARRIER	ROUTE AVAIL (%)	BH TRAFFIC	CAPACITY	UTILIZATION (%)	CONGESTION (%)	ASR (%)
AT&T1	100	26.92	22.83	117.93	11.90%	31.51%
AT&T2	100	97.61	110.35	88.45	0.15%	35.63%
BHARTI INDIA1	100	181.42	387.89	46.77	0.00%	51.05%
BHARTI INDIA2	87.87	183.94	311.71	59.01	0.00%	45.45%
BELGACOM1	100	112.39	229.85	48.9	0.00%	36.40%
BELGACOM2	100	86.14	229.85	37.48	0.00%	35.46%

4.2 Data Analysis

As already mentioned, the data used in this study are all network performance indicators used in the mobile telecommunication network. The Analysis of the data obtained is done using the PROMETHEE method.

4.2.1 Alternatives

In this study, the six international carriers (alternatives) were considered and represented as:

$A_1, A_2, A_3, A_4, A_5, A_6$

4.2.2 Criteria (C_i)

The criteria identified by the Network Performance team for measuring performance are:

i. Route Availability (C_1)

ii. Busy Hour Traffic (C_2)

iii. Capacity (C_3)

iv. Utilization (C_4)

v. Congestion (C_5)

vi. Answer to Seizure Ratio (C_6)

Route (Trunk) Availability (C_1)

Trunk is a logical connection between two switching nodes, in other words the telephone lines connecting one telephone switch or exchange with another are called trunks, see Figure 4.1. Route availability is expressed as the ratio of the number of trunk outage to total number of trunks expressed as a percentage.

Mathematically it is defined as:

$$\text{Route Availability [\%]} = \frac{\text{Number of trunk outage}}{\text{Total number of trunks}} \text{X100} \qquad (4.1)$$

Condition: Route availability should not be less than 100%. This is maximizing criterion and has the following set of data for the alternatives (A_j), see Table 4.1

$\{X_{ij}\} = \{X_{11}, X_{12}, X_{13}, X_{14}, X_{15}, X_{16}\}$, for i=1

$\{ X_{1j} \} = \{100, 100, 100, 87.87, 100, 100\}$

X_{ij} = Score of alternative j over criterion i

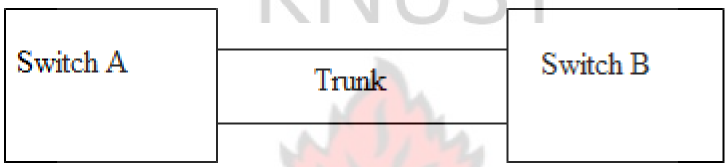


Figure 4.1: Diagram of a trunk connecting two switches A and B

Busy Hour Traffic (C_2)

Telephone traffic may fluctuate throughout the day, and may have a "busy hour" which is the hour that has the most number of calls. Busy hour depends on various factors such as stock market, weather and international events. Because "calls bunch up," all traffic planning has to focus on peak periods. It is not acceptable to provide excellent service most of the time and inadequate service just when customers want to make calls. It should be noted that all routes have their own busy hour.

In telecommunication system, traffic is defined as the occupancy of the server in the network. There are two types of traffic ie voice traffic and data traffic. For voice traffic, the calling rate is defined as the number of calls per traffic path during the busy hour. In a day, the 60 minutes interval in which the traffic is highest is called busy hour (BH).

If the average number of calls to and from a terminal during a period T second is '*n*' and the average holding time is '*h*' seconds, the average occupancy of the terminal is given by

$$A = \frac{nh}{T} \tag{4.2}$$

Condition: The busy hour traffic should be high and therefore it is a maximizing criterion with the following set of data for the alternatives (A_j).

$\{ X_{ij}\} = \{X_{21}, X_{22}, X_{23}, X_{24}, X_{25}, X_{26}\}$, for i=2

$\{ X_{2j} \} = \{22.92, 97.61, 181.42, 183.94, 112.39, 86.14\}$

Capacity (C_3)

The capacity of a given carrier is measured in terms of the subscribers or the traffic load that it can handle. The Erlang B formula is the most commonly used figure in any telecommunication capacity calculation, (Baldiwala, 2011). The Erlang B formula assumes an infinite population of sources (such as telephone subscribers), which jointly offer traffic to *N* servers (such as links in a trunk group). The rate of arrival of new calls (birth rate) is constant, not depending on the number of active sources, because the total number of sources is assumed to be infinite. The rate of call departure (death rate) is equal to the number of calls in progress divided by the mean call holding time. The formula calculates blocking probability in a loss system, where if a request is not served immediately when it tries to use a resource, it is aborted. Requests are therefore not queued. Blocking occurs when there is a new request from a source, but all the servers are already busy. The formula assumes that blocked traffic is immediately cleared. The formula provides the GoS (grade of service) which is the probability *P* that a new call arriving at the circuit group is rejected because all servers (circuits) are busy.

Mathematically it is expressed as:

$$p_g = \frac{\frac{s^k}{k!}}{\sum_{i=0}^{k} \frac{s^i}{i!}} \qquad (4.3)$$

p_g probability of call failure (“loss”)

s offered traffic in Erlangs (number of simultaneous calls)

k number of channels to carry the traffic (calls)

Condition: The capacity of the carrier should be high and thus a maximizing criterion with the following set of data for the alternatives (A_j)

$\{X_{ij}\} = \{X_{31}, X_{32}, X_{33}, X_{34}, X_{35}, X_{36}\}$, for i=3

$\{X_{3j}\} = \{22.83, 110.35, 387.89, 311.71, 229.85, 229.85\}$

Utilization (C_4)

The utilization of the trunk is calculated as a ratio of the total traffic to the capacity expressed as a percentage

$$Utilisation[\%] = \frac{Total\ traffic}{Capacity} x100 \qquad (4.4)$$

Condition: The utilization should not exceed 80%, it is therefore a minimizing criterion and has the set of data for the alternatives (A_j)

$\{ X_{ij}\} =\{X_{41}, X_{42}, X_{43}, X_{44}, X_{45}, X_{46}\}$, for i=4

$\{ X_{4j}\} =\{117.93, 88.45, 46.77, 59.01, 48.9, 37.48\}$

Congestion (C_5)

It is the condition in a switching center when a caller cannot obtain a connection to the wanted to end user immediately. In a circuit switching system, there will be a period of congestion during which no new calls can be accepted. There are two ways of specifying congestion.

- Time Congestion: cumulative time where all resources are busy or occupied
- Call congestion: Ratio of calls rejected due to insufficient resources (Switching and /or Transmission

It is the proportion of calls arising that do not find a free server. Call congestion is a loss system and also known as the probability of loss while in a delay system it is referred to as the probability of waiting. If the number of sources is equal to the number of servers, the time congestion is finite but the call congestion is zero.

When the number of sources is large in comparison with servers, the probability of a new call arising is independent of the number already in progress and therefore the call congestion is equal to the time congestion. In general, time and call congestions are different but in most practical cases, the discrepancies are small. Mathematically, it is expressed below:

$$Congestion[\%] = \frac{Number\ of\ connected\ failed\ calls}{Total\ number\ of\ call\ attempts} x100 \quad (4.5)$$

Condition: Congestion should be less than or equal to 1%, it is a minimizing criterion and has the following set of data for the alternatives ($A_{j,}$)

$\{ X_{ij} \} = \{X_{51}, X_{52}, X_{53}, X_{54}, X_{55}, X_{56}\}$, for i=5

$\{ X_{5j}\} = \{11.9, 0.15, 0, 0, 0, 0\}$

Answer to Seizure Ratio (C_6)

The answer/seizure ratio (ASR) is a measurement of network quality and call success rate in telecommunications. It is the percentage of answered telephone calls with respect to the total call volume.

Mathematically the answer/seizure ratio is defined as 100 times the ratio of successfully answered calls divided by the total number of call attempts (seizures):

$$ASR[\%] = \frac{answered\ calls}{seizures} x100 \quad (4.6)$$

Condition: This value must not be less than 40%, it is a maximizing criterion which has set of data for the alternatives (A_j)

$\{ X_{ij} \} = \{X_{61}, X_{62}, X_{63}, X_{64}, X_{65}, X_{66}\}$, for i=6

$\{ X_{6j} \} = \{31.51, 35.36, 51.05, 45.46, 36.4, 35.46\}$

4.2.3 Weight of a Criterion

Weight of the criterion W_i for i= 1,2,3…,6 are taken by the network performance team to be the same. Thus each criterion is weighed 1/6 which sum up to be 1. This signifies that all the criteria are of equal value of importance.

4.3 Format uses for input data

In this study, the values of the various criteria obtained from the network operator will be assigned weights based on the importance of each criterion to the network operator. These weights and criteria values will serve as the input data in the PROMETHEE method.

4.4 Model Formulation

The carrier linkage problem is modeled using a decision table, the decision table showing the performance of carriers as of January, 2012 by a mobile operator in Ghana is shown in Table 4.2. The first column, labeled criteria (C_1,…, C_6) is the column for the six criteria.

The second column, type of criteria indicates whether a give criterion is a maximizing or minimizing criterion. The third column alternatives is a 6 X 6 matrix in which each of the 6 rows represents ($C_1, \ldots, C_6$), whiles each of the 6 columns represents one of the six alternatives ($A_1, \ldots A_6$). The entries of the matrix X_{ij} where i= 1,2…,6 and j= 1,2…,6 are the scores of the various alternatives under each criterion for all the criteria.

Table 4.2: Decision Table showing Performance of Carriers

Alternatives							
Criteria	Type	A_1	A_2	A_3	A_4	A_5	A_6
C_1	Max	100	100	100	87.87	100	100
C_2	Max	26.92	97.61	181.42	183.94	112.39	86.14
C_3	Max	22.83	110.35	387.89	311.71	229.85	229.85
C_4	Min	117.93	88.45	46.77	59.01	48.9	37.48
C_5	Min	11.9	0.15	0	0	0	0
C_6	Min	31.51	35.36	51.05	45.45	36.4	35.46

4.5 Multiple Criteria Optimization

This involves the evaluation and ranking of the six alternatives on the six criteria concurrently. In this case the ranking is done by considering all the criteria at the same time.

The PROMETHEE method which is one of the multiple criteria approaches is used in this study and elaborated in subsequent pages.

4.6 Computational Method

The ranking involves a series of computations that are done in the following steps:

Step One: The Preference Function: The data used was sampled from a continuous set and as a result the Gaussian preference function is chosen for this study. The Gaussian preference function is often chosen in PROMETHEE methodology for evaluating continuous data (Villota, 2009). This is expressed below as:

$$p(d) = \begin{cases} 0 \ , when \ d \leq 0 \\ 1 - e^{-\frac{d^2}{2\sigma^2}}, \ when \ d > 0 \end{cases} \quad (4.7)$$

Using this function the only parameter to be defined is the Standard deviation σ. This is computed using the decision matrix of Table 4.2 with the formula below:

$$\sigma^2{}_i = \sum_{j=1}^{n} \left(\frac{(X_{ij} - \mu)^2}{n-1} \right), i = 1, ..6 \quad (4.8)$$

$$\mu_i = \frac{1}{n} \sum_{j=1}^{n} X_{ij} \quad (4.9)$$

Where μ_i is the mean of data

The standard deviation ($\sigma^2{}_i$) and mean (μ_i) for each of the criterion C_i are tabulated in Table 4.3.

Table 4.3: The Mean and Standard deviation of Criteria

Criteria	Mean	Standard Deviation
C_1	97.9783	4.952051763
C_2	114.737	60.10679518
C_3	215.413	132.3364859
C_4	66.4233	30.73794246
C_5	2.00833	4.846278641
C_6	39.205	7.418815943

1. **Calculation of Deviations**

The deviations $d_i(A_k, A_l)$ is obtained through a pair wise comparison of the values of the alternatives on each criterion over all the criteria. It should be noted that the deviations are obtained as below:

$\{X_{ik}-X_{il}\}$ - maximization (4.9a)

$\{-(X_{ik}-X_{il})\}$ – minimization (4.9b)

Where X_{ik} and X_{il} correspond to values of two alternatives on a criterion as provided in the decision matrix of Table 4.2

Table 4.4 presents all possible deviations $d_i(A_k, A_l)$ from the pair wise comparison of all the alternatives on each criterion.

Table 4.4a: Deviations $d_1(A_k, A_l)$ on the maximizing criterion C_1

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	0	0	12.13	0	0
A_2	0	0	0	12.13	0	0
A_3	0	0	0	12.13	0	0
A_4	-12.13	-12.13	-12.13	0	-12.13	-12.13
A_5	0	0	0	12.3	0	0
A_6	0	0	0	12.3	0	0

Table 4.4b: Deviations $d_2(A_k, A_l)$ on the maximizing criterion C_2

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	-70.69	-154.5	-157.02	-85.47	-59.22
A_2	70.69	0	-83.81	-86.33	-14.78	11/47
A_3	154.5	83.81	0	-2.52	69.03	95.28
A_4	157.02	86.33	2.52	0	71.55	26.25
A_5	84.47	14.78	-69.03	-71.55	0	26.25
A_6	59.22	-11.47	-95.28	-97.8	-26.25	0

Table 4.4c: Deviations $d_3(A_k, A_l)$ on the maximizing criterion C_3

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	-87.52	-365.06	-288.88	-207.02	-207.02
A_2	87.52	0	-277.54	-201.36	-11.95	-11.95
A_3	365.06	227.54	0	76.18	158.04	158.04
A_4	288.88	201.36	-76.18	0	81.86	81.86
A_5	207.02	119.5	-158.04	-81.86	0	0
A_6	207.02	119.5	-158.04	-81.86	0	0

Table 4.4d: Deviations $d_4(A_k, A_l)$ on the minimizing criterion C_4

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	-29.48	-71.16	-58.92	-69.03	-80.45
A_2	29.48	0	-41.68	-29.44	-39.55	-50.97
A_3	71.16	41.68	0	12.24	2.13	-9.29
A_4	58.92	29.44	-12.24	0	-10.11	-21.53
A_5	69.03	39.55	-2.13	10.11	0	-11.42
A_6	80.45	50.97	9.29	21.53	11.42	0

Table 4.4e: Deviations $d_5(A_k, A_l)$ on the minimizing criterion C_5

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	-11.75	-11.9	-11.9	-11.9	-11.9
A_2	11.75	0	-0.15	-0.15	-0.15	-0.15
A_3	11.9	0.15	0	0	0	0
A_4	11.9	0.15	0	0	0	0
A_5	11.9	0.15	0	0	0	0
A_6	11.9	0.15	0	0	0	0

Table 4.4f: Deviations $d_6(A_k, A_l)$ on the maximizing criterion C_6

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	0	-3.85	-19.54	-13.94	-4.89	-3.95
A_2	3.85	0	-15.69	-10.09	-1.04	-0.1
A_3	19.54	15.69	0	5.6	14.65	15.59
A_4	13.94	10.09	-5.6	0	9.05	9.99
A_5	4.89	1.04	0	-9.05	0	0.94
A_6	3.95	0.1	-15.59	-9.99	-0.94	0

Step Two: Preference Evaluation

After the computation of the deviations $d_i(A_k, A_l)$, the evaluation $P_i(A_k, A_l)$ which measures the intensity of the decision maker's preference of A_k over A_l should be done. This is done by using mathematical formula stated below:

$$p(d) = \begin{cases} 0 \ , when \ d \leq 0 \\ 1 - e^{-\frac{d^2}{2\sigma^2}}, \ when \ d > 0 \end{cases} \tag{4.10}$$

$d = d_i(A_k, A_l)$

$\sigma^2 = \sigma^2{}_i$

Table 4.5 summarizes the values of $p_i(A_k, A_l)$ for each criterion $C_1 \in C$

Table 4.5a: Values of $p_1(A_k, A_l)$ for criterion C_1

	I=1	I=2	I=3	I=4	I=5	I=6
K=1	0	0	0	0.9502	0	0
K=2	0	0	0	0.9502	0	0
K=3	0	0	0	0.9502	0	0
K=4	0	0	0	0	0	0
K=5	0	0	0	0.9502	0	0
K=6	0	0	0	0.9502	0	0

Table 4.5b: Values of $p_2(A_k, A_l)$ for criterion C_2

	I=1	I=2	I=3	I=4	I=5	I=6
K=1	0	0	0	0	0	0
K=2	0.4992	0	0	0	0	0.0180
K=3	0.9632	0.62171	0	0	0.4829	0.7153
K=4	0.9670	0.6435	0.0008	0	0.5076	0.7339
K=5	0.6361	0.0298	0	0	0	0.0910
K=6	0.3845	0	0	0	0	0

Table 4.5c: Values of $p_3(A_k, A_l)$ for criterion C_3

	I=1	I=2	I=3	I=4	I=5	I=6
K=1	0	0	0	0	0	0
K=2	0.1964	0	0	0	0	0
K=3	0.9777	0.7720	0	0.1527	0.5099	0.5099
K=4	0.9077	0.6858	0	0	0.1741	0.1741
K=5	0.7058	0.3348	0	0	0	0
K=6	0.7058	0.3348	0	0	0	0

Table 4.5d: Values of $p_4(A_k, A_l)$ for criterion C_4

	I=1	I=2	*I*=3	I=4	*I*=5	I=6
K=1	0	0	0	0	0	0
K=2	0.3687	0	0	0	0	0
K=3	0.9314	0.6012	0	0.0762	0.0023	0
K=4	0.8407	0.3679	0	0	0	0
K=5	0.9197	0.5629	0	0.0527	0	0
K=6	0.9675	0.7471	0.0446	0.2175	0/0667	0

Table 4.5e: Values of $p_5(A_k, A_l)$ for criterion C_5

	I=1	I=2	I=3	I=4	I=5	I=6
K=1	0	0	0	0	0	0
K=2	0.9471	0	0	0	0	0
K=3	0.9510	0.0004	0	0	0	0
K=4	0.9510	0.0004	0	0	0	0
K=5	0.9510	0.0004	0	0	0	0
K=6	0.9510	0.0004	0	0	0	0

Table 4.5f: Values of $p_6(A_k, A_l)$ for criterion C_6

	I=1	I=2	I=3	I=4	I=5	I=6
K=1	0	0	0	0	0	0
K=2	0.1260	0	0	0	0	0
K=3	0.9688	0.8932	0	0.2479	0.8577	0.8901
K=4	0.8289	0.6034	0	0	0.5248	0.5961
K=5	0.1952	0.0098	0	0	0	0.0080
K=6	0.1322	0	0	0	0	0

Step Four: Aggregate Preference Index

The result to be used in further analysis is obtained by the computation of the aggregate preference index. As already discussed the aggregate preference index denoted by $\pi(A_k, A_l)$ is mathematically expressed below:

$$\pi(A_k, A_l) = \sum_1^n (W_i P_i(A_k, A_l) \tag{4.11}$$

With k, l = 1,…, n and w_i = 0.16667 is the weight of each criterion. Table 4.6 shows the values of $\pi(A_k, A_l)$ for all six alternatives.

Table 4.6 Aggregate Preference Indices $\pi(A_k, A_l)$

	I=1	I=2	I=3	I=4	I=5	I=6
K=1	0	0	0	0.1584	0	0
K=2	0.3562	0	0	0.1584	0	0
K=3	0.7987	0.4818	0	0.2378	0.3088	0.3526
K=4	0.7492	0.3835	0.0001	0	0.2010	0.2507
K=5	0.5680	0.1563	0	0.1672	0	0.0165
K=6	0.5235	0.18038	0.0074	0.1946	0.0111	0

Step Five: Partial Ranking

From the aggregate preference indices the following analysis can be made:

The partial ranking of the finite set of alternatives is obtained through the equations below:

Positive Outranking flow:

$$\emptyset^+(A_j) = \frac{1}{n-1}\sum_{k=1}^{n} \pi(A_j, A_k) \tag{4.11}$$

Negative Outranking flow:

$$\emptyset^{-}(A_j) = \frac{1}{n-1} \sum_{k=1}^{n} \pi(A_k, A_j) \qquad (4.12)$$

The values of the positive and negative flow are tabulated in Table 4.7

Table 4.7: Values of the positive and negative flow

A_j	$\emptyset^{+}(A_j)$	$\emptyset^{-}(A_j)$
A_1	0.03168	0.5991
A_2	0.10292	0.2403
A_3	0.43583	0.0015
A_4	0.3169	0.1833
A_5	0.1816	0.10418
A_6	0.1834	0.1240

The following conditions below should be well noted when using the partial ranking:

1. A_k is preferred to A_l if and only if one of the following three conditions is satisfied

i. $\emptyset^{+}(A_k) > \emptyset^{+}(A_l)$ *and* $\emptyset^{-}(A_k) < \emptyset^{-}(A_l)$

ii. $\emptyset^{+}(A_k) > \emptyset^{+}(A_l)$ *and* $\emptyset^{-}(A_k) = \emptyset^{-}(A_l)$

iii. $\emptyset^{+}(A_k) = \emptyset^{+}(A_l)$ *and* $\emptyset^{-}(A_k) < \emptyset^{-}(A_l)$

From Table 4.8a, the entries denoted by the dash (-) indicates no preference between any pair of alternatives while entries with the value one (1) show preference of alternative A_k over A_l.

Table 4.8a: Preference table for the six alternatives

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	-	-	-	-	-	-
A_2	1	-	-	-	-	-
A_3	1	1	-	1	1	1
A_4	1	1	-	-	-	-
A_5	1	1	-		-	-
A_6	1	1	-		-	-

2. Indifference: indifference exists between any pair of alternative if and only if the condition below is satisfied:

   $\emptyset^+(A_k) = \emptyset^+(A_l)$ *and* $\emptyset^-(A_k) = \emptyset^-(A_l)$

   There exists no indifference in this case

3. Incomparability: two of the alternatives are incomparable if and only if

4. $\emptyset^+(A_k) > \emptyset^+(A_l)$ *and* $\emptyset^-(A_k) > \emptyset^-(A_l)$

Table 4.8b presents the incomparability between pairs of alternative in this study, entry 1 means A_k is incomparable to A_l

Table 4.8b: Incomparability between pairs of alternative

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	-	-	-	-	-	-
A_2	-	-	-	-	-	-
A_3	-	-	-	-	-	-
A_4	-	-	-	-	1	1
A_5	-	-	-	-	-	-
A_6	-	-	-	-	1	-

The incidence Table 4.8c is considered the same as Table 4.8a

Table 4.8c: the incidence table for alternatives

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	-	-	-	-	-	-
A_2	1	-	-	-	-	-
A_3	1	1	-	1	1	1
A_4	1	1	-	-	-	-
A_5	1	1	-		-	-
A_6	1	1	-		-	-

From the incidence table, the row with the highest number of one's is the row with the highest number of directed arcs and the corresponding alternative in the row is the best alternative.

The graph of partial ranking based on Table 4.8c is given in Figure 4.1 below:

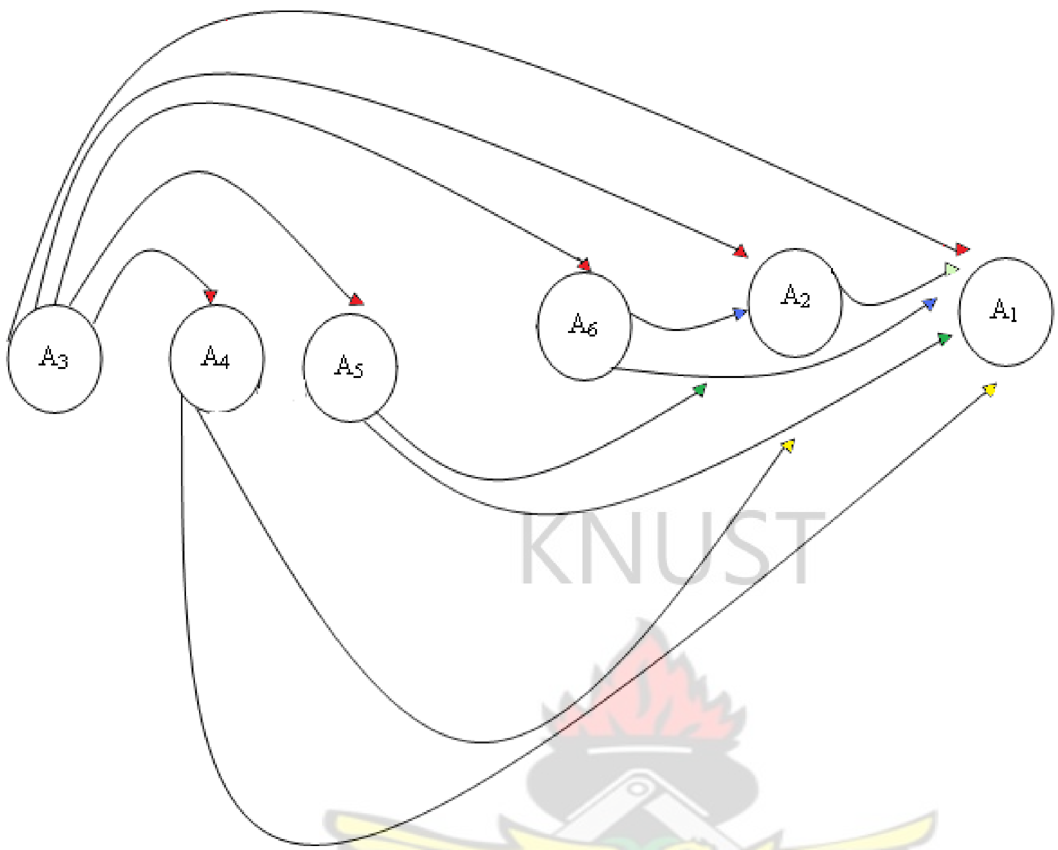


Figure 4.2: Graph of partial ranking

From Figure 4.1 graph of partial ranking, it is realized that there are no connections between A_4, A_5 and A_6 thus three alternatives are incomparable, therefore we proceed to use the complete ranking method.

Step Six: Complete Ranking

In the complete ranking, the alternatives are analyzed using the net flow which is mathematically given below:

$$\phi(A_j) = \phi^+(A_j) - \phi^-(A_j) \quad (4.13)$$

The values of the net flow for all six alternatives are summarized in Table 4.9

Table 4.9: Values of the net flow for all six alternatives

A_j	$\emptyset^{+}(A_j)$	$\emptyset^{-}(A_j)$	$\phi(A_j)$
A_1	0.03168	0.5991	-0.5674
A_2	0.10292	0.2403	-0.1374
A_3	0.43583	0.0015	0.4343
A_4	0.3169	0.1833	0.1336
A_5	0.1816	0.10418	0.0774
A_6	0.1834	0.1240	0.0594

1. Preference exists between a pair of alternatives (A_k, A_l) if

$$\emptyset\ (A_k) \neq \emptyset\ (A_l)$$

Considering the alternative (A_k, A_l), alternative A_k is preferred to alternative A_l if and only if $\emptyset\ (A_k) > \emptyset\ (A_l)$ otherwise A_k is not preferred to alternative A_k

From Table 4.10, the entries denoted by the dash (-) indicates no preference between any pair of alternatives while entries with the value one (1) show preference of alternative A_k over A_l.

Table 4.10: Pair wise comparison of net flow

	A_1	A_2	A_3	A_4	A_5	A_6
A_1	-	-	-	-	-	-
A_2	1	-	-	-	-	-
A_3	1	1	-	1	1	1
A_4	1	1	-	-	1	1
A_5	1	1	-		-	1
A_6	1	1	-	-	-	-

2. Indifference exists between two alternatives if and only if

$$\emptyset^{+}(A_k) = \emptyset^{+}(A_l)$$

In this case, no indifference exists

The graph of the complete ranking based on Table 4.10 is shown in Figure 4.2 below:

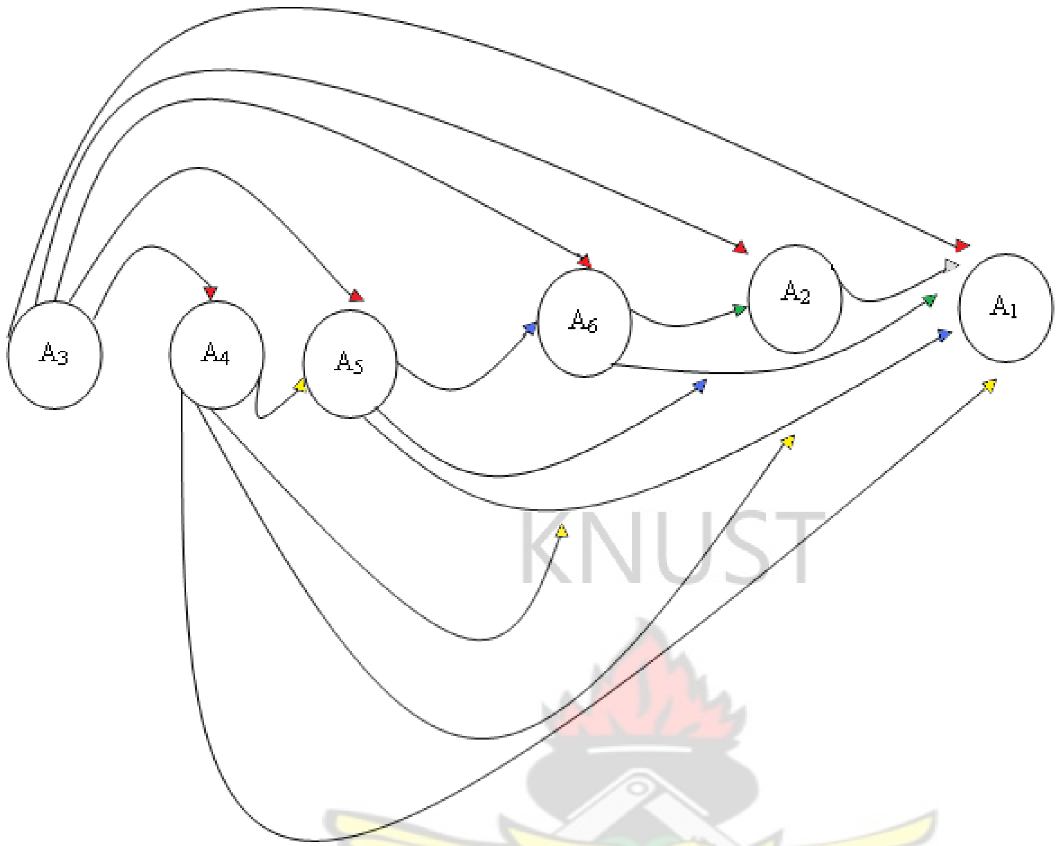


Figure 4.3: the graph of the complete ranking

The ranking is done based on the number of directed arcs that is recorded by each alternative such that the best alternative A_3 is the one with the highest number of directed arcs and the alternative A_1 with no directed arc becomes the worst alternative.

From Table 4.11, Column one, contains all alternatives (A_1 .., A_6), the entries in the second column 0…,5 denotes the number of arcs emanating from nodes (A_1 .., A_6) in Figure 4.3. The third column is the position of alternatives based on the number of arcs.

Table 4.11: Ranking of Six alternatives using Promethee

Alternatives	Number of directed Arcs	Ranking Position
A_1	0	6th
A_2	1	5th
A_3	5	1st
A_4	4	2nd
A_5	3	3rd
A_6	2	4th

From table above $A_3 > A_4 > A_5 > A_6 > A_2 > A_1$ where ">" means "ranks better than "

Hence, the alternative A_3 is the best alternative to be used by the telecommunication provider to get the best performance for call terminations

4.7 Discussion

Results obtained ranked all carriers from best to worst using the complete ranking in the Promethee method since there was incomparability in the system the partial raking could not be used to rank alternatives.

CHAPTER 5

CONCLUSIONS AND RECOMMENDATIONS

5.1 Conclusion

From the multi criteria ranking of Telecommunications carriers using the Promethee Method the following conclusions can be drawn:

1. The multi carrier linkage problem can be modeled as a multi decision making problem using a decision table in the Promethee method.
2. The promethee method can be used to rank all carriers linking a network to a given destination when the criteria for the decision making are known, and this is achieved using either the partial or complete rankng.
3. From results obtained, incomparability existed when using the partial ranking in Promethee method. In the Promethee complete ranking, no incomparability occurred.
4. The promethee method ranked all alternatives from best to worst using pair wise comparisons of alternatives.

5.2 Recommendations

Based on the research the following are recommended

1. The Promethee method can aid telecommunication operators to rank carriers in the telecommunication network with high level of accuracy.

2. Network operators need a simple understanding of pair wise comparison used in the Promethee method which makes implementation of this method very simple.

3. I recommend that further studies be made using other methods.

4. I also recommend that this approach can be helpful in making any multi criteria decision in any industrial field.

5. This study is limited to the Global System for Mobile (GSM) telecommunication network.

6. The Promethee Method was chosen for this study because the use of deviations and preference function gives optimal results.

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