KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOGY

Forecasting Inflation in Ghana using Particle Swarm Optimization (PSO)

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Declaration

I hereby declare that this submission is my own work towards the award of the M.Phil degree and that, to the best of my knowledge, it contains no material previously published by another person nor material which had been accepted for the award of any other degree of the university, except where due acknowledgement had been made in the text.

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Dedication

This book is dedicated to the Atiso and Agbasse families.

Abstract

The phenomenon of inflation has proven to be both important and unpredictable. The goal of every Central Bank is to achieve and maintain a desirable rate of inflation in a fiscal year. Major macroeconomic policies pertaining to minimizing inflation are implemented based on predictions made into the future. In this study the Particle Swarm Optimization (PSO) intelligent method was used to make in-sample forecasts of inflation figures over a period. The Generalized Autoregressive Conditional Heteroscedastic (GARCH) model was used to obtain an inflation model. The proposed method was implemented using Matlab (2012a) and forecasts error were measured using MSE, MAPE and MAD. The results obtained revealed that the method yielded lower errors for the inflationary data sets used.

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Chapter 1

INTRODUCTION

1.1 Background of Study

The traditional function of every Central Bank is to achieve and maintain price stability, which is a pre-requisite for sustainable development. Inflation has been a bigger opposing force affecting government and policy makers decision in the world.

Ghana as a nation was in the single digit inflation measure during her early stages of development, thus after her independence until she had her first taste of double-digit inflation in 1964, Sowa and Kwakye (1993). This was followed by a short period with no pressure on inflationary rates from 1967 to 1971 with inflation below 10 % per annum. After this period, levels of inflation remained generally ranging between 10 % to 123 % from 1972 to 1983. However, Ghana suffered a massive increase in inflation level in 1983 which caused government and policy makers to introduce control measures to reduces the increasing levels. This inflation rate was attributed to excessive demand pressures fueled by monetary growth attendant upon fiscal deficit financing. Upon government and policy makers intervention at stabilizing the increment by policies like "credit squeeze" on the banking system, inflation levels remained in the double-digit zone till 2011 when Ghana recorded another single-digit again which lasted till 2013. The economy of Ghana has been experiencing rapid surges in the inflation rate after this period due to factors like large public sector borrowing and government spending, increase in prices of goods and services which could lead the country back to the 1983 surge.

Laidler and Parkin (1975), defines Inflation as a process of continuously rising prices or falling in the value of the currency of an economy.

Inflation is mainly linked to monetary issues of the economy both locally and globally. When prices become increasingly unstable in an economy, it leads to high inflationary rates, which in the long run affects the currency value. Currently Inflation in Ghana has led to several negative effects on the economy most especially the employment sector. Inflation alongside other macroeconomic indicators like Employment, Consumer price index (CPI), Balance of Payments (BOP), Interest Rates etc., are usually considered when governments and policy makers want to measure the state of the economy. Recent hikes in inflation values can be attributed to governments large public sector borrowing and negative net export rate. During the first quarter of 2015, the government of Ghana borrowed a total of about GH¢16.08 billion from the domestic market, this included contingent liability from Bank of Ghana and Ghana Cocoa Board. Government is to borrow GH¢25.42 billion from the domestic market by end of first half of the year. Government's inability to manage its borrowing culture clearly depicts a struggling government appetite for funds. The government of Ghana was expected to comply with its target for first quarter of the year 2015 by borrowing GH¢12.71 billion, however as mostly expected, it overshot it targets by GH¢3.37 billion (First Quarter Economic Report, 2015).

Inflation is measured in several ways but Ghana measures its inflation based on consumer price index (CPI). CPI inflation generally picked up during the year, exceeding the limit target band of 11.56 % to end the year at 13.5 %, from 8.8 % in 2012. The rise in CPI inflation was largely driven by the non-food component. Non-food inflation increased from 11.8 % at the beginning of the year to 18.1 % in December while food inflation declined to 7.2 % from 8.0 % in 2012. The increase in CPI inflation during the year was due to several factors including petroleum and utility price adjustments and depreciation of the cedi (BOG, 2013). The study seeks to forecast inflation of the Ghanaian economy using intelligent optimization methods. Forecasting future trends in inflation is a very difficult task as it is influenced by a lot of economical factors like unstable volatility in the financial market. A little has been done to the best of my knowledge in using intelligent search methods as forecasting tools especially in the area of inflation.

1.1.1 Problem Statement

Inflation is a major indicator of the well being of an economy, hence governments and policy makers rely on its prediction in making decision and formulating policies to further improve the economy. Yet for generations of governments and investors the phenomenon of inflation has proven to be unpredictable. The desired inflation target of below 10% is expressed in terms of an annual rate of inflation based on the Consumer Price Index (CPI). Although the Bank is not bound by law to explain to the Ministry of Finance or to parliament if the target is not achieved, the Governor may be summoned to the Finance Committee of parliament to explain developments within the economy. However inflation targets have barely been achieved by many central banks.

In predicting inflation, it is very important to develop a reliable model with minimal error measures. Even though there are statistical model construction tools that have proven to be effective, the accuracy of existing models needs improvement through exploratory studies. Intelligent optimization methods have rarely been explored to best of my knowledge in predicting financial time series data even though Particle Swarm Optimization has been successful in forecasting in other areas like weather temperature forecasts.

1.1.2 Justification

Over the last quarter of the 20th century, a consensus developed that price stability should be the primary focus of monetary policy. It is now agreed that the economic well-being of the general population is best served by keeping inflation low and stable and that, in order to deliver on this objective, central banks should be independent of political authorities, but receive a clear mandate for which they are then held accountable (BIS, 2009).

When it comes to economic indicators like inflation, decision making for the fiscal year by governments and policy makers becomes very critical. Hence to make the best of decisions, the past must revisited and the future sought to be known. Making very accurate predictions into the future will go a long way to help governments and policy makers to arrive at the best decisions on monetary and macroeconomic policies to curtail the growth of inflation.

This study will also add to the already existing knowledge on predicting inflation and will help other researchers in making further developments with different approaches.

1.1.3 Objectives

The objectives were to :

1. Develop an inflation model.
2. Forecast using Particle Swarm Optimization.

1.1.4 Summary

The study has five chapters. Chapter one captures the background and scope of the study. It also highlights on the objective, problem statement, justification and summary of the study.

Chapter two stresses more on the existing literature on the study. It tackles forecasting methods in the financial time series environment and related works already done considering the methodology of the study. This chapter gives examples of some works already done using statistical methods and a few works on the proposed method by the study.

Chapter three gives accounts of the proposed methodology of the study. Particle Swarm Optimization method uses an inflation model obtained using time series data from the Bank of Ghana.

Chapter four of the study gives full results on the study and discussion. Some relevant information and facts revealing is done in this chapter. This chapter focuses on bringing out the relevance of the study.

Chapter five draws conclusion from the analysis and deductions made from chapter four and make recommendations.

Chapter 2

REVIEW OF LITERATURE

2.1 Introduction

A lot of work has been done on forecasting inflation in Ghana with some time series models like Autorgressive Integrated Moving Average (ARIMA), Autoregressive Conditional Heteroscedastic (ARCH), Generalized Autorgressive Conditional Heteroscedastic (GARCH), Exponential GARCH and many more models but a few research has been done using intelligent methods like the Particle Swarm Optimization method.

2.1.1 Forecasting in Practice

In Burruss and Kuettner (2003), a forecasting method, called the prediction method of the Product Life Cycle (PLC) is proposed to more accurate forecasting products with high uncertainty, a steep curve of obsolescence, and a short life cycle. A short life cycle is usually a life cycle from 9 to 18 months. The article describes three requirements for products to be provided by this method adequately within the industry of electronic consumer products. They should have lifecycle phases defined in the introduction to maturity, then at the end of life, peak demand during the introductory phase, followed by a gradual decline over settlement maturity, and a steep End - Of - Life (EOL) drop-off that is often caused by roll-overs products provided. A summary step by step method of forecasting the life cycle of the product, as used by Hewlett-Packard, is described. The first step is to analyze historical data to generate a life cycle form of commodities for the product family or group for which the forecast is to be created.

The second step is to develop a model of seasonality and adjust the forecast model accordingly. The third step is to develop a model for the planned price cuts. The fourth step is to use the drop of price model that was developed in step three to readjust forecasts seasonally adjusted. The fifth step is to develop a model of how product shipments are affected by special events. The sixth and final step is to apply the model of special events for the forecast model, which should already be set to seasonality and lower prices. The product life cycle prediction method has many advantages in forecasting. It gives forecasters the ability to track the impact of factors such as seasonality, price cuts, special events and sales, both individually and collectively. It also improves forecast accuracy for products with high uncertainty and a short life cycle. Hewlett-Packard believes that the company is to save $ 15 million annually as a result of improved forecast accuracy because of the prediction method of the life cycle of the product.

There are many examples of prediction methods in practice. With Fisher et al (1994), prediction is used to estimate future sales adequately at Sport Obermeyer Limited., a fashion clothing company of ski-apparel business. Due to changing fashion trends and a growing need to generate accurate forecasts, Sport Obermeyer Ltd. decides to adopt a new approach called the ”forecast accurate response”. The approach incorporates two basic elements that many other forecasting systems abound. The first of these elements is that the approach takes into account the amount of lost sales. The second element is to distinguish between products for which demand is easily predictable and products for which demand is unpredictable. By including these elements in the forecasting methodology, the company gains the ability to use the flexible production capacity and faster cycle time more efficiently. With the implementation of the ”precise response” approach, Sport Obermeyer was able to almost completely eliminate the ski clothing production costs that customers do not want (overproduction) and the cost of not producing clothes ski that customers want (under production).

2.1.2 Modern Perspective on Forecasting

According Fildes and Godwin (2007), 149 surveys were collected from forecasts doctors from a wide range of industries, to consider the use of judgment in the forecast and whether the procedures for predicting the business were consistent with the principles. The forecasters surveyed were responsible for the provision of a number of elements from one point to 34 million items , with a median of 400 articles . The survey showed that the majority of forecasters forecasts on a monthly basis . The forecast on a weekly basis was the second most common ; However, there were more than double the amount of forecasters forecast on a monthly basis than any other period of time. The principles established at this time to exercise judgment , as in the study were as follows: Principle 1: Use quantitative rather than qualitative methods;

Principle 2: Limit the subjective adjustments of quantitative forecasting;

Principle 3: Adjust for expected events in the future.

Principle 4: Ask experts to justify their forecasts in writing;

Principle 5: Use structured procedures to integrate methods and quantitative judgment.

Principle 6: Combine forecasts approaches that differ; and

Principle 7: If combining forecasts, start with equal weights.

Principle 8: Compare past performance of various forecasting methods; and

Principle 9: Solicit feedback on forecasts.

Principle 10 is to use error measurements that adjust scale in the data.

Finally, Principle 11 is to use several measures of forecast accuracy. The results of this study show that many organizations are falling short of best practice in forecasting. Many rely heavily on the informal judgment and insufficiently on statistical forecasting methods and often blur with their decisions. Many organizations could improve forecast accuracy if they were following the basic principles such as the limitation of judgment quantitative forecast adjustments, which requires managers to justify their adjustments in writing, and evaluate the results of interventions judgement.

Jain, (2008a) explains that there are three characteristics which indicate forecast how much a company forecast error can afford. These three characteristics are the cost of an error, the adaptability of a business, and industry benchmarks. The cost of forecast error comes from two types of forecast error; over-forecasting and underestimation. Over estimation error results in excess inventory in anticipation that leads to discounts on the product to try to sell excess stocks. Underestimation results in lost sales and production costs increased due to an increase in production rates. It is very beneficial for a company to have the ability to adjust quickly to an error. The adaptability of a company and its products depends on the delivery. The more time passes, the more a company can adapt to predict errors, enabling enterprises with shorter deadlines luxury greatest forecasting error. Industry benchmarks show how society error they can afford. Comparing the forecast errors with other companies in the industry, a company can determine whether they provide with the precision necessary to stay competitive. This also allows the company to set goals for how forecast error is reasonable and where the company should focus on their forecast errors. A result observed in this study was that during the forecast calculation error, the error will be less when calculating the error for major product groups (with a larger total volume) rather than a product itself (with relatively low volume). In general, the larger the group the error is calculated for, the error will be more compared to the forecast error of each product. This is due to the combination of total sales and related forecasts for the major product groups, allowing the offset of more than forecast by underestimating between different products. Also, as expected, the study showed that the error increases as the forecast horizon of the forecast is increased.

Jain (2008b) explains that there are three types of predictive models; time series (univariate) and of cause and effect, and judgment. In the time-series modeling, past data is used to determine the best statistical adjustment. Time series models include: simple moving averages, simple trend, exponential smoothing, decomposition and Autoregressive Integrated Moving Average (ARIMA). Patterns and cause and effect are used when there is a cause (independent variable) and an effect (dependent variable). For example, if the number of vehicle sales are dependent on the amount of money spent on advertising then the cause is the amount of money spent on advertising and the effect is the number of vehicle sales. A model for forecasting cause and effect is generally appropriate in scenarios where there is a strong relationship between the variables of cause and effect.

Models normally used include: regression, econometric, and the neural network. Judgment models are used when there is no historical data or the data that exist are not applicable. This scenario is in play when a forecast for the new product is being prepared or in cases concerning the sale of luxury goods (fashion products may follow different trends). Judgment models include: Analog, Delphi, Broadcast, Performance Evaluation Review Technique (PERT), Investigation, and the

scenario.

He noted further that the most common type of prediction model used in the industry today are time series models , which account for 61 % of all forecast models used . Time series models are followed by cause and effect models at 18 % and judgment models at 15 %. Five percent of companies surveyed use custom ”home-grown” models . Further analysis of time series models show that average or simple trend models account for 57 % of the time series models . This is followed by exponential smoothing at 29 % , ARIMA at7 % , and the decomposition at 6 %.

Saffo (2007) explained that the difference between prediction and the forecast is that the prediction is concerned with certainty the future while forecasting looks at how hidden currents in the present, influence the possible changes in direction. The main aim of the forecast is to identify the range of possibilities. Six rules are given for effective forecasting. Rule 1 is to define a cone of uncertainty. The cone of uncertainty is used to help the decision maker to exercise a strategic judgment. The most important factor with the cone of uncertainty is to define its width, which is a measure of the overall uncertainty. When you make the cone of uncertainty, a cone that is too small is worse than that is too broad. Initially defining a cone rises too much on the ability to generate hypotheses about the results and any replies. A cone that is too narrow can cause unpleasant surprises. To create a cone of uncertainty, we must be able to make the proper distinction between highly improbable outliers and wildly impossible outliers. Rule 2 is in search of the curve S. Many important developments generally follow the shape of S-curve of a power law: ”Change starts slowly and gradually, putters long silence, then suddenly explodes, possibly gradual decrease. and even down ”It is important to identify an S curve pattern as it begins to emerge well before the inflection point. Rule 3 is to embrace the things that do not match. All that portion of the S curve to the left of the inflection point is paved with indicators that are subtle pointers when aggregated become powerful notes of things to come. The best way for forecasters to identify an emerging S curve is to become attentive to the things that do not match. Because of our aversion to uncertainty and concern at this time, we tend to ignore indicators that do not fit into familiar boxes. Rule 4 is to have strong opinions weakly. The author claims that one of the biggest mistakes a forecaster can do is to rely on a strong information component apparently because she happens to reinforce the conclusion that he or she has already achieved. In anticipation, many locking weakness of information is more reliable than a point or two of strong information. Rule 5 is to look back twice as far as you look forward.

When looking for parallels, always look back at least twice as far as you are impatient. The hardest part of looking back is when the story does not match. Rule 6 is knowing when not to make forecasts. There are times when the forecast is easy, and other times when it is impossible. The cone of uncertainty is not static; it expands and contracts as current events are held. Thus, there are moments of uncertainty when the cone expands to a point where the forecaster should avoid making a forecast. When the forecast, the amount of the forecast error greatly affects the profitability of the product. Thus, being able to predict with as little error as possible prediction is desired.

2.2 Consumer Price Index (CPI) as a Measure of Inflation

Halka and Leszczynska (2012) in their paper titled ”What Does the Consumer Price Index Measure? Bias Estimates for Poland” addressed the problem of bias in the measure of inflation as provided by the price index of consumer goods and services (CPI) in Poland. They estimated the size of the bias resulting from two sources: substitution effect and the application of plutocratic weights in index calculation. Their study involved a comparison of the official consumer price index in Poland with superlative indices and the democratic index in 2005-2011 period. The survey did not identify an upward CPI bias and the findings indicated a slight understatement of the CPI stemming from both sources (respectively: 0.1 and 0.3 pp. per annum). A downward bias due to substitution effect was rather unusual. A deeper analysis pointed to two possible explanations to this phenomenon. On the one hand, overstatement may be absent due to frequent adjustments in the weights used for CPI calculation, which resulted in a better match between the index and the changes that occurred in the consumption structure. On the other hand, it was proved that in the period analyzed, there was a faster-than-CPI rise in the prices of those goods and services demanded for which was relatively inelastic, and a positive growth of households' real income was observed over the recent decade. When looking into the "plutocratic gap", it was found that the CPI (plutocratic) index for Poland was lower than the democratic index. Such a result of the "plutocratic gap" survey was in line with the research conducted for other countries.

Ocran M.K. (2007) in his paper examined the causes of inflation in Ghana between 1960 and 2003. Stylized facts about the inflation experience indicated that following the exit from the West African Currency Board inflation management had been ineffective despite two decades of vigorous reforms. He used the Johansen cointegration test and an error correction model which identified inflation inertia, changes in money supply, changes in Government of Ghana Treasury bill rates as well as changes in exchange rate as determinants of inflation in the shortrun. Inflation inertia was found to be the dominant determinant of inflation in Ghana. His findings suggested that to make Treasury bill rates more effective as nominal anchor inflation, expectations ought to be reduced considerably.

The choice of this formula is justified with the ease of its calculation and publication. On the other hand, this method of construction and computation the index is not free from certain limitations. When collecting price data, it is difficult to keep up with changes in consumers' behaviour (i.e. the substitution of goods becoming more expensive with their cheaper equivalents), account for the changing quality of the purchased goods or for the arrival of new goods in the market in the period between the weight-setting and the price survey. Moreover, the construction of weights may lead to an overrepresentation of households with the highest consumption spending. Therefore, the CPI may be flawed with a certain bias and thus it may fail to fully reflect the changes in the cost of living. The fact that this problem is by no means purely methodological, as it might seem, is given so much attention stems from the essential role of inflation in economic processes. Failure to measure inflation accurately with CPI may have a distorting influence on economic policy, including monetary policy. The role of the CPI in monetary policy is particularly crucial in countries where central banks rely on direct inflation targeting. Between 1990 and 2010, approximately 10 developed countries and 15 developing ones embarked on this strategy (Svensson, 2010).

Silver M. and Armknecht (2012) in their working paper compiled Consumer price indexes (CPIs) at the higher (weighted) level using Laspeyres-type arithmetic averages. Their paper questioned the suitability of such formulasand considered two counterpart alternatives that used geometric averaging, the Geometric Young and the (price-updated) Geometric Lowe. The paper provided a formal decomposition and understanding of the differences between the two. Empirical results was provided using United States CPI data. The findings led to an advocacy of variants of a hybrid formula suggested by Lent and Dorfman (2009) that substantially reduces bias from Laspeyres-type indexes.

According to Al-Hamidy from the BIS (2009), there are several measures of inflation conceptually, each having its own merits and shortcomings, but the one that is most appropriate and commonly used for monitoring inflation is the Consumer Price Index (CPI). It covers prices of those items that enter into the representative consumption basket of the household sector and is typically available on a monthly basis with short time lags. Once published, it is rarely revised. It is also widely known and used in revising contracts for inflation. Thus, on grounds of transparency and timeliness, the CPI is the preferred index for monitoring inflationary trends. However, to make the CPI credible, it is important that it should be computed by an independent national statistical agency, separate from the central bank, that should have an elaborate organizational set-up to collect detailed, reliable and up to date data on prices on a frequent basis and to undertake family budget surveys periodically to incorporate in the representative consumption basket the changes that take place in consumers' needs and preferences over reasonable time periods. It should also make suitable adjustments in the CPI should there be a substantial quality change in any item that is included in the consumption basket.

Wiesiolek and Kosoir in the same report suggested that, the notion of core inflation is one of the most important concepts for the conduct of monetary policy. Core inflation measures are frequently referred to in discussions about monetary policy decisions because of their usefulness as analytical tools and as guides for these decisions. They are also commonly used to communicate and explain monetary policy decisions to the public. Finally, core inflation measures are also sometimes used to specify inflation targets. The usefulness of core inflation measures for monetary policy stems from the fact that they should in principle distinguish between permanent and transitory price movements, or between generalized inflation and relative price movements.

However, despite the widespread presence of core inflation in monetary policy conduct, its measurement is not unproblematic. There are a plethora of different methods for computing core inflation and of different criteria that may be used to evaluate the core inflation measures. Moreover, different core inflation measures can show a varying degree of usefulness for distinct policy purposes. In addition, their usefulness can vary over time, with the changes in the nature of inflationary developments. In the Ghana Economic Review and Outlook (2013), the Ghana Statistical Service (GSS) suggested that the CPI basket, which was established about 10 years ago, is now obsolete and becoming increasingly inappropriate. Rapid technological changes have also led to the appearance of new items which are substitutes for the original ones. In the nature of things, however, the new items differ, sometimes significantly, in quality - compact discs and audio cassette tapes or mobile phones increasingly replacing land lines. The improved quality has typically meant higher prices and hence automatic substitution would impart an upward bias to CPI inflation. On the other hand, deletions could create the so-called ”zero-entry” problem which could result in a downward bias in the CPI inflation. The GSS plans to replace the consumer basket later this year or early next year. CEPA urges that this be done. Further, to avoid the credibility issue in future the GSS should improve its communications and keep users better informed about changes in data gathering and methodology and their likely consequences for the interpretation of the statistics.

2.3 Review of Related Work

Wang and Zhao (2009) presented a paper on an ARIMA model which used particle swarm optimization algorithm (PSO) for model estimation. Because the traditional estimation method was complex and may obtain very bad results, PSO which can be implemented with ease and has a powerful optimizing performance is employed to optimize the coefficients of ARIMA. In recent years, inflation and deflation plague the world moreover the consumer price index (CPI) which is a measure of the average price of consumer goods and services purchased by households was usually observed as an important indicator of the level of inflation, so the forecast of CPI was focused on by both scientific community and relevant authorities. Furthermore, taking the forecast of CPI as a case, it was illustrated that, the improvement of accuracy and efficiency of the new method and the result showed it was predominant in forecasting.

Behnamian and Ghomi (2010) stated that forecasting has always been a crucial challenge for organizations as they play an important role in making many critical decisions. Much effort has been devoted over the past several decades to develop and improve the time-series forecasting models. In these models most researchers assumed linear relationship among the past values of the forecast variable. Although the linear assumption makes it easier to manipulate the models mathematically, it can lead to inappropriate representation of many real-world patterns in which non-linear relationship is prevalent. This paper introduces a new time-series forecasting model based on non linear regression which has high flexibility to fit any number of data without pre-assumptions about real patterns of data and its fitness function. To estimate the model parameters, they used hybrid meta-heuristic which had the ability to estimate the optimal value of model parameters. The proposed hybrid approach was simply structured, and comprised two components: a Particle Swarm Optimization (PSO) and a simulated annealing (SA). The hybridization of a PSO with SA, combining the advantages of these two individual components, was the key innovative aspect of the approach. The performance of the proposed method was evaluated using standard test problems and compared with those of related methods in literature, ARIMA and SARIMA models. The results in solving on eleven (11) problems with different structure revealed that the proposed model yields lower errors for these data sets.

Eshun Nunoo (2013) used both the econometric and ANN methods to predict inflation in Ghana. The econometric models (AR and VAR) and the ANN models (NAR and NARX) were applied to the monthly year-on-year inflation data from Jan. 1991 to Dec. 2011. The models were estimated using the data from Jan. 1991 to Dec. 2010 so as to forecast for the period Jan. 2011 to Dec. 2011. It was found that the forecast errors of the ANN models were lower than those of the econometric models. Thus, the ANN predicts inflation better than the econometric models.

Engle (1982) used the Autoregressive Conditional Heteroscedastic (ARCH) model to estimate the means and variances of inflation in the United Kingdom. He employed the Lagrange Multiplier model validity test based simply on the autocorrelation of the squared Ordinary Least Square (OLS) residuals against maximum likelihood. It was realized that the OLS gave better inflation variances as compared to the maximum likelihood even though it was biased at some point. The ARCH model proved to be very useful in improving the performance of the least square models and for obtaining more realistic forecast variances.

Frimpong and Oteng-Abayie (2006) in studying volatility of returns on the Ghana Stock Exchange (GSE) used the random walk (RW), GARCH, EGARCH and TGARCH models. The unique 'three days a week' Databank Stock Index (DSI) was used to study the dynamics of the GSE volatility over a 10-year period. Their results revealed that the DSI exhibited the stylized facts such as volatility clustering, leptokurtosis and asymmetry effects associated with stock market returns on more advanced stock markets. The random walk hypothesis was also rejected and overall, the GARCH (1,1) model outperformed the other models under the assumption that the innovation follows a normal distribution.

Abbod and Deshpande (2008) in their paper showed how the performance of the basic algorithm of the Group Method of Data Handling (GMDH) can be improved using Genetic Algorithms (GA) and Particle Swarm Optimization (PSO). The new improved GMDH was then used to predict currency exchange rates: the US Dollar to the Euros. The performance of the hybrid GMDHs were compared with that of the conventional GMDH. Two performance measures, the Root Mean Squared Error (RMSE) and the Mean Absolute Percentage Errors (MAPE) showed that the hybrid GMDH algorithm gives more accurate predictions than the conventional GMDH algorithm.

Although Particle Swarm Optimization (PSO) is used in variety of applications; it has limitations in the training phase. In this work, a new enhancement for PSO was proposed to overcome such limitations. The proposed PSO optimization consists of two stages. In the first stage, a Gaussian Maximum Likelihood (GML) was added to PSO to update the last 25% of swarm particles, while in the second stage, a Genetic Algorithm was applied whenever there is lethargy or no change in the fitness evaluation for two consecutive iterations. Finally, the proposed PSO was applied in time series predictions using Local Linear Wavelet Neural Network (LLWNN). The work was evaluated with three different data sets.

Implementation of the proposed PSO showed better results than conventional PSO and many other hybrid PSOs proposed by others (Albehadili et al, 2014).

Mohapatra et al (2013) wrote a paper on a comparative study of particle swarm optimization (PSO) based hybrid swarmnet and simple Functional Link Artificial Neural Network (FLANN) model. Here both the models are initially trained with Least Mean Square (LMS) algorithm, then with PSO algorithm. The models were forecasting the stock indices of two different datasets on different time horizons i.e. one day, one week, and one month ahead. The performance was evaluated on the basis of Root Mean Square Error (RMSE) and Mean Absolute Percentage Error (MAPE). It was verified that PSO based hybrid swarmnet performed better in comparison to PSO based FLANN model, simple hybrid model trained with LMS and simple FLANN model trained with LMS.

Mahnam and Ghomi (2012) presented a hybrid algorithm to deal with the forecasting problem based on time variant fuzzy time series and particle swarm optimization algorithm, as a highly efficient and a new evolutionary computation technique inspired by birds' flight and communication behaviors. The proposed algorithm determined the length of each interval in the universe of discourse and degree of membership values, simultaneously. Two numerical data sets were selected to illustrate the proposed method and compare the forecasting accuracy with four fuzzy time series methods. The results indicated that the proposed algorithm satisfactorily competes well with similar approaches.

Ngailo's (2011) study was based on financial time series modelling with special application to modelling inflation data for Tanzania. In particular the theory of univariate non linear time series analysis was explored and applied to the inflation data spanning from January 1997 to December 2010. The data was obtained from Tanzania National Bureau of Statistics. Time series models namely, the autoregressive conditional heteroscedastic (ARCH) (with their extensions to the generalized ARCH (GARCH)) models were fitted to the data. The stages in the model building namely, identification, estimation and checking was explored and applied to the data. A best fitting model was selected based on how well the model captures the stochastic variation in the data (goodness of fit). The goodness of fit is assessed through the Akaike information criteria (AIC) , Bayesian information criteria (BIC) and standard error (SE): Based on minimum AIC and BIC values, the best fit GARCH models tend to be GARCH(1, 1) and GARCH(1, 2). After estimation of the parameters of selected models, a series of diagnostic and forecast accuracy test were performed. Having satisfied with all the model assumptions, GARCH(1, 1) model was judged to be the best model for forecasting.

Based on the selected model, we forecasted twelve (12) months inflation rates of Tanzania in-sample period (that is from January 2010 to December 2010). From the results, it was observed that the forecasted series are close to the actual series.

Ying-Wong and Sang-Kuck (2009) introduced a time series model that captured both long memory and conditional heteroskedasticity and assessed their ability to describe the US inflation data. Specifically, the model allowed for long memory in the conditional mean formulation and used a normal mixture GARCH process to characterize conditional heteroskedasticity.The proposed model yielded a good description of the salient features, including skewness and heteroskedasticity, of the US inflation data. Further, the performance of the proposed model compared quite favorably with, for example, ARMA and ARFIMA models with GARCH
errors characterized by normal, symmetric and skewed Student-t distributions.

Javed et al (2012) examined the relationship between Inflation and Inflation uncertainty for Pakistan using monthly data over 1957:1-2007:12. ARMA-GARCH model was applied to estimated conditional volatility of inflation. Findings of the study supported Friedman-Ball hypothesis for Pakistan as Granger-causality test revealed that inflation affects inflation uncertainty positively. There was no evidence for inflation uncertainty affecting inflation rates and only unidirectional relation was evident with causality running from inflation to inflation uncertainty. High volatility persistence for inflation was also confirmed. Results of the study may be useful for policymakers at central bank to devise more efficient monetary policy.

Mbeah-Baiden (2013) carried out a study on Inflation in Ghana. Secondary data consisting of year-on-year inflation data for each month from January 1965 to December 2012 was used in this study. The total number of data points is therefore 576. The year-on-year inflation is the percentage change in the consumer price index (CPI) over a twelve-month period was used to measure changes over time in the general price level of goods and services that households acquire for the purpose of consumption. The monthly year-on-year inflation was collected by the Ghana Statistical Service. The data was analyzed and the three selected time series models (i.e. the ARCH, GARCH and the EGARCH) for the non-constant conditional variance series were estimated using the maximum likelihood estimation process. The ARCH effects were tested using the Ljung-Box statistics Q (m) test (McLeod and Li, 1983) and the Lagrange multiplier test of Engle (1982) as this forms the basis for building ARCH-type models. The partial autocorrelation function (PACF) of the squared residuals was used to determine the order. Next the estimation of the parameters for the tentative models was carried out using the maximum likelihood estimation method. In this study, it was assumed that the residuals are normally distributed since it is the most commonly used distribution and it makes the estimation of the parameters relatively easier. Lastly, the estimated models were checked to verify if it adequately represents the series. Diagnostic checks were performed on the residuals to see the validity of the distribution assumption. In particular, the measure of skewness, kurtosis and Quantile-to Quantile plot (Q-Q plot) of the residuals was used to check for the validity of the distribution assumption.

All the analyses were carried out with statistical software MINITAB 16.0 and EVIEWS 5.0.

The ARCH (2) model was selected as the best fit model for predicting the monthly rate of inflation amongst the ARCH (m) models. In the case of the GARCH (m,n) and EGARCH (m,n) models, the order (1,2) was the best choice amongst the four different order combinations. Thus the GARCH (1,2) and EGARCH (1,2) models were selected as the best fit models amongst the GARCH (m,n) and EGARCH (m,n) models respectively. With respect to the Box and Jenkins models, the ARIMA (2,1,1) model was adjudged the best fit model for modelling monthly rates of inflation in Ghana. Subsequently, the three selected autoregressive Heteroscedastic best fit models; AR (2), GARCH (2,1) and EGARCH (2,1) were compared based on their forecast performance. The goodness of fit models that were used included the root mean squared error, mean absolute error, mean absolute percent error and the Theil's Inequality coefficient. The EGARCH (2,1) was adjudged the most appropriate model amongst the three best fit models in modelling the monthly rates of inflation in Ghana as it had the minimum value for all the goodness of fit statistics. An asymmetric effect was also evident in the volatility in the monthly rates of inflation. However, there was an absence of leverage effects as positive shock changed the volatility in the monthly rate of inflation more than a negative shock of equal magnitude. Finally, when the EGACRH (2,1) model was compared to the ARIMA (2,1,1) model, the EGACRH (2,1) was found to be superior in modelling the rate of inflation in Ghana.

Zakaria et al (2013) stated in their paper that, Gold has been considered a safe return investment because of its characteristic to hedge against inflation. As a result, the models to forecast gold must reflect its structure and pattern. Gold prices follow a natural univariate time series data and one of the methods to forecast gold prices is Box-Jenkins, specifically the autoregressive integrated moving average (ARIMA) models. This is due to its statistical properties, accurate forecasting over a short period of time, ease of implementation and able to handle nonstationary data. Despite the fact that ARIMA is powerful and flexible in forecasting, however it is not able to handle the volatility and nonlinearity that are present in the data series. Previous studies showed that generalized autoregressive conditional heteroskedatic (GARCH) models are used in time series forecasting to handle volatility in the commodity data series including gold prices. Hence, this study investigate the performance of hybridization of potential univariate time series specifically ARIMA models with the superior volatility model, GARCH incorporates with the formula of Box-Cox transformation in analyzing and forecasting gold price.

The Box-Cox transformation is used as the data transformation due to its power in normalizing data, stabilizing variance and reducing heteroscedasticity. There is a two-phase procedure in the proposed hybrid model of ARIMA and GARCH.

In the first phase, the best of the ARIMA models is used to model the linear data of time series and the residual of this linear model will contain only the nonlinear data. In the second phase, the GARCH is used to model the nonlinear patterns of the residuals. This hybrid model which combines an ARIMA model with GARCH error components is applied to analyze the univariate series and to predict the values of approximation. In this procedure, the error term of the ARIMA model is said to follow a GARCH process of orders r and s. The performance of the proposed hybrid model is analyzed by employing similar 40 daily gold price data series used by Asadi et al. (2012), Hadavandi et al. (2010), Khashei et al. (2009) and Khashei et al. (2008). From the plotting in-sample series, the gold price series does not vary in a fixed level which indicates that the series is nonstationary in both mean and variance, exhibits upward and nonseasonal trends which reflect the ARIMA models. The hybridization of ARIMA(1,1,1)-GARCH(0,2) revealed significant result at 1% significance level and satisfied the diagnostic checking including the heteroskedasticity test. The plotting of forecast and actual data exhibited the trend of forecast prices follows closely the actual data including for the simulation part of five days out-sample period. Consequently, the hybrid model of ARIMA(1,1,1)- GARCH(0,2) for the transformed data is given by:

$$y_t^* = 0.274y_{t-1}^* + 0.726y_{t-2}^* + \epsilon_t - 0.992\epsilon_{t-1} \ where \ \epsilon_t \ iidN(0,1)$$

$$\sigma_t^2 = 1.16 * 10^{-5} + 1.992\sigma_{t-1}^2 - 1.025\sigma_{t-2}^2$$

Empirical results indicate that the proposed hybrid model ARIMA-GARCH has improved the estimating and forecasting accuracy by fivefold compared to the previously selected forecasting method. The findings suggest that combination of ARIMA (powerful and flexibility) and GARCH (strength of models in handling volatility and risk in the data series) have potential to overcome the linear and data limitation in the ARIMA models. Thus, this hybridization of ARIMAGARCH is a novel and promising approach in gold price modeling and forecasting.

Omane-Adjepong et al (2013) in their paper examined the most appropriate shortterm forecasting method for Ghana's inflation. A monthly inflation data which spanned from January 1971 to October 2012 was obtained from Ghana Statistical Service. The data was divided into two sets: the first set was used for modelling and forecasting, whiles the second was used as test set. Seasonal-ARIMA and

Holt-Winters approaches were used to achieve short-term out-of-sample forecast.

The accuracy of the out-of-sample forecast was measured using MAE, RMSE,

MAPE and MASE. Empirical results from the study indicated that the SeasonalARIMA forecast from $ARIMA(2,1,1)(0,0,1)_{12}$ recorded MAE, RMSE, MAPE and MASE of 0.1787, 0.2104, 1.9123 and 0.0073 respectively; that of the Seasonal

Additive HW was 1.8329, 2.0176, 19.996, 0.0745 and the Seasonal Multiplicative HW forecast recorded 2.2305, 2.4274, 24.000, 0.0911 respectively. Based on these results, they concluded by proposing the Seasonal-ARIMA process as the most appropriate short-term forecasting method for Ghana's inflation.

Harvey and Cushing (2014) analyzed how the information contained in the disaggregate components of aggregate inflation helps improve the forecasts of the aggregate series. Direct univariate forecasting of the aggregate inflation data by an autoregressive (AR) model was used as the benchmark where all autoregressive (AR), moving average (MA) and vector autoregressive (VAR) models of the disaggregates were compared. The results showed that directly forecasting the aggregate series from the benchmark model was generally superior to aggregating forecasts from the disaggregate components. Additionally, including information from the disaggregates in the aggregate model rather than aggregating forecasts from the disaggregates performed best in all forecast horizons when appropriate disaggregates were used. The implication of these results was that better inflation forecasts for Ghana was produced by using information from relevant disaggregates in the aggregate model rather than direct forecasts of the aggregate or aggregating forecasts from the disaggregates.

Ofori and Ephraim (2012) in a paper made a representation of and comparison of the best Exponential Smoothing Technique via no transformation, Square Root transformation and Natural Log transformation of the dataset. Analysis was done based on the monthly inflation rates of Ghana from January, 2000 to December, 2011. The result showed that the predicted rates of inflation were consistent with observed time series. The Damped-Trend Exponential Smoothing technique was found as the most suitable with least Normalized Bayesian Information Criterion

(BIC) of 1.373, Mean Absolute Percentage Error (MAPE) of 5.652, Root Mean Square of 1.846 and a high value of R-Square of 0.951. A forecast of inflation was made for the year 2012, which showed that inflation rates will be between 8% and 11%.

Chapter 3

METHODOLOGY

3.1 Introduction

The study seeks to use intelligent optimization methods in making forecast on inflation in the Ghanaian economy. The Particle Swarm Optimization method was used with some time series models in formulating and objective function for inflation. ARIMA, ARCH and GARCH models was used in obtaining an Inflation model based on the data from the Bank of Ghana. The PSO was then used to forecast inflation based on the model developed by the time series models above. Model evaluation was done using the Mean Absolute Deviation (MAD), Mean Absolute Percentage Error (MAPE) and Mean Square Error (MSE).

3.2 Some Time Series Forecasting Techniques

Modelling financial time series is a complex problem. This complexity is not only due to the variety of the series in use (stocks, exchange rates, interest rates, etc.), to the importance of the frequency of the observation (second, minute, hour, day, etc) or to the availability of very large data sets. It is mainly due to the existence of statistical regularities (stylized facts) which are common to a large number of financial series and are difficult to reproduce artificially using stochastic models, Francq and Zakoian (2010).

Financial variables like stock prices and inflation have heteroscedasticity (unequal variance) assumption due to volatility and usually have the following character-

istics;

(i) The distribution of a financial time series X_t has heavier tails than normal.

(ii) Values of X_t do not have much correlation, but values of X_t^2 are highly correlated.

(iii) The changes in X_t tends to cluster. Volatility changes in X_t tend to be followed by large or small changes, Mandelbrot (1963).

Forecasting is the process of making statements about events whose actual outcomes (typically) have not yet been observed. An example might be estimation of the expected value for some variable of interest at some specified future date. Prediction is a similar, but more general term. Usage can differ between areas of application. For example in hydrology, the terms "forecast" and "forecasting" are sometimes reserved for estimates of values at certain specific future times, while the term "prediction" is used for more general estimates, such as the number of times floods will occur over a long period. In short, Forecasting can be described as predicting what the future will look like. Different forecasting models work best for different situations; the nature of the business, the nature of data, forecast granularity, forecast horizon, shelf life of the model and the expected accuracy of the forecasts. Some time series forecasting techniques commonly used include averaging, exponential smoothing and indexing techniques, Winter's method, Holt's method, ARIMA ect..

Auto Regressive Integrated Moving Average (ARIMA) is probably the most powerful of all forecasting models, but is expensive in terms of the time to build a model. Both ARIMA and Winter's model take into account the seasonality but ARIMA needs more data (at least 4 seasons) than the latter. Table 3.1 below shows the various time series models and the data types they fit well.

The time series models above have been around for sometime now but researchers have tend to use the Autoregressive Integrated Moving Average (ARIMA), Autoregressive Conditional Heteroscedasticity (ARCH) and Generalized Autoregressive Conditional Heteroscedasticity (GARCH) models for forecasting since they have proven to be better over the years.

Table 3.1: A cross-section of some time series models and their characteristics

Model Type	Most Suited Data Types	Forecast Period	Shelflife
Moving Averages	No Trend, No Seasonality	Short	Short
Exponential Smoothing	No Trend, Varying Levels	Short	Short
Holt's Method	Varying Trend, Varying Levels, No Seasonality	Short	Short
Winter's Mehtod	Varying Trend, Varying Levels and Seasonality	Short-Medium	Medium
ARIMA	Varying Trend, Varying Levels, Seasonality	Short-Medium	Long

3.2.1 Autoregressive Integrated Moving Average (ARIMA)

Autoregressive Integrated Moving Average (ARIMA) model was first introduced by Box and Jenkins and has been among most popular time series forecasting models. The model is based on the assumption that time series is stationary, and that mean and autocorrelation structure are constant. It is known as an integrated model because the stationary model which is fitted to the differenced data set has to be summed up or integrated to generate a model for the non-stationary data. The ARIMA model is a stochastic model for time series forecasting where the future value of a variable is a linear function of past observations and random errors as expressed below:

$$y_t = \theta_0 + \varphi_1 y_{t-1} + \varphi_2 y_{t-2} + \dots + \varphi_p y_{t-p} + \varepsilon_t - \theta_1 \varepsilon_{t-1} - \theta_2 \varepsilon_{t-2} - \dots - \theta_q \varepsilon_{t-q} \quad (3.1)$$

Where; $i = 1,2,\dots,p$ $j = 1,2,\dots,q$

y_t - actual value ε_t - random error at time t

φ_i *and* θ_j - model parameters p *and* q - order of the model

3.2.2 Autoregressive Conditional Heteroscedasticity (ARCH) Model

The ARCH model was developed by R. F. Engle in 1982 to provide a framework for volatility modelling taking into consideration the dependence of the conditional second moments.

Let $\{X_t\}$ be the mean-corrected return, ε_t be the Gaussian white noise zero mean and unit variance and I_t be the information set time t, where $I_t = \{x_1, x_2, ..., x_{t-1}\}$, then the ARCH model is specified as:

$$X_t = \sigma_t \varepsilon_t \tag{3.2}$$

$$\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2 + \alpha_2 X_{t-2}^2 + ... + \alpha_m X_{t-m}^2 \tag{3.3}$$

Where, $\alpha_0 > 0$, $\alpha_i \geq 0$, $i = 1,2,..,m$ and

$E(X_t|I_t) = E[E(X_t|I_t)] = E[\sigma_t E(\varepsilon_t)] = 0$

$V(X_t|I_t) = E(X_{t2}) = \sigma_{t2} = \alpha_0 + \Sigma_{mi=1}\alpha_i X_{t2-1}$

$E(\varepsilon_t|I_t) = 0$

The model structure makes it clear that the dependence of the present volatility X_t is a simple quadratic function of its lagged values. The estimated parameters (α_i, $i = 0,1,2,...,m$) of the model are obtained by regressing X_t on $X_{t-1}^2,\ X_{t-2}^2,\ \dots,\ X_{t-m}^2$. Conditional variance σ_t^2 must always be positive which implies that the estimated parameters must be non-negative, i.e, $\alpha_0 > 0$ and $\alpha_i \geq 0$ where $i = 1,2,...,m$.

For the ARCH model to be valid, the presence of ARCH effects should be statistically significant and tested for. The presence of conditional heteroscedasticity implies there exist ARCH effects in a data set. Two popular formal statistical test methods is used to test for the presence of ARCH effects namely Ljung-Box Statistics Q(m) test and the Lagrange Multiplier (LM) test.

3.2.3 Generalized Autoregressive Conditional Heteroskedastic (GARCH) Model

Generalized Autoregressive Conditional Heteroscedastic model, an extension of the ARCH model was developed by Bollerslev in 1986 with the key concept being the conditional variance on past observations. Classical GARCH model expresses the conditional variance as a linear function of the squared past values of the series. This makes it possible to capture the main sytlized facts characterizing the series.One of the earliest time series models allowing for heteroskedasticity or time-varying variance is the Autoregressive Conditional Heteroscedastic (ARCH) model introduced by Engle (1982). The ARCH models have the ability to capture all the above characteristics in financial market variables. Bollerslev (1986) extended this idea into Generalized Autoregressive Conditional Heteroscedastic (GARCH) models which give more parsimonious results than ARCH models, similar to the situation where Autoregressive Moving Average (ARMA) models are preferred over Autoregressive (AR) models.

3.2.4 GARCH(p,q) Model

A process ${}_t$ is called Generalized Autoregressive Conditional Heteroscedastic model of order p and q, if its two conditional moments exist. Thus if ${}_t$ given and information set F_t has a mean of zero and a conditional variance h_t as stated below;

(i) $E(\epsilon_t|\epsilon_u,\ u < t) = 0, t \in Z$

(ii) $h_t = \alpha_0 + \alpha_1\epsilon_{t-1}^2 + \dots + \alpha_q\epsilon_{t-q}^2 + \beta_1 h_{t-1} + \dots + \beta_p h_{t-p}$

$$\sigma_t^2 = Var(\epsilon_t|\epsilon_u, u < t) = \alpha_0 + \sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2 + \sum_{j}^{p} \beta_j \sigma_{t-j}^2, t \in Z \tag{3.4}$$

Where α_0 is a constant, $i = 1,2...,q$ and $j = 1,2,...,p$.

$\sum_{i=1}^{q} \alpha_i \epsilon_{t-i}^2$ - ARCH term

$\sum_{j=1}^{p} \beta_j h_{t-j}$ - GARCH term

ϵ_{t-i}^2 - past i period's squared residual from mean equation.

h_{t-j} - past j period's forecast variance. α_0 α_i β_j -

Unknown parameters to be estimated for.

For conditional variance to be guaranteed, $h_t > 0 \alpha_0 > 0 \alpha_i \geq 0 and \beta_j \geq 0$

3.2.5 Model Selection

Before engaging in the construction of a model, we must accept that there are no true models. Indeed, models only approximate reality. A model in essence mimics the behaviour pattern in an event and not the exactness of the pattern. Therefore, it is every researcher's aim to minimize information loss when modelling a situation. Several models are formed mostly based on observed data over a period in trying to predict the future. It becomes a challenge to select the one that would give the best results and outcome. According to Burnham and Anderson (2001), simplicity and parsimony, several hypotheses, and strength of evidence are the three principles that regulates the ability to make inferences in research. Simplicity and Parsimony are well regarded due to the quality they possess for a better and reliable research deductions.

Parsimony is particularly evident in issues of model building, where the investigator must make a compromise between model bias and variance. Here, bias corresponds to the difference between the estimated value and true unknown value of a parameter, whereas variance reflects the precision of these estimates; a common measure of precision is the Standard Error (SE) of the estimate. Thus, a model with too many variables will have low precision whereas a model with too few variables will be biased (Burnham and Anderson 2002). Various model validation tests like the Standard Error, Akaike Information Criterion (AIC), Bayesian Information Criterion (BIC) etc. measures the effectiveness of the model parameters estimated. The parameter standard error measures the shortfall of the parameter in the model constructed but most studies chooses the AIC to measure the efficiency of a model.

Akaike Information Criterion (AIC)

Kullback and Leibler (1951) addressed the issue of measuring model efficiency and avoiding information loss by developing the Kullback-Leibler Information to represent information lost during model construction to approximate reality. Hirotogu Akaike later in 1973 introduced the Akaike Information Criterion (AIC) based on the Kullback-Leibler Information. The author established a relationship between the Kullback-Leibler Information and the maximum likelihood used as an estimation method a statistical analysis. By so doing, Akaike in essence developed an information criterion to estimate the Kullback-Leibler Information. The AIC is defined as:

$$AIC = 2K - 2L \tag{3.5}$$

Where L is the log-likelihood and K is the number of parameters generated by the model. The second term of the AIC in equation (3.50) measures the goodness of fit of the model whereas the first term is called the penalty function . The AIC penalizes for the addition of parameters, and thus selects a model that fits well but has a minimum number of parameters (i.e., simplicity and parsimony).

In cases where analyses are based on more conventional least squares regression for normally distributed errors, one can compute readily the AIC with the following formula (where arbitrary constants have been deleted):

$$AIC = n\log(\sigma^1) + 2K \tag{3.6}$$

the model parameter count.

In the case of smaller sample size where $\frac{n}{K} \leq 40$, the second-order Akaike Information Criterion (AIC_c) is used.

$$AIC_c = -2(log - likelihood) + 2K + \frac{2K(K+1)}{(n-K-1)} \tag{3.7}$$

The AIC_c will tend to AIC as the sample size increases and will therefore give the same conclusion as the AIC.

After the AIC computation for each model constructed, is then up to us to select the model that will give a better forecast. The selection of a good model among a group of models is done by picking the model with the least AIC. In the case where some models have the same AIC, the principle of parsimony is used. The principle of parsimony states that a model with fewer parameters is usually better than a complex model. Alternatively to the use of the principle of parsimony, forecast accuracy tests between the competing models can be used (Aidoo, 2010). The main advantage of the AIC is its usefulness for both in-sample and out-ofsample forecasting performance of a model. In-sample forecasting indicates how the chosen model fits the data in a given sample while out-of-sample forecasting is concerned with determining how a fitted model forecast future values.

The AIC has been criticized of inconsistency and over-fitting of model despite the advantages stated. As a result, Schwartz (1978) proposed the Bayesian Information Criterion to curtail for the inconsistency of the AIC.

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[1] = Residual Sum of Squares and n is the sample size. K still remains Where, σ
n

Bayesian Information Criterion (BIC)

The Bayesian Information Criterion often called the Schwarz Information Criterion was introduced by Schwartz (1978). The author derived it to serve as a competitor to AIC and also to serve as an asymptotic approximation to a transformation of the Bayesian posterior probability of a candidate model. BIC was justified by Schwarz, "for the case of independent, identically distributed (iid) observations, and linear models", under the assumption that the likelihood is from the regular exponential family. The Bayes factor in relation makes it a good method of selecting the most appropriate model for forecasting. The BIC is obtained by replacing the non-negative factor in equation 3.5 by $K \ln(n)$ as expressed below.

$$BIC = K \ln(n) - 2L \tag{3.8}$$

Where, K continues to be the number of parameters in the model and n is the sample size or the length of time of series. Again L represents the log-likelihood which is used in fitting the model. The minimum of all computed BIC models is selected and the corresponding is adjudged the appropriate model. As compared to the AIC, the penalty term for the BIC is more stringent (for $n \geq 8$, $K \ln(n)$ exceeds $2K$). The BIC is equally used for comparing in-sample and out-of-sample forecasting performance of a model.

3.2.6 Model Diagnostics

After model construction, one has to test for the validity of the model in order to use it in making predictions. The study uses two model validity methods normally used by researchers namely the Jarque Bera and Box-Ljung tests. The model diagnostic checks are done to measure the accuracy or goodness of fit of a proposed model. The residuals and more specifically standardized residuals are considered during the check. The residuals are normally assumed to be independently and identically distributed (iid) following a normal distribution. Plots of the residuals such as the histogram, the normal probability plot and the time plot of residuals can be used. If the model fits the data well the histogram of residuals should be approximately symmetric. The Auto-Correlation Factor (ACF) and the Partial Auto-Correlation Factor (PACF) of the standardized residuals are used for checking the adequacy of the conditional variance model. The Jarque Bera and the Box-Ljung Q-test are used to check the validity of the ARCH effects as well as test for autocorrelation in the data. To test the presence of ARCH effects, the null hypothesis of no ARCH effects is rejected if the significance probability value (p-value) is less than specified level of significance. In case of testing for the presence of autocorrelation, the null hypothesis of no autocorrelation is rejected if the Ljung-Box (Q) statistics of some of the lags are significant. Thus if the probability value of Ljung-Box (Q) statistics of some of the lags are less than the specified level of significance, then the null hypothesis of no autocorrelation is rejected. Once the estimated model satisfies all model assumptions, then the model adjudged to have a proper representation of the data. Having established this fact, the model can then be used to make forecasts of the series under consideration. Table 3.2 shows the model diagnostic tests that was done for the various GARCH models.

Table 3.2: Model diagnostic tests on data

	Jarque Bera Test					Q(M)-Test	
GARCH Model	X-Squared	df	P-Value		X-Squared	df	P-Value
(1,1)	6899.231	2	2.2e-16		0.2815	1	0.5957
(1,2)	4163.194	2	2.2e-16		0.8441	1	0.3582
(2,1)	6952.809	2	2.2e-16		0.1487	1	0.6998
(2,2)	6881.125	2	2.2e-16		0.0535	1	0.8171

3.3 Particle Swarm Optimization (PSO)

Particle Swarm Optimization (PSO) is an intelligent algorithm, which was first introduced by Kennedy and Eberhart (1995). PSO was inspired by social dynamics and evolving behaviour of movement that arises in socially organized colonies of some animals likes birds, fishes, flocks, insects, etc. Bird flocks, fish schools, and animal herds constitute representative examples of natural systems where aggregated behaviors are met, producing impressive, collision-free, synchronized movements. In such systems, the behavior of each group member is based on simple inherent responses, although their outcome is rather complex from a macroscopic point of view, Parsopoulos and Vrahatis (2010). PSO was initially derived based on the behaviour of swarm in search of food and later developed based on the topological aspect to make it a multidimensional search method. In the preliminary stages of PSO, the nearest-neighbour velocity matching and acceleration by distance were the main rules used to generate swarming behaviour. The PSO is a population-based algorithm with the population called the swarm and its individual candidates called particles. By population-based, it means the algorithm spans the population simultaneously for potential candidate
solutions.

Let $B \subset R^n$ be the feasible search space and $f : B \, 7\!\rightarrow Y \subseteq R$, be the objective function where B is assumed to be a feasible search space because further explicit constraints are posed on feasible solutions.

The swarm is defined as a set :

$$S = (x_1, x_2, \ldots, x_N)$$

Where N is the number of candidate particles and is defined as:

$$x_i = (x_{i1}, x_{i2}, \ldots, x_{iN})^T \in B, \qquad i = 1,2,\ldots,N$$

Indices are arbitrary assigned to particles, while N is a user-defined parameter of the algorithm. The objective function $f(x)$ is assumed to be available for all points in B which implies each particle has a unique functional value of $f_i = f(x_i) \in Y$.

The particles are assumed to move iteratively within the feasible search space B, which is made possible by adjusting their positions using a position shift called velocity denoted by:

$$v_i = (v_{i1}, v_{i2}, \ldots, v_{iN})^T, \qquad i = 1,2,\ldots,N$$

The PSO algorithm allows the velocity to be updated based on previous steps.

It also allows the particles to store their positions visited during each iterative search. These positions are stored in memory set P for retrieval in future search.

$$P = (p_1, p_2, \ldots, p_N)$$

where P is the memory set.

$$P_i = (p_{i1}, p_{i2}, \ldots, p_{in}), \ i = 1,2,\ldots,N$$

P_i is the position of each particle at every iterative event. The algorithm again approximates the best position ever visited during the iterative search in the feasible region known as Pbest and is denoted by:

$$P_g(t) = argminf(p_i(t)), i = 1,2,\ldots,N$$

Where g is the index of the best position with the lowest functional value in P at a given iteration t.

The mathematical representation of the velocity and position vector update at every iteration in the search space is given below ;

$$v_{ij}(t+1) = v_{ij}(t) + c_1R_1(p_{ij}(t) - x_{ij}(t) + c_2R_2(p_{gj}(t) - x_{ij}(t) \qquad (3.9)$$

$$x_{ij}(t+1) = x_{ij}(t) + v_{ij}(t+1) \qquad (3.10)$$

Where c_1 and c_2 are positive constants and R_1 and R_2 are two uniform random variables in the range [0,1].

c_1 and c_2 are weighting factors that represent the cognitive and social parameters, respectively. Initially, an acceleration constant $c = c_1 = c_2$ was used as single weight but it was later realized that the double weights gave better results. The iteration counter in this case is t, while $i = 1,2,...,N$ and $j = 1,2,...,n$. The best position (*pbest*) of the particles after every update, evaluation and iteration are also updated in memory. The new best position of x_i at the next iteration is given below;

$$P_i(t+1) = \begin{cases} x_i(t+1), & \text{if } f(x_i(t+1)) \le f(p_i(t)) \\ p_i(t), & \text{otherwise} \end{cases}$$

A step by step algorithm of PSO is given below.

(i) Initialize a population of particles with random positions and velocities withD-dimensional problem space.

(ii) For each particle, evaluate the desired objective value fitness function in thesearch space.

(iii) Compare each particle's fitness evaluation with its pbest. If the current valueis better than pbest, then set pbest equal to the current value, and p_i equals to the current location x_i in D-dimensional space.

(iv) Identify the particle in the neighbourhood with the best success so far knownas global best (gbest) position of particles, and assign its index to the variable g. Set gbest to the position of this particle in the search space.

(v) Update the velocity and position of the particles according to Equations (3.9) and (3.10).

(vi) Loop to step two (2) until a stopping criterion is met, usually a sufficientlygood fitness, or a maximum number of iterations.

3.3.1 Further Improvements of PSO

Although PSO has proven to be a very successful intelligent search method, it had some deficiencies which resulted in further research for improvement on the algorithm. It worked best for simple optimization problems but experienced its worst deficiency termed as swarm explosion effect when it was applied to much complex optimization problems with larger search spaces and a multitude of local minima. The swarm explosion effect refers to the uncontrolled increase of magnitude of the velocities of particles which resulted in swarm divergence as was significantly verified by a lot of researchers. This was due to the fact that, there was no velocity threshold mechanism in place to prevent the particles in the swarm from diverging from the pbest and gbest. This problem was addressed by velocity clamping at desirable levels which will prevent particles from taking extreme steps away for their current positions. As a result, a user-defined maximum velocity threshold was introduced, v_{max}. Thus, after velocity update in equation (3.9), the maximum velocity constraint is applied to the new velocity before using it in the position update $x_i(t + 1)$ in equation (3.10).

In the threshold constraint, the magnitude of the new velocity must be less than or equal to v_{max} as shown below:

$$|v_{ij}(t + 1)| \leq v_{max},\ i = 1,2,...,N\ j = 1,2,...,n$$

In case of violation of the threshold velocity (v_{max}), the new velocity is set directly equal to the closest velocity bound with respect to v_{max}.

$$|v_{ij}(t{+}1)| = \{v_{max},\ if\ v_{ij}(t + 1) \geq v_{max}\ or - v_{max},\ if\ v_{ij}(t + 1) \leq -v_{max}$$

3.3.2 Inertia Optimizer

Despite the refinement of the PSO algorithm by the velocity clamping method to curtail swarm explosion effect, it was not enough to maintain the stability of the swarm as particles still deviated from their possible best solutions in the last phase of the optimization procedure in a complex problem thereby resulting in local minima solutions instead of global minima. To correct this weakness in the algorithm, the concept of inertia weight w was introduced into the velocity update equation to keep the magnitude of the swarm velocities within feasible search space and closer to potential solutions, Shi and Eberhart (1998):

$$v_{ij}(t + 1) = wv_{ij}(t) + c_1R_1(p_{ij}(t) - x_{ij}(t) + c_2R_2(p_{gj}(t) - x_{ij}(t) \qquad (3.11)$$

$$x_{ij}(t + 1) = x_{ij}(t) + v_{ij}(t + 1) \qquad (3.12)$$

$$i = 1,2,...,N \text{ and } j = 1,2,...,n$$

The inertia weight is selected such that the effect of the velocity fades during the execution of the algorithm. A preferable weight w could be in the interval [0.1, 1.2] to promote spanning in early optimization stages and will provide a linear decrease towards zero to eliminate oscillatory behaviours in later stages. In general, a linearly decreasing scheme for w is described mathematically below;

$$w(t) = w_{up} - (w_{up} - w_{low})\frac{t}{T_{max}} \tag{3.13}$$

Where, t is the iteration counter, w_{low} *and* w_{up} are desirable lower and upper bounds of inertia weight and T_{max} is the total number of iterations allowed.

Experimental results show that PSO has the biggest speed of convergence when w is between 0.8 and 1.2. While experimenting, w is confined from 0.9 to 0.4 according to the linear decrease, which makes PSO search for the bigger space at the beginning and locate the position quickly where there is the most optimist solution. As w is decreasing, the speed of the particle will also slow down to search for the delicate partial. The method quickens the speed of the convergence, and the function of the PSO is improved. When the problem that is to be solved is very complex, this method makes PSO's searching ability for the whole at the later period after several generation is not adequate, the most optimist solution cannot be found, so the inertia weights can be used to work out the problem, Bai(2010).

3.3.3 Constriction or Increase Convergence Factor

Clerc (1999) introduced a new parameter called the convergence factor χ to further improve the velocity equation as given below;

$$v_{ij}(t+1) = \chi\{v_{ij}(t) + c_1R_1(p_{ij}(t) - x_{ij}(t) + c_2R_2(p_{gj}(t) - x_{ij}(t)\} \quad (3.14)$$

Where $\chi = \frac{2}{2-\varphi-\sqrt{\varphi^2-4\varphi}}$ and $\phi = c_1 + c_2$

The convergence parameter has proven to be much quicker in convergence than the inertia weight. For efficient improvement of the convergence factor, $\phi > 4$.

3.3.4 Optimization Model Formulation of PSO

The focus of the study is to forecast inflation rate based on fitted time series model. The Inflation model will have only one parameter for the prediction which reveals the need to use the one dimensional PSO algorithm as shown below:

$$v_i(t+1) = \chi\{wv_i(t) + c_1R_1(p_i(t) - x_i(t) + c_2R_2(p_g(t) - x_i(t)\} \quad (3.15)$$

$$x_i(t+1) = x_i(t) + v_i(t+1) \quad (3.16)$$

Let $F(x)$ be an inflation model, then the optimization model for it is stated below;

$$Min \quad F(x)$$

Subject to

$$-V_{max} \leq V_i(t+1) \leq V_{max}$$

$$x_{min} \leq x_i(t) \leq x_{max}$$

Where, $-V_{max}$ and V_{max} are the minimum and maximum swarm velocity respectively. x_{min} and x_{max} are also the swarm's population lower and upper bounds. Let the best position and global best position of swarm be *Pbest*) and *Gbest* respectively, then the steps in implementing the method is outlined below:

(i) Set initial population of the particles, X_n where n is the total population size.

(ii) Set initial velocity equals zero.

(iii) Evaluate the fitness function $F(x)$ for all particles.

(iv) Set $Pbest_i(t)$ for $i = 1,2,...,n$. Thus if $F(x_i(t + 1)) \leq F(x_i(t))$ then $Pbest_i(t) = x_i(t + 1)$ and $x_i(t)$ otherwise.

(v) Evaluate Gbest. $Gbest = min\ F(Pbest)$. If $F(Gbest) \leq min\ F(Pbest_i)$, where $i = 1,2,...,n$, then $Gbest = Pbest_i$

(vi) Calculate the maximum velocity of the particles using

$V_{max} = \dfrac{x_{max} - x_{min}}{n}$, then $V_{min} = -V_{max}$

(vii) Evaluate the swarm velocity V_i and compare it the $V_{max} and V_{min}$ to control swarm velocity explosion effects. Thus if $V_i \leq V_{min}$, set $V_i = V_{min}$ while

$V_i = V_{max}$ when $V_i \geq V_{max}$.

(viii) Update the particles position in swarm. If the new update $x_i(t + 1)$ lies within x_{min} *and* x_{max}, then we keep it, but keep $x_i(t)$ when $x_i(t + 1)$ is outside the range.

(ix) Repeat step 3 to 8 until stopping criterion is met.

3.3.5 Implementation of the Proposed Method

Secondary data obtained from the Bank of Ghana on inflation between the periods of January, 2002 and August, 2014 was used for the study. Based on the data obtained, an inflation model was derived using time series models with the help of a R statistical software. A model selection was done after analysing the data and knowing the characteristics being exhibited by the data. The inflation model obtained at this stage was used as an objective function by the Particle Swarm

Optimization algorithm to predict monthly values of inflation for the data period. A mathematical software known as Matlab was used in arriving at the forecasts of the PSO algorithm. Statistical error measurements was then be done between the forecast values and existing values to know the efficiency of the method being used.

3.4 Proposed Model

A set of constructed GARCH models were derived from the data set obtained. GARCH (2,1) model was selected as the appropriate model using the AIC model selection criterion which was used as the objective function of the PSO. Table 3.3 shows the residuals and the AIC values of the various GARCH models that were constructed.

Table 3.3: Residuals and AIC values on the various GARCH models obtained

Model	Min	1Q	Median	3Q	Max	AIC
(1,1)	0.4182	0.9363	0.9692	1.0268	2.3044	1277.738
(1,2)	0.4501	0.7948	0.8306	0.8811	2.0144	1287.517
(2,1)	0.4255	0.9375	0.9695	1.0258	2.3116	1270.799
(2,2)	0.4334	0.9389	0.9715	1.0278	2.3221	1272.876

3.5 Model Adequacy

A fundamental concern in forecasting is the measurement of forecasting error for a given data set and a given forecasting method. Accuracy can be defined as "goodness of fit" or how well the forecasting model is able to reproduce data that is already known, Makridakis and Wheelwright (1989).

A forecast error actually is the difference between forecast value and the actual data value at the same instance. In selecting a better measurement of accuracy of forecasts, the following should be taken into accounts as suggested by Armstrong (2001) and Ahlburg (2001);

> The error term of a model should not rely more on outliers and should be obtained from reliable test cases that mimics the actual forecasting situation.

The term should not depend on any scale

There should be significant sensitivity analysis of error measures when the model under consideration is subjected to perturbation.

The measure should be reliable and valid

For this reason the study considers the following three standard forecast accuracy measurement: Mean Square Error (MSE), Mean Absolute Percentage Error (MAPE) and Mean Absolute Deviation (MAD).

Mean Square Error (MSE)

The Mean Square Error forecast accuracy method of measurement seems to be the most popular of the three measurements mentioned above regarding the fact that it measures the dispersion of forecast errors by taking the average of squared individual errors. In this method, the smaller the MSE value, the more stable the model under consideration. Nevertheless, the MSE method stresses much on the large error terms. Thus, it gives greater weight to large error terms than to smaller error terms due to the squaring of errors before they are summed. Even though some researchers argue that the MSE is not reliable as compared to other measures, it is yet the most popularly used amongst the others. The MSE is mathematically represented below;

$$MSE = \frac{\sum_{i=1}^{n}(X_i - \overset{*}{X}_i)^2}{n} \quad (3.17)$$

Where,

X_i - the actual being forecast

$\overset{*}{X}_i$ - the forecast i - period of each forecast made n - the number of periods of data

Mean Absolute Percentage Error (MAPE)

The Mean Absolute Percentage Error is measured as the average of the sum of all the percentage errors of a data set in absolute terms to avoid equal predicted and actual values from cancelling out. MAPE is less sensitive to disturbances from outliers which allows direct comparison of the MAPE to other methods. It does not show any bias towards smaller error terms which makes it better measure to the MSE. However, it is bias towards favouring under-forecasts but not limited to errors on over-forecasts. Thus zero forecasts can never have over a 100% MAPE.

$$MAPE = \frac{P}{n} \times 100\% \tag{3.18}$$

Where,
$P = \frac{\sum_{i=1}^{n}(|X_i - \overset{*}{X}_i|)}{X_i}$

Mean Absolute Deviation (MAD)

Also known as the Mean Absolute Error (MAE) does not consider whether the error measurement was an overestimate or underestimate. It measures error by taking the average or mean of the absolute value of error. It has proven to be useful when linked to revenue or some other independent measure of value. MAD is expressed as:

$$MAD = \frac{\sum_{i=1}^{n}(|X_i - \overset{*}{X}_i|)}{n} \tag{3.19}$$

Chapter 4

Results and Discussion

4.1 Data

Monthly inflation figures from January, 2002 to August, 2014 as shown in appendix B of this study was used to fit a model using the GARCH time series model. A time plot of the data under consideration is shown by Figure 5.1 in Appendix A. The data showed seasonal moving averages when ARIMA was modeled, resulting in several parameters (parsimony) for the inflation model.

Estimation of time series data using a model requires that the data under consideration be stationary to avoid spurious results. The Kwiatkowski-PhilipsSchmidt-Shin (KPSS) and Augmented Dicky-Fuller (ADF) stationarity test were used on the times series data considered in this study which revealed that an ADF value of -3.0783 with a lag order of 5 at 5% significance without trend. Critical ADF values are shown in Figure 4.1.

Th KPSS test failed to reject the null hypothesis:"Level Sationarity" at a significant p-value of 0.01681 < 0.05. The data was also tested for ARCH effects using the ARCH LM-test with the null hypothesis: no ARCH effects. The data also shown significant ARCH effects but the GARCH model was used due to its flexibility in taking care of volatility in financial time series data. Table 4.1 shows the various test results generated.

Table 4.1: KPSS,ADF and ARCH LM-tests results

	KPSS Level	Dicky-Fuller	ARCH LM -test (Chi-Square)
Levels	0.6641	-3.0783	99.7412
P-Value	0.01681	0.1271	6.661 e-16

Critical values for Dickey–Fuller *t*-distribution.				
	Without trend		With trend	
Sample size	1%	5%	1%	5%
T = 25	- 3.75	- 3.00	- 4.38	- 3.60
T = 50	- 3.58	- 2.93	- 4.15	- 3.50
T = 100	- 3.51	- 2.89	- 4.04	- 3.45
T = 250	- 3.46	- 2.88	- 3.99	- 3.43
T = 500	- 3.44	- 2.87	- 3.98	- 3.42
T = 8	- 3.43	- 2.86	- 3.96	- 3.41

Figure 4.1: Critical ADF Values: Source(Fuller, W. A. (1976))

4.1.1 Model Construction

The GARCH model and parameter estimation was done using R-soft statistical software and with a minimum AIC value of 1270.799 among other models, the GARCH(2,1) model was adjudged the appropriate model for forecast as given already in Table 3.3.

The GARCH model chosen is represented below where equation 3.2 remains the same:

$$\sigma_t^2 = \alpha_0 + \alpha_1 X_{t-1}^2 + \beta_1 \sigma_{t-1}^2 + \beta_2 \sigma_{t-2}^2 \tag{4.1}$$

Table 4.2 shows the parameter estimates of the various GARCH models constructed.

Table 4.2: Parameters estimates of the various GARCH models

Model	Parameter	Estimate	Std. Error	t-value	Pr(> |t|)
(1,1)	a_0	3.221e+01	4.192e+02	0.077	0.939
	a_1	8.978e-01	1.943e+00	0.462	0.644
	b_1	3.392e-13	2.400e+00	0.00	1.00
(1,2)	a_0	3.042e+01	3.316e+02	0.092	0.927
	a_1	6.606e-01	1.579e+00	0.418	0.676
	a_2	6.215e-01	3.110e+00	0.200	0.842
	b_1	4.839e-13	2.611e+00	0.00	1.000
(2,1)	a_0	3.041e+01	4.053e+00	0.075	0.940
	a_1	8.611e-01	1.867e+00	0.461	0.645
	b_1	1.581e-13	2.948e+00	0.000	1.000
	b_2	4.034e-02	1.670e+00	0.024	0.981
(2,2)	a_0	2.863e+01	5.487e+02	0.052	0.958
	a_1	8.231e-01	1.674e+00	0.492	0.623
	a_2	2.973e-02	1.059e+01	0.003	0.998
	b_1	1.488e-13	1.277e+01	0.000	1.000
	b_2	5.082e-02	1.897e+00	0.027	0.979

PSO Model

As proposed in chapter three, the PSO takes over from the GARCH model selected from Table 4.2 above using the minimum AIC. The GARCH model gave the following expression as the forecast value and the conditional variance of the model. This is shown as;

$$X_t = \varepsilon\sigma_t \tag{4.2}$$

Where ε is the sum of error made on each of the parameters estimated.

$$\sigma_t^2 = (3.041e{+}01){+}(8.611e{-}01)X_{t-1}^2{+}(1.581e{-}13)\sigma_{t-1}^2{+}(4.034e{-}02)\sigma_{t-2}^2 \quad (4.3)$$

The PSO representation of the model is as stated below; Let X_t be the forecast

value for inflation. Then a function $F(X_t) = F(X_0, X_1, X_2, X_3)$, where the elements of F represents the estimated parameters in equation 4.3.

The PSO took the GARCH model generated as its objective function to make forecast since it modelled the data well.

$$X_t = \varepsilon\sqrt{h} \quad (4.4)$$

Where, $h = (3.041e + 01) + (8.611e - 01)X_1^2 + (1.581e - 13)X_2^2 + (4.034e - 02)X_3^2$

4.2 Results

A PSO matlab codes was used to generate forecast values to find the parameter that gave the minimum errors in making a forecast. Three parameters were used to make predictions using the total periods of the data under consideration. A forecast of 152 periods was made and the various model adequacy methods were used to measure the errors. A maximum of one iteration was done due to the fact that, iterations more than one generated swarm explosion and that forecasts resulted in only the minimum swarm restrictions.

It was realized that the parameter X_2 had the minimum error measured as compared to X_1 and X_3 in table 4.3 below. The results as shown in Appendix B revealed that, even though the parameter X_2 had the minimum error measure, the parameters X_1 and X_3 made one and three exact predictions respectively for August, 2008 and January, 2004, March, 2005, June, 2008 respectively.

Since all the parameters had very accurate forecast values, the measure of deviation of the parameters were compared and the one with the minimum deviation measure was selected to represent the actual forecast as shown in Appendix B. It was realized that the error measure reduced drastically after the best forecast values were selected. About 31.58%, 35.53% and 32.89% forecast values were selected for the parameters X_1, X_2 and X_3 respectively. Table 4.3: Parameter adequacy

ERROR MEASURE	X_1	X_2	X_3	(X_1,X_2,X_3)
MSE	145.55	105.52	105.57	32.34
MAPE (%)	67.30	56.67	58.75	28.85
MAD	9.79	8.43	8.61	4.21

Chapter 5

Conclusion and Recommendation

5.1 Conclusion

An inflation model was developed using the GARCH statistical model construction tool. After careful model diagnostics and other validity tests, the GARCH (2,1) model was selected which in turn represented the objective function of the proposed PSO algorithm. Based on the results obtained, the study concludes that the proposed method for the study which is the Particle Swarm Optimization together with the Generalized Autoregressive Heteroscedasticity (GARCH) model performed very well with minimal errors measures recorded at MSE, MAPE and MAD of 32.34, 28.58% and 4.21 respectively.

5.2 Recommendation

With respect to the analysis and results obtained;

It is recommended that central banks take a look at using the PSO intelligent methods in forecasting inflation values considering the minimal forecast error. It is also recommended that further research be conducted in this area of study to improve the proposed method of forecast.

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Appendix A

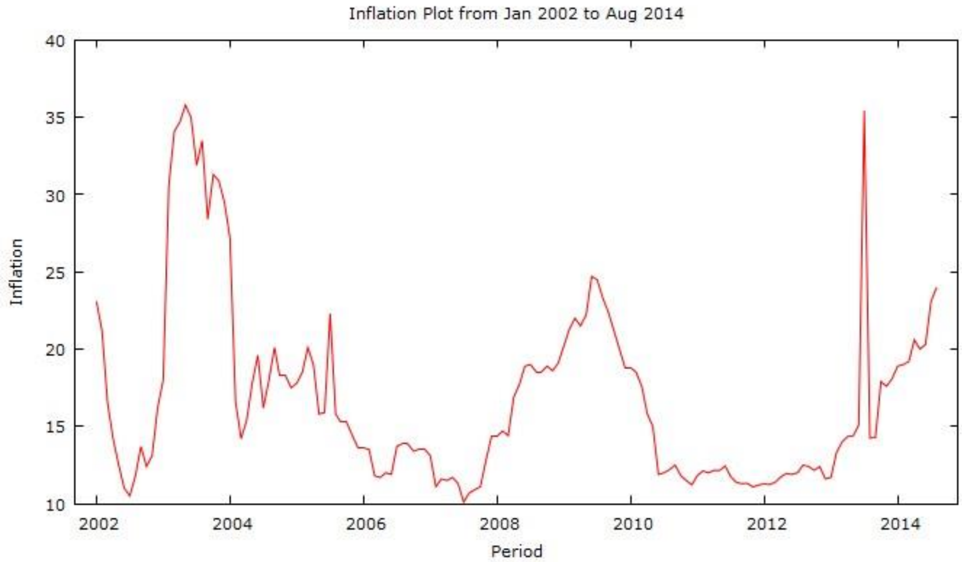

Figure 5.1: Time Plot of Inflation Data

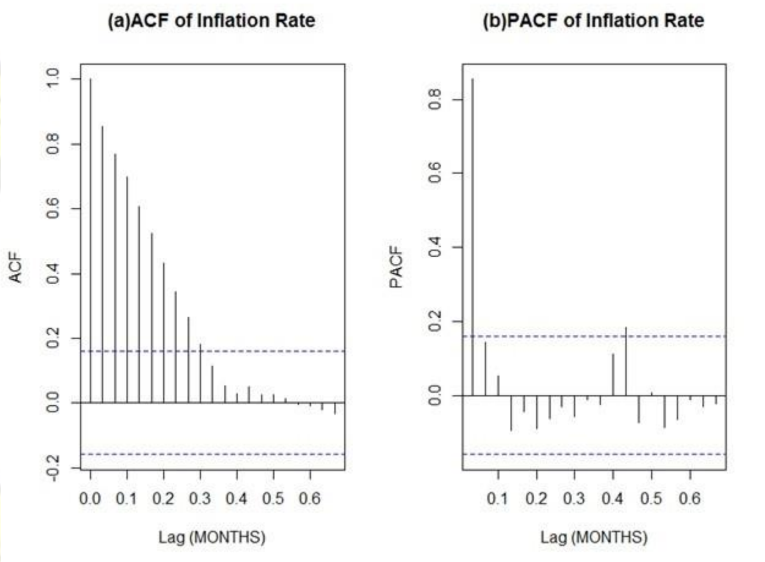

Figure 5.2: ACF anf PACF Plot of Inflation Rate

Appendix B

Monthly Inflation Data from BOG

Year	Month	SP	EX RATE	INF
2002	Jan	957.3	0.7357	23.1
2002	Feb	969.9	0.7545	21.2
2002	Mar	1018	0.769	16.5

2002	Apr	1041	0.7803	14.2
2002	May	1132.7	0.791	12.5
2002	Jun	1223.7	0.8043	11
2002	Jul	1257.1	0.8136	10.5
2002	Aug	1309.7	0.8164	11.8
2002	Sep	1310.7	0.8188	13.7
2002	Oct	1339.8	0.8275	12.4
2002	Nov	1362.7	0.8339	13.1
2002	Dec	1395.3	0.8439	16.2
2003	Jan	1434.7	0.8537	18
2003	Feb	1491	0.856	30.6
2003	Mar	1643.7	0.86	34.1
2003	Apr	1766.4	0.869	34.7
2003	May	1865	0.8684	35.8
2003	Jun	2084.7	0.87	35
2003	Jul	2315.3	0.8722	31.9
2003	Aug	2535.6	0.8736	33.5
2003	Sep	2643.3	0.8732	28.4
2003	Oct	2899	0.8754	31.3
2003	Nov	3300.8	0.8805	30.9
2003	Dec	3553.4	0.8852	29.5

Year	Month	SP	EX RATE	INF
2004	Jan	3798.1	0.888	27.2
2004	Feb	4633.1	0.8915	16.6
2004	Mar	5665	0.9018	14.2
2004	Apr	6544	0.9049	15.4
2004	May	6575.9	0.9029	17.8
2004	Jun	7045.4	0.9047	19.6

Year	Month	SP	EX RATE	INF
2004	Jul	7125	0.9042	16.2
2004	Aug	7316.3	0.9046	18
2004	Sep	6997.8	0.9052	20.1
2004	Oct	6932.9	0.9049	18.3
2004	Nov	6747.4	0.9055	18.3
2004	Dec	6798.5	0.9051	17.5
2005	Jan	6889.4	0.905	17.8
2005	Feb	6737.2	0.9058	18.5
2005	Mar	6453.8	0.9075	20.1
2005	Apr	6108.2	0.9081	19
2005	May	6050	0.9066	15.8
2005	Jun	5862.7	0.9075	15.9
2005	Jul	5019.7	0.9077	22.3
2005	Aug	4842.3	0.9086	15.8
2005	Sep	4878.3	0.9086	15.3
2005	Oct	4894.7	0.9084	15.3
2005	Nov	4793.1	0.9099	14.4
2005	Dec	4769	0.9131	13.6
2006	Jan	4692.8	0.9129	13.6
2006	Feb	4730.2	0.9119	13.5
2006	Mar	4764.1	0.9139	11.8
2006	Apr	4780.2	0.9141	11.7
2006	May	4843.8	0.9145	12
2006	Jun	4833.3	0.9191	11.9
2006	Jul	4885.3	0.9198	13.7
2006	Aug	4913.3	0.9198	13.9
2006	Sep	4943.5	0.921	13.9

Year	Month	SP	EX RATE	INF
2006	Oct	4973.3	0.9224	13.4
2006	Nov	4992.9	0.9229	13.5
2006	Dec	5006	0.9235	13.5
2007	Jan	5012.2	0.9235	13.1
2007	Feb	5044.9	0.9256	11.1
2007	Mar	5092.3	0.9269	11.6
2007	Apr	5139.7	0.9274	11.5
2007	May	5224.5	0.9274	11.7
2007	Jun	5294.6	0.9285	11.3
2007	Jul	5341.8	0.93	10.1
2007	Aug	5557.4	0.9355	10.7
2007	Sep	5676.8	0.9428	10.9
2007	Oct	5839.6	0.9455	11.1
2007	Nov	6387.2	0.968	12.8
2007	Dec	6599.8	0.9704	14.4
2008	Jan	6718.9	0.9759	14.4
2008	Feb	7005.3	0.9751	14.7
2008	Mar	7848.1	0.978	14.4
2008	Apr	9349.6	0.9872	16.9
2008	May	9815.2	1.0024	17.7
2008	Jun	10346.3	1.0325	18.9
2008	Jul	10650.7	1.0692	19
2008	Aug	10791	1.1161	18.5
2008	Sep	10890.8	1.1345	18.5
2008	Oct	10788.3	1.1565	18.9
2008	Nov	10573.4	1.1777	18.6
2008	Dec	10431.6	1.2141	19.1

2009	Jan	10221	1.2828	20.2
2009	Feb	9836.8	1.3402	21.3
2009	Mar	9247.2	1.3832	22
2009	Apr	8822.9	1.4042	21.5
2009	May	7496	1.4396	22.2
2009	Jun	5424	1.4725	24.7
2009	Jul	5230.5	1.4858	24.5
2009	Aug	5900.4	1.4619	23.3
2009	Sep	6292.1	1.4514	22.4
2009	Oct	5378.7	1.4416	21.2
2009	Nov	5386.5	1.4322	20
2009	Dec	5572.3	1.4287	18.8
2010	Jan	5625.4	1.4257	18.8
2010	Feb	5541.2	1.4266	18.5
2010	Mar	6014.3	1.4168	17.6
2010	Apr	6518.9	1.417	15.8
2010	May	7172.1	1.4206	15
2010	Jun	6591.1	1.4267	11.9
2010	Jul	6394	1.4353	12
2010	Aug	6821.8	1.4307	12.2
2010	Sep	6835.7	1.4269	12.5

Year	Month	SP	EX RATE	INF
2010	Oct	-	1.4293	11.82
2010	Nov	-	1.4367	11.5
2010	Dec	-	1.4738	11.22
2011	Jan	-	1.5013	11.82
2011	Feb	-	1.4937	12.12
2011	Mar	-	1.5021	12

Year	Month	SP	EX RATE	INF
2011	Apr	-	1.4972	12.16
2011	May	-	1.5018	12.15
2011	Jun	-	1.5064	12.44
2011	Jul	-	1.5055	11.75
2011	Aug	-	1.5104	11.38
2011	Sep	-	1.5224	11.3
2011	Oct	-	1.5328	11.32
2011	Nov	-	1.5412	11.08
2011	Dec	-	1.5505	11.21
2012	Jan	-	1.6475	11.27
2012	Feb	-	1.6735	11.24
2012	Mar	-	1.6888	11.39
2012	Apr	-	1.703	11.73
2012	May	-	1.8103	11.95
2012	Jun	-	1.8735	11.9
2012	Jul	-	1.8843	12
2012	Aug	-	1.8907	12.5
2012	Sep	-	1.8887	12.4
2012	Oct	-	1.8789	12.16
2012	Nov	-	1.8772	12.4
2012	Dec	-	1.88	11.6
2013	Jan	-	1.884	11.7
2013	Feb	-	1.8864	13.3
2013	Mar	-	1.901	13.99
2013	Apr	-	1.9126	14.35
2013	May	-	1.9408	14.39
2013	Jun	-	1.9469	15.08

2013	Jul	-	1.9494	35.43
2013	Aug	-	1.9559	14.24
2013	Sep	-	1.9608	14.29
2013	Oct	-	2.0291	17.9
2013	Nov	-	2.0822	17.6
2013	Dec	-	2.2	18.1
2014	Jan	-	2.3975	18.9
2014	Feb	-	2.5232	19
2014	Mar	-	2.68	19.2
2014	Apr	-	2.7939	20.6
2014	May	-	2.892	20
2014	Jun	-	-	20.3
2014	Jul	-	-	23.1
2014	Aug	-	-	24

Parameter Forecast (X_1^*, X_2^*, X_3^*) and Selected Forecast (X^*)

Year	Month	Data(X)	X_1^*	X_2^*	X_3^*	Forecast (X^*)
2002	Jan	23.1	26.92	16.88	30.71	26.92
2002	Feb	21.2	20.16	27.87	16.88	20.16
2002	Mar	16.5	20.17	16.95	23.68	16.95
2002	Apr	14.2	19.79	13.27	13.76	13.76
2002	May	12.5	28.28	16.66	30.14	16.66
2002	Jun	11	14.49	26.69	15.93	14.49
2002	Jul	10.5	26.74	18.29	21.18	18.29
2002	Aug	11.8	28.26	18.84	17.48	17.48
2002	Sep	13.7	29.14	29.31	22.96	22.96
2002	Oct	12.4	15.60	19.50	24.53	15.60
2002	Nov	13.1	30.88	12.43	28.53	12.43
2002	Dec	16.2	32.55	30.46	32.11	30.46
2003	Jan	18	20.62	31.75	21.56	20.62

2003	Feb	30.6	10.90	33.57	23.37	33.57
2003	Mar	34.1	17.40	19.88	17.78	19.88
2003	Apr	34.7	30.25	10.24	27.74	30.25
2003	May	35.8	25.67	26.41	30.17	30.17
2003	Jun	35	12.27	27.13	18.33	27.13
2003	Jul	31.9	29.45	13.32	27.40	29.45
2003	Aug	33.5	10.74	20.01	30.70	30.70
2003	Sep	28.4	31.21	18.27	25.75	25.75
2003	Oct	31.3	20.14	20.93	24.53	24.53
2003	Nov	30.9	35.51	11.93	18.65	35.51
2003	Dec	29.5	15.40	19.98	18.77	19.98
2004	Jan	27.2	30.32	23.46	27.13	27.13
2004	Feb	16.6	23.59	24.41	15.39	15.39

Year	Month	Data(X)	X_1^*	X_2^*	X_3^*	Forecast (X^*)
2004	Mar	14.2	27.64	18.29	26.65	18.29
2004	Apr	15.4	35.16	27.39	24.48	24.48
2004	May	17.8	14.50	30.75	21.86	14.50
2004	Jun	19.6	16.77	15.10	32.11	16.77
2004	Jul	16.2	29.99	29.72	13.19	13.19
2004	Aug	18	18.67	18.41	15.51	18.41
2004	Sep	20.1	35.71	15.68	16.63	16.63
2004	Oct	18.3	19.05	19.10	23.08	19.05
2004	Nov	18.3	24.20	12.71	21.75	21.75
2004	Dec	17.5	19.74	23.19	28.39	19.74
2005	Jan	17.8	24.77	14.07	14.82	14.82

2005	Feb	18.5	10.88	22.13	12.91	22.13
2005	Mar	20.1	24.33	19.14	20.03	20.03
2005	Apr	19	17.95	19.16	17.10	19.16
2005	May	15.8	32.55	25.86	28.13	25.86
2005	Jun	15.9	31.14	29.95	19.46	19.46
2005	Jul	22.3	11.24	10.69	31.02	31.02
2005	Aug	15.8	33.38	17.86	28.36	17.86
2005	Sep	15.3	26.78	30.26	30.54	26.78
2005	Oct	15.3	17.23	12.91	24.72	17.23
2005	Nov	14.4	14.11	11.41	19.11	14.11
2005	Dec	13.6	34.15	15.08	20.41	15.08
2006	Jan	13.6	23.04	15.01	25.16	15.01
2006	Feb	13.5	10.58	30.14	18.21	10.58
2006	Mar	11.8	14.65	16.78	26.37	14.65
2006	Apr	11.7	21.60	32.08	16.91	16.91
2006	May	12	24.44	17.40	32.78	17.40

Year	Month	Data(X)	X_1^*	X_2^*	X_3^*	Forecast (X^*)
2006	Jun	11.9	29.54	24.21	20.56	20.56
2006	Jul	13.7	30.92	25.33	18.78	18.78
2006	Aug	13.9	11.61	28.44	34.08	11.61
2006	Sep	13.9	25.12	15.11	28.50	15.11
2006	Oct	13.4	35.64	18.05	27.87	18.05
2006	Nov	13.5	29.41	28.11	26.29	26.29

2006	Dec	13.5	35.55	20.88	27.00	20.88
2007	Jan	13.1	32.44	11.67	14.57	11.67
2007	Feb	11.1	10.24	31.30	23.55	10.24
2007	Mar	11.6	12.40	20.49	11.32	11.32
2007	Apr	11.5	15.28	13.64	21.15	13.64
2007	May	11.7	30.69	12.84	19.31	12.84
2007	Jun	11.3	31.77	20.72	33.89	20.72
2007	Jul	10.1	19.44	28.99	26.15	19.44
2007	Aug	10.7	13.70	12.47	33.48	12.47
2007	Sep	10.9	13.91	18.83	14.18	13.91
2007	Oct	11.1	29.98	28.56	14.92	14.92
2007	Nov	12.8	32.88	20.88	20.63	20.63
2007	Dec	14.4	27.68	12.84	23.39	12.84
2008	Jan	14.4	35.03	24.88	25.79	24.88
2008	Feb	14.7	27.28	20.74	22.34	20.74
2008	Mar	14.4	16.01	23.61	24.27	16.01
2008	Apr	16.9	30.99	30.09	15.26	15.26
2008	May	17.7	28.14	31.67	16.47	16.47
2008	Jun	18.9	31.75	29.82	18.88	18.88
2008	Jul	19	28.91	24.04	17.71	17.71
2008	Aug	18.5	18.50	19.83	12.53	18.50

Year	Month	Data(X)	X_1^*	X_2^*	X_3^*	Forecast (X^*)
2008	Sep	18.5	17.07	10.85	25.99	17.07

2008	Oct	18.9	33.40	19.30	16.29	19.30
2008	Nov	18.6	32.60	28.38	16.50	16.50
2008	Dec	19.1	19.44	29.08	30.51	19.44
2009	Jan	20.2	27.27	16.88	15.25	16.88
2009	Feb	21.3	23.87	25.74	11.06	23.87
2009	Mar	22	34.19	28.96	19.55	19.55
2009	Apr	21.5	26.49	23.93	18.54	23.93
2009	May	22.2	15.62	34.51	27.01	27.01
2009	Jun	24.7	23.60	20.24	15.13	23.60
2009	Jul	24.5	30.80	19.99	12.57	19.99
2009	Aug	23.3	32.54	25.19	25.07	25.07
2009	Sep	22.4	17.73	26.86	20.29	20.29
2009	Oct	21.2	28.88	25.61	32.06	25.61
2009	Nov	20	16.93	10.30	27.57	16.93
2009	Dec	18.8	30.91	19.33	17.77	19.33
2010	Jan	18.8	33.06	21.56	32.23	21.56
2010	Feb	18.5	27.38	13.15	21.04	21.04
2010	Mar	17.6	18.77	29.82	22.73	18.77
2010	Apr	15.8	12.61	21.24	33.18	12.61
2010	May	15	21.59	13.99	24.10	13.99
2010	Jun	11.9	34.63	30.86	12.17	12.17
2010	Jul	12	32.14	33.20	33.89	32.14
2010	Aug	12.2	30.93	16.18	16.49	16.18

Year	Month	Data(X)	X_1^*	X_2^*	X_3^*	Forecast (X^*)
2010	Sep	12.5	12.81	17.20	13.97	12.81
2010	Oct	11.82	28.98	20.95	27.90	20.95
2010	Nov	11.5	35.78	30.31	23.94	23.94
2010	Dec	11.22	22.76	15.83	13.79	13.79
2011	Jan	11.82	28.56	24.81	22.44	22.44
2011	Feb	12.12	31.73	17.05	22.12	17.05
2011	Mar	12	12.85	18.13	25.86	12.85
2011	Apr	12.16	32.63	28.77	29.58	28.77
2011	May	12.15	22.39	17.08	20.98	17.08
2011	Jun	12.44	17.48	18.86	24.81	17.48
2011	Jul	11.75	20.75	32.67	17.81	17.81
2011	Aug	11.38	29.40	15.13	17.39	15.13
2011	Sep	11.3	17.00	27.08	14.10	14.10
2011	Oct	11.32	23.76	15.03	13.87	13.87
2011	Nov	11.08	29.55	32.51	15.37	15.37
2011	Dec	11.21	14.44	29.68	24.77	14.44
2012	Jan	11.27	29.98	23.67	17.15	17.15
2012	Feb	11.24	25.28	14.11	19.01	14.11
2012	Mar	11.39	33.28	20.16	30.02	20.16
2012	Apr	11.73	35.12	23.85	26.51	23.85
2012	May	11.95	24.13	28.45	25.54	24.13
2012	Jun	11.9	13.85	30.63	13.22	13.22

2012	Jul	12	27.74	33.14	21.03	21.03
2012	Aug	12.5	12.96	11.61	28.73	12.96
2012	Sep	12.4	10.17	24.97	18.67	10.17
2012	Oct	12.16	20.24	19.82	20.16	19.82
2012	Nov	12.4	31.35	27.89	31.83	27.89
2012	Dec	11.6	30.21	18.11	14.94	14.94
2013	Jan	11.7	26.93	12.77	31.36	12.77
2013	Feb	13.3	25.19	29.24	24.42	24.42

Year	Month	Data(X)	X_1^*	X_2^*	X_3^*	Forecast (X^*)
2013	Mar	13.99	14.58	12.07	29.99	14.58
2013	Apr	14.35	14.04	11.61	24.26	14.04
2013	May	14.39	22.06	21.43	31.92	21.43
2013	Jun	15.08	14.20	29.55	26.25	14.20
2013	Jul	35.43	33.72	23.86	14.93	33.72
2013	Aug	14.24	16.66	25.34	19.12	16.66
2013	Sep	14.29	19.33	10.80	28.52	10.80
2013	Oct	17.9	16.20	26.41	28.06	16.20
2013	Nov	17.6	28.82	30.05	27.86	27.86
2013	Dec	18.1	26.66	12.53	34.09	12.53
2014	Jan	18.9	20.85	11.96	27.89	20.85
2014	Feb	19	35.79	23.55	24.34	23.55
2014	Mar	19.2	14.07	25.70	23.21	23.21
2014	Apr	20.6	17.41	26.21	33.96	17.41

2014	May	20	23.20	15.13	25.28	23.20
2014	Jun	20.3	27.66	21.25	12.13	21.25
2014	Jul	23.1	35.56	11.06	27.89	27.89
2014	Aug	24	25.18	33.54	19.70	25.18