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KUMASI

CREW ROUTE SCHEDULING USING THE MULTIPLE TRAVELING SALESMAN PROBLEM

By

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I hereby declare that this submission is my own work towards the MSc and that, to the best of my knowledge, it contains no material previously published by another person nor material which has been accepted for the award of any other degree of the University, except where due acknowledgement has been made in the text.

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ABSTRACT

Most of the coverage of maintenance operations of the technical men of the Electricity Company of Ghana (ECG) is unplanned. They have to move from one substation to the other to work on faults and cables as well as perform routine maintenance of which the maintenance schedule needs planning. This thesis uses the multiple traveling salesman problem model to plan the routes of a given number of maintenance crew so as to reduce the cost of traveling. Data used was distances of cities from ECG map of substations and transformers location in the Makola district from period 2003 to date. An algorithm was used to input the distances in Matlab. The algorithm was run ten different times varying the number of maintenance crew from two to five. The maintenance crew of two after all the tours covered the distance of 21.8100km being the minimized distance and routes for each of the maintenance crew was modeled with the two maintenance crew being the best.

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LIST OF ABBREVIATIONS

MTSP	Multiple Traveling Salesman Problem
ECG	Electricity Company of Ghana
VRA	Volta River Authority
NED	Northern Electrification Department
LV	Low Voltage
GA	Genetic Algorithm
MATLAB	Matrix Laboratory
MGA	Modified Genetic Algorithm
SDMTSP	Single Depot Multiple Traveling Salesman Problem
GT	Group Technology
DNA	Deoxyribo Nucleic Acid
BDMM	Basic Differential Multiplier Method
MCSP	Maximum Covering Shortest Path
DPPC	Dynamic Pre-Populated Crossover
MLTS	Multiple Longest Traveling Salesman
NP	Non Polynomial
ACO	Ant Colony Optimization
MDMTSP	Multiple Depot Multiple Traveling Salesman Problem
MTRP	Multiple Traveling Robot Problem
DPSO	Discrete Particle Swarm Optimization
PSO	Particle Swarm Optimization
TSP	Traveling Salesman Problem

DEDICATION

I dedicate this work to God, for the strength. Also to my parents for all the investments they have made towards my education.

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CHAPTER ONE

INTRODUCTION

1.0 BACKGROUND OF STUDY

Scheduling plays a vital role in every organization since it helps to plan and manage the resources by minimizing cost. Many literatures exist on scheduling. Svestka and Huckfeldt (1973) report an application for deposit carrying between different branch banks. Here, deposits need to be picked up at branch banks and returned to the central office by a crew of messengers. The problem is to determine the routes of messengers with a total minimum cost. Lenstra and Rinnooy Kan (1975) describe two similar applications, where the first application consists of finding the routes of a technical crew, which has to visit telephone boxes in North Holland. The second application involves designing the routes of vehicles to visit 200 mailboxes in Utrecht, such that the number of vehicles used is minimum. Another application of the multiple traveling salesman problem (mTSP) in crew scheduling is reported by Zhang et al.(1999), who investigate the problem of scheduling multiple teams of photographers to a large number of elementary and secondary schools.(Bektas,2006)

Over the years, electricity has played a vital role in the development of Ghana. With a customer base of approximately 1.4 million, it has been estimated that 45-47 percent of Ghanaians, including 15-17 percent of the rural population, have access to grid electricity. It is estimated that about half of electricity produced is consumed by domestic (or residential) consumers for household uses such as lighting, ironing, refrigeration, air conditioning, television, and the like. Commercial and industrial users account for the rest consumed.

1.0.1 History of Electric Power in Ghana

There are three main periods of electricity in Ghana."Before Akosombo", refers to the period before the construction of the Akosombo Hydroelectric Power Plant in 1966. This is a period of isolated generation facilities with low rates of electrification. The second period, "the Hydro Years", covers the 1966 to the mid eighties, also called the Volta

Development era. The Volta Development includes the Akosombo Hydroelectric Power Plant commissioned in 1966 and the Kpong Hydroelectric Plant, completed in 1982. By the mid eighties, demand for electricity had exceeded the firm capability of the Akosombo and the Kpong Hydro Power Plants. The third period, "Thermal Complementation", from the mid eighties to date, is characterized by efforts to expand power generation through the implementation of the Takoradi Thermal Power Plant as well as the development of the West African Gas Pipeline to provide a secure and economic fuel source for power generation. (RCEER, 2005)

In Ghana, Electricity Company of Ghana Limited (ECG) is the entity that is responsible for distribution of electricity to consumers. It is the entity that consumers interact with when they receive and pay their bills or when they have service questions (billing, metering, line connection, etc.). Electricity Company of Ghana Limited is responsible for distribution of electricity to consumers in southern Ghana, namely Ashanti, Central, Greater Accra, Eastern and Volta Regions of Ghana. The Northern Electrification Department (NED), a subsidiary of Volta River Authority (VRA) is responsible for power distribution in northern Ghana namely, Brong-Ahafo, Northern, Upper East and Upper West Regions.ECG has its head office in Accra. The organization structure is made up of the Board of Directors, Managing Director, with the sections engineering, operations, finance, materials and transport, human resource, customer services, legal and audit. The total staff strength as at June 2010 was 5,393. In Greater Accra, the organization is divided into two, namely the Accra East Region and the Accra West Region. Accra West Region is divided into five districts namely Kaneshie, Achimota, Dansoman, Korle-bu and Avenor.

1.0.2 Accra East Region

The staff strength of Accra East as at June 2010 was 529 with 387 males and 142 females. The organizational structure is made up of the Regional Director, who the Regional Accounts Manager, District Manager(Legon), Regional Human Resource Manager, Regional Commercial Manager, District Manager(Makola),Regional Engineer, District Manager(Roman Ridge), District Manager(Teshie), District Manager(Kwabenya), District Manager(Mampong) report to. District Communication Officer, District Technical Officer, District Engineer and Customer Relations Officer work under the District Managers. Accra East region has six districts which are Legon, Makola, Roman Ridge, Teshie, Kwabenya and Mampong. Some of the cities under which Legon falls include Madina, East Legon, and"Shiashi". Makola has Cantonments, La, and Osu under it. Roman Ridge has Kanda, Dzorwulu. Teshie has Teshie Nungua Estate,"Baatsonaa", and Spintex Road. Kwabenya has Atomic, Haatso, and Dome under it and Mampong has Aburi, Oyibi, and Mampong.

1.0.3 District 'A' Makola District

The Makola district also has its structure as Regional Director who District 'A' Manager reports to. District Engineer also reports to the District 'A' Manager. Under the District Engineer are Chief Technical Engineers, Principal Technical Engineers and Senior Technical Engineers and

Technical Engineers. The Makola district has about 250 substations spread across the cities. The Operations department, technical section plans the schedule for the technical men and engineers to visit substations for maintenance and repairs. There are two types of maintenance, planned and unplanned. The planned is mostly for preventive maintenance. There are two types of maintaince. Preventive maintenance is when the technical men carry out preventive measures such as oiling rusty transformer to prevent the transformers from being destroyed. This is usually planned for by the technical section. Corrective maintenance is when the technical men carry out corrective measures on faulty transformers to restore it. Most often, the technical men carry out the unplanned and corrective maintenance because of reported faults.

There are two divisions under the District Engineer which are Low Voltage (LV), Network maintenance (Cables and Lines) and Low Voltage, Operations and Faults. Each division work on their particular problem reported. The LV Operations and Faults go with a Senior Technical Engineer and a gang per shift making up of one foreman and three artisans. The LV Network maintenance is subdivided into LV Lines and LV Cables. Each group goes out with a Technical Engineer, one foreman, four artisans for the Lines gang and one foreman, two artisans, and two tradesmen for Cable gang. Some of the challenges the teams face include fault tracking, procedures to follow, and because the teams mostly use unplanned maintenance, it makes their routes scattered and makes their distances longer. (Dwamena and Opoku, 2010)

1.1 STATEMENT OF PROBLEM

Most of the operations of the technical men are unplanned maintenance. They have to move from one substation to the other to work on faults and cables which makes their routes scattered. This makes it impossible for the technical men to visit all of the substations in other cities. Also when there is a planned routine check to be carried out, what would be the best route to minimize total distance?

1.2 OBJECTIVE OF THE STUDY

The objectives of the study are to:

- Model a route path for the technical men.
- Minimize the route (distance) travelled by technical men.

1.3 METHODOLOGY

Given a set of cities, let there be m salesmen located at a single city. The remaining cities that are to be visited are called intermediate cities. Then, the multiple traveling salesman problem (mTSP) consists of finding tours for all m salesmen, who all start and end at the city, such that each intermediate city is visited exactly once and the total cost of visiting all nodes is minimized. The model to be used is the multiple traveling salesman problem.

The data to be used is the distances of the cities, from the ECG map of substation locations. Heuristic solution to be applied is Genetic Algorithm (GA) which is coded in computer language and Matrix Laboratory (Matlab) be used for data analysis. Data period is from 2003 to date. Information for the study is obtained from oral interview of personnel from Electricity Company Of Ghana (ECG) and also published books on the internet.

1.4 SCOPE OF STUDY

The study is confined to the Makola district of the Electricity Company of Ghana (Accra East Region).

1.5 JUSTIFICATION OF STUDY

The mTSP can be used to model many real life applications. When the routes of the technical men are modelled, it would help them visit all the cities assigned them to work on faults and service substations and transformers. This would help industries and individual organizations to function and increase economic growth, and production would be increased. Also it is going to add to academic knowledge and lastly going to help ECG provide better electricity service to the people of Ghana.

1.6 ORGANISATION OF THESIS

The thesis is organized in five chapters. Chapter one describes the introduction and the history of electricity in Ghana and the Electricity Company of Ghana. Chapter two describes the literature review of the thesis. Chapter three deals with the methodology. Chapter four deals with data collection and analysis. Chapter five presents the summary, conclusions and recommendations.

1.7 SUMMARY OF THE CHAPTER

Chapter one dealt with the introduction, background of the study, problem statement, objectives of the study, methodology, scope of the study, justification of the study and organization of the thesis.

The next chapter deals with the literature review, work done by others.

CHAPTER TWO

2.0 LITERATURE REVIEW

Tang et al. (2000) proposed a parallel strategy to model the scheduling problem for hot rolling production scheduling, of major iron and steel companies in China, and solve it using a new modified genetic algorithm (MGA). Combing the model and man–machine interactive method, a scheduling system was developed. The result of one year's running in Shanghai Baoshan Iron and Steel Complex showed 20 percent improvement over the previous manual based system.

Byungsoo (2007) proposed several construction heuristic algorithms, including greedy algorithms , cluster first and route second algorithms, and route first and cluster second algorithms for no depot minmax Multiple Traveling Salesmen Problem (mTSP)with simulated annealing method used to prevent the drawback of trapping in the local minima. Among the construction algorithms, route first and cluster second algorithms method performed best. In terms of running time, clustering first and routing second algorithms took shorter time on large scale instances. To evaluate the performance of the proposed heuristic methods, their solutions were compared with the optimal solutions obtained using mixed integer programming formulation of the problem. For small scale problems, the heuristic solutions were equal to the optimal solution.

Plebe and Anile (2002) proposed a self-organizing map structure for the optimization of the harvest sequence for the two independent arms of a fruit-harvesting robot. Results of tests indicate that the proposed approach is efficient and reliable for harvest sequence planning.

Chen and Hsu (2004) have proposed an immune algorithm for solving traders' cooperative strategy of periodic marketing problem. Numerical examples indicate that the proposed approach can efficiently and effectively search over promising solution regions to exhibit the superior performance of the proposed methodology for finding the optimal or near optimal itinerant strategy during the specified planning horizon.

Yamamoto et al. (2010) proposed an adaptive routing method in the cruising taxis. Simulation experiment showed that the method was able to pick up more customers than the existing means of cruising taxis.

Paydar et al. (2010) formulated the cell formation problem as a single depot multiple travelling salesman problem (SDmTSP). Computational results compared the model with a set of group technology (GT) problems. The approach produced solutions with better grouping efficacy.

Senthilkumar and Bharadwaj (2008) presented a hybrid approach that combines clustering and Genetic Algorithm (GA) to solve the Multi Robot Path Exploration Problem. Experimental results are presented to illustrate the performance of the proposed scheme.

Zoraida et al. (2010) proposed to solve Capacitated Vehicle Routing Problem. They utilized thermodynamic properties of Deoxyribo Nucleic Acid(DNA) for the first time, along with other bio-chemical operations to obtain the optimal solution. The method exhibits the ability to solve complex combinatorial problems using molecular computing.

Wacholder et al. (1989) developed an efficient neural network algorithm for solving the Multiple Traveling Salesmen Problem (mTSP). The dynamic model associated with the problem is based on the Basic Differential Multiplier Method (BDMM) which evaluates Lagrange multipliers simultaneously with the problem's state variables. In all test cases, the algorithm always converged to valid solutions.

Abdel-Hameed and Abdel-Lateef (1988) developed the use of dual-based approaches for solving Multi-objective Location Problems such as the Maximum Covering Shortest Path (MCSP) problem. For the first of these a dual-based scheme is developed and applied to an example with considerable success but further experimentation is required to establish the merits of the approach more generally.

Sofge et al. (2008) worked on to solve the Multiple Traveling Salesman Problem (mTSP), and compares a variety of evolutionary computation algorithms and paradigms for solving it. Techniques implemented, analyzed, and discussed include use of a neighborhood attractor schema (a variation on k-means clustering).

Zhou and Li (2010) worked on an improved genetic algorithm to provide an alternative and effective solution to the mTSP. The initial population was generated by greedy strategy, the simulation results based on the algorithm showed that the improved method is effective and feasible.

Modares et al.(1999) introduced a new algorithm in competition based network to solve the minmax multiple travelling salesmen problem (mTSP), the generalized 2 opt exchange heuristic algorithms and the elastic net algorithm are applied to the minmax mTSP problem solution. The adaptive approach obtained the superior solution in all instances, compared to the generalized 2opt exchange heuristic and the elastic net.

Nallusamy et al.(2009) worked on generating of an optimized route for multiple Traveling Salesman Problems."Tabu Search" and "Simulated Annealing" were extensively used. From the results, Simulated Annealing generated optimized route, covering less distance than Tabu search.

Oberlin et al.(2009) worked on the well known LKH heuristic, a multiple depot, multiple traveling salesman problem transformed into a single, asymmetric traveling salesman problem for instances involving Dubins vehicles. Results show that the transformation is effective and high quality solutions can be found for large instances in a relatively short time.

Koh et al. (2006) developed a new approach to solve multi-TSP problem, using Genetic Algorithm (GA). The performance of the new operators GA Inspection Module and DPPC (Dynamic Pre-Populated Crossover) for a better evolutionary approach to the time-based problem were discussed.

Estevez-Fernandez et al. (2003) worked on multiple longest traveling salesman (mLTS) games. First, it is shown that the value of a coalition of an mLTS game is determined by taking the maximum of suitable combinations of one and two person coalitions. they provide relations between the structure of the core and the underlying network.

Junjie and Dingwei (2006) worked on and made the attempt to show how the ant colony optimization (ACO) can be applied to the mTSP with ability constraint. They compare it with MGA by testing several standard problems. The computational results show that the proposed algorithm can find competitive solutions even not all of the best solutions within rational time, especially for large scale problems.

Király and Abonyi (2010) proposed a novel, easily interpretable representation based GA to solve the complex combinatorial optimization problem, mTSP. The result showed the extension of classical GA tools for mTSP is not a trivial problem, it requires special, interpretable encoding to ensure efficiency.

Aljanaby et al. (2008) proposed a new multiple ant colonies optimization algorithm of using multiple ant colonies, as an extension of the Ant Colony Optimization framework. The new algorithm is based on the ant colony system and utilizes average and maximum pheromone evaluation mechanisms. The new algorithm can effectively be used to tackle large scale optimization problems. Computational tests show promises of the new algorithm.

Borovska et al. (2007) worked on strategies for designing parallel genetic algorithms on multicomputer platforms. They considered two parallel genetic computational models based on the manager/workers and Single Program Multiple Data parallel paradigms. Parallelism profiling and analysis of parallel system performance were made for the different parallel computational models in respect to the scalability of the application and the scalability of the parallel machine size.

Carter (2003) worked on a new modeling methodology for setting up the mTSP to be solved using a GA. The model was also used to model and solve a multi line production problem in a spreadsheet environment. The method used effectively created a schedule while considering all required constraints.

Xu and Rodrigues (2010) developed a 3/2-approximation algorithm, which runs in polynomial time when the number of depots is a constant on the multiple depot multiple traveling salesman problem (mDMTSP).

Sariel-Talay et al. (2009) proposed a multi robot cooperation framework employing a dynamic task selection scheme to solve mTRP. The proposed framework carried out an incremental task allocation method that dynamically adapts to current conditions of the environment, thus handling diverse contingencies. Globally efficient solutions are obtained through mechanisms that resulted in the allocation of the most suitable tasks from dynamically generated priority-based rough schedules. The efficiency and the robustness of the proposed scheme were evaluated through experiments both in simulations and on real robots.

Kara et al.(2006) worked on classical multiple traveling salesman problem (mTSP) by imposing a minimal number of nodes that a traveler must visit as a side condition. Computational analysis showed that the solution of the multi depot mTSP with the proposed formulation was significantly superior to previous approaches.

Chandran et al.(2006) have in a departure from other methodologies that have been employed for the multiple traveling salesman problem (mTSP), proposed a clustering approach to solve the balancing of workloads amongst salespersons. When tested over a range of data-sets, the proposed method was found to achieve a good balance of workloads among the clusters, each of which was visited by a salesperson.

Tsutome et al.(2004) have proposed a heuristic solution method to make an ordered route for the basic problem on logistic engineering, obtained actual delivering routes by using belonging degree based on c-means clustering. As features of the proposed method, it listed up to be applicable to problems on about 1,000 towns, to be intended to equalize shared route lengths of every salesmen.

Bezalel et al.(1980) developed Lagrangean relaxation as new optimal solution method for the Multiple Travelling Salesman Problem, in which required the computation of a degree-constrained minimal spanning tree was utilized. The algorithm was tested on problems with up to 400 cities and 10 salesmen. They are working on an improved version of the algorithm that holds promise of being able to solve even larger problems.

Feng et al.(2009) employed discrete particle swarm optimization (DPSO) algorithm in the scheduling of jobs on multiple parallel production lines and the scheduling of multiple vehicles transporting goods in logistics. When applying particle swarm optimization (PSO) for the MTSP, a difficulty rises, which is to find a suitable mapping between sequence and continuous position of particles in particle swarm optimization. For overcoming this difficulty, PSO is combined with ant colony optimization (ACO), and the mapping between sequence and continuous position of particles is established. To verify the efficiency of the DPSO algorithm, it is used to solve the mTSP and its performance is compared with the ACO and some traditional DPSO algorithms. The computational results show that the proposed DPSO algorithm is efficient.

Discenza (1981) showed how to transform to a TSP with $n + m + 4$ nodes, where m is the number of salesmen. The four extra nodes were included to prevent the transformed problem from escaping part of the fixed costs. They showed that two of these nodes can be eliminated without sacrificing the formulation as a TSP. All formulations produce equivalent solutions.

Ran et al.(2006) proposed a new algorithm to improve the working efficiency of locomotives. A locomotive working diagram was transformed to a multiple traveling salesmen problem, and its mathematical model was set up to gain the optimum solution. Actual train diagram data proved that the algorithm was useful to obtain the optimal equilibrium of a locomotive working diagram with a minimum locomotive number.

Mitrovi et al. (2002) presented a lower and an upper bound for the minimum number of vehicles needed to serve all locations of the multiple traveling salesman problem with time windows. They introduced the start-time precedence graph and the end-time precedence graph. The bounds are generated by covering the precedence graph with minimum number of paths. They presented instances for which bounds are tight, as well as instances for which bounds can be arbitrary bad. The closeness of such instances was discussed.

CHAPTER THREE

METHODOLOGY

3.0 TRAVELING SALESMAN PROBLEM MODEL

The traveling salesperson problem (TSP) is a classic model for various production and scheduling problems. Many production and scheduling problems ultimately can be reduced to the simple concept that there is a salesperson who must travel from city to city (visiting each city exactly once) and wishes to minimize the total distance traveled during his tour of all *n* cities. Obtaining a solution to the problem of a salesperson visiting *n* cities while minimizing the total distance traveled is one of the most studied combinatorial optimization problems. While there are variations of the TSP, the Euclidean TSP is NP-hard (Schmitt et al., 1998). The interest in this particular type of problem is due to how common the problem is and how difficult the problem is to solve when *n* becomes sufficiently large.

The TSP can be formulated as an integer linear programming model as follows:

$$Min \qquad \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij} \qquad (3.1)$$

s.t

$$\sum_{i=1}^{n} X_{ij} = 1, \qquad j = 1, \dots, n \qquad (3.2)$$

$$\sum_{j=1}^{n} X_{ij} = 1, \qquad i = 1, \dots, n \qquad (3.3)$$

$$\{(i,j) | i,j = 2, \dots, n; X_{ij} = 1\} \qquad (3.4)$$

$$X_{\text{ij}} = \{0,1\}\ for\ all\ i, j = 1, \dots, n \qquad (3.5)$$

where c_{ij} is the distance from city *i* to city *j*. If $c_{ij} = c_{ji}$ for all *i* and *j* then the problem is symmetric, otherwise it is asymmetric. The binary variable X_{ij} equals one if the route from city *i* to *j* is in the solution and zero otherwise. The objective in (3.1) is to minimize the total distance traveled. Constraints (3.2) and (3.3) ensure, respectively, that each city is entered and exited exactly once.

3.1 MULTIPLE TRAVELING SALESMAN PROBLEM

A generalization of the well-known traveling salesman problem (TSP) is the multiple traveling salesman problem (mTSP), which consists of determining a set of routes for m salesmen who all start from and turn back to a home city (depot). Although the TSP has received a great deal of attention, the research on the mTSP is limited. The mTSP can in general be defined as follows: Given a set of nodes, let there be m salesmen located at a single depot node. The remaining nodes (cities) that are to be visited are called intermediate nodes. Then, the mTSP consists of finding tours for all m salesmen, who all start and end at the depot, such that each intermediate node is visited exactly once by only one of the salesmen and the total cost of visiting all nodes is minimized. The cost metric can be defined in terms of distance, time, cost etc. In the single depot situation, all salesmen start from and end their tours at a single point. On the other hand, if there exist multiple depots with a number of salesmen located at each, the salesmen can either return to their original depot after completing their tour or return to any depot with the restriction that the initial number of salesmen at each depot remains the same after all the travel. (Bektas, 2006) The main goal is to minimize the total traveling cost of the above problem that is often formulated as assignment based integer linear programming.

$$Min \qquad \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij} \tag{3.6}$$

s.t

$$\sum_{j=2}^{n} x_{1j} = m \tag{3.7}$$

$$\sum_{j=2}^{n} x_{j1} = m \tag{3.8}$$

$$\sum_{i=1}^{n} X_{ij} = 1, \qquad j = 1, \dots, n \tag{3.9}$$

$$\sum_{j=1}^{n} X_{ij} = 1, \qquad i = 1, \dots, n \tag{3.10}$$

$$\{(i,j) | i,j = 2, \dots, n; X_{ij} = 1\} \tag{3.11}$$

$$X_{ij} = \{0,1\}\ for\ all\ i,j = 1, \dots, n \tag{3.12}$$

where $X_{ij} \in \{0, 1\}$ is a binary variable used to represent that an arch is used on the tour, C_{ij} is the cost associated to the distances between the i th and j th nodes, m is the number of salesmen. (Abonyi and Király,2009).

A number of solution procedures have been proposed, that are tailored exclusively for the mTSP. These procedures generally consist of algorithms based on integer linear programming formulations and transformations of the problem to the TSP. Also, heuristics procedures have been proposed to solve the mTSP. Examples of this heuristics solutions include simple heuristics, simulated annealing,tabu search, genetic algorithms and others. This thesis is going to focus on the use of the heuristics procedure particularly genetic algorithm to solve the mTSP.

3.2 Genetic Algorithm

Every genetic algorithm starts with an initial solution set consists of randomly created chromosomes. This is called population. The individuals in the new population are generated from the previous population's individuals by the predetermined genetic operators. The algorithm finishes if the stop criteria is satisfied. Obviously for a specific problem it is a much more complex task, we need to define the encoding, the specific operators and selection method.

3.2.1 Fitness Function

The fitness function assigns a numeric value to each individual in the population. This value define some kind of goodness, thus it determines the ranking of the individuals. The fitness function is always problem dependent. In this case the fitness value is the total cost of the transportation, i.e. the total length of each round trip. The fitness function calculates the total length for each chromosome, and summarizes these values for each individual. This sum is the fitness value of a solution. Obviously it is a minimization problem, thus the smallest value is the best.

3.2.2 Selection

Individuals are selected according to their fitness. The better the chromosomes are, the more chances they have to be selected. The selected individuals can be presented in the new population without any changes (usually with the best fitness), or can be selected to be a parent for a crossover. We use the so-called tournament selection because of its efficiency. In the course of tournament selection, a few individuals are selected from the population randomly. The winner of the tournament is the individual with the best fitness value. Some of the first participants in the ranking are selected into the new population.

3.2.3 Operators

Because of our new representation, implementation of new genetic operators becomes necessary. There are two sets of mutation operators, the so-called In-route mutations and the Cross-route mutations.

3.2.4 In-route mutation

In-route mutation operators work inside one chromosome. The first operator chooses a random subsection of a chromosome and inverts the order of the genes inside it. The second operator reverses two randomly chosen genes in the given chromosome and the third put a randomly chosen gene into a given place.

3.2.5 Cross-route mutation

Cross-route mutation operates on multiple chromosomes. If we think about the distinct chromosomes as individuals, this method could be similar to the regular crossover operator. The method when randomly chosen subparts of two chromosomes are transposed. If the length of one of the chosen subsections is equal to zero, the operator could transform into an interpolation. (Arthur, 2003)

3.3 Proposed Genetic Algorithm

1. Create an initial population of P chromosomes.

2. Evaluate the fitness of each chromosome in the population.

3. Select the best two parent chromosomes from the current population.

4. Exchange bit strings with the uniform order based crossover to create two offspring.

5. Process each offspring by the exchange mutation operator, and evaluate them.

6. Replace the worst parent with the best offspring out of the two offsprings in the population.

7. Go back to Step 3 if the number of generations is less than some upper bound. Otherwise, the final result is the best chromosome created during the search. (Senthilkumar and Bharadwaj, 2008)

An example of function optimization by processing a population to get a new population. To illustrate the selection procedure, we take the crossover probability to be Pc=1.

Min $f(x) = \frac{x^4}{4} - \frac{7}{3}x^3 + 7x^2 - 8x + 15, \quad x = 0,1,2,3,\ldots,10$

Step 1

The search space is $x = 0,1,2,\ldots,10$. We encode elements of the search space in a binary sequence. Express $x = 10$ and $x = 0$ in binary sequence to obtain $10=1010_2$ and $0=0000_2$.Thus $x = 10$ is an individual solution and 1010 is its chromosome representation. The chromosome has 4 genes placeholders for the allele. The allele information in the genes will be the binary numbers 0 and 1.The chromosomes for $x = 9$ is therefore

1	0	0	1

The objective evaluation is $f(9)$ =449.25. There are 2^4 permutations for a binary string of length 4.These 2^4 permutations consist of both infeasible and feasible solutions. There are 11 feasible solutions, which constitute our search space and the rest form the infeasible set. Since the solution set is restricted to the integers we look for suboptimal solution.

Step 2

Generation counter for populations: we set $g = 1$ to set the counter tag for the initial population. Since we are processing only the initial population there will be no further increment of the counter.

Step 3 and 4

Initial population: we select at random 4 individual (solutions). We choose the number to be the same as the length of a chromosome string. Take $x_1 = 5, x_2 = 1, x_3 = 3, x_4 = 9$

TABLE 3.1 SHOWING SERIAL NUMBERS, CHROMOSOME REPRESENTATION, SOLUTION AND OBJECTIVE FUNCTION OF EACH SOLUTION

Serial No	Chromosome $g(x)$	Solution (x)	Objective $f(x)$
1	0101	5	14.58
2	0001	1	11.92
3	0011	3	11.25
4	1001	9	449.25

Step 5

Selection of parents: Since the problem is a minimization problem, we use the fitness formula $F = (F_{\max} - f_{\mathrm{i}}) + 1$ (Cox, 2005) to reserve the magnitude of the objective. We then use fitness proportionate selection rule that requires the calculation of: (i) Probability of selection= $\frac{F(x)}{\sum F(x)}$

(ii) Expected count = $\frac{F(x)}{F(x)}$

(iii) Actual count= Round up of expected count to nearest integer.

Step 6

Apply crossover operation: The pairings(1,5),(3,5),(1,3),(9,9) are used and the space number(2) representing the crossover point between the second and third binary digits is chosen.

Step 7

Apply mutation operation on offspring: A random number is assigned to each allele of a chromosome offspring. Mutation (exchange of 0 and 1) is done if Rand (i, j) < 0.3: where ‘i’ is serial number of chromosomes and ‘j’ is the loci of the allele or binary bit.

Step 8

Evaluation of mutated offspring. $x = 13$ and $x = 11$ are not evaluated because they fall outside the domain or solution space. They are infeasible solutions.

TABLE 3.2 SHOWING MUTATED OFFSPRING, INDIVIDUAL REPRESENTATION AND EVALUATION OF OBJECTIVE FUNCTION

Number	Mutated chromosomes	x	$f(x)$
1	0001	1	11.92
2	0101	5	14.58
3	0111	7	101.92
4	1101	13	-
5	0010	2	12.33
6	0001	1	11.92
7	1011	11	-

Repeated mutated chromosomes are deleted and join the rest to members of the old generation to get the recombination. In the table below, the serial numbers with asterisks are for chromosomes from the offsprings. The chromosomes $g(x) = 0010$ representing $x = 2$ will be added to the old generation of population to form a new generation. The chromosome $g(x) = 0001$ representing $x = 1$ and which has the lowest $f(x)$ is already part of the old generation of population, so is the chromosome $g(x) = 0101$ representing $x = 5$.

TABLE 3.3 SHOWING CHROMOSOMES FROM OFFSPRINGS

Serial No	Chromosomes $g(x)$	Solution (x)	Objective $f(x)$
1	0101	5	14.58
2	0001	1	11.92
3	0011	3	11.25
4	1001	9	449.25
5*	0001	1	11.92
6*	0101	5	14.58
7*	0111	7	101.92
8*	0010	2	12.33

Repeating chromosomes from the recombination to get the intermediate generation in the next table and ordered in ascending order.

TABLE 3.4 SHOWING THE INTERMEDIATE GENERATION

Serial No	Chromosomes $g(x)$	Solution (x)	Objective $f(x)$
1	0011	3	11.25
2	0001	1	11.92
3	0010	2	12.33
4	0101	5	14.58
5	0111	7	101.92
6	1001	9	449.25

Step 9

Chromosome 4 in the old generation is deleted and chromosome 5 in the offspring list used to replace it. The new generation is given in the table below.

TABLE 3.5 SHOWING THE NEW GENERATION

Serial No.	Chromosomes $g(x)$	Solution (x)	Objective $f(x)$
1	0011	3	11.25
2	0001	1	11.92
3	0010	2	12.33
4	0101	5	14.58

Return to step 2 to continue with the process until the termination condition is satisfied. (Amponsah and Darkwah, 2007)

3.4 GENETIC ALGORITHMS AND THE MULTIPLE TRAVELING SALESPERSON PROBLEM

Researchers have proposed two methods of modeling the mTSP such that a TSP operator can be used. One method uses two chromosomes to model the mTSP. The first chromosome provides a permutation of the cities and the second chromosome assigns a salesperson to the city in the corresponding position of the first chromosome (Malmborg, 1996; Park, 2001). This method requires two chromosomes of length n to represent a solution.

Cities

2	5	14	6	1	11	8	13	4	10	3	12	15	9	7

Salespersons

2	1	1	3	4	3	2	4	4	1	3	2	1	2	3

Figure 3.1
Example of a Two Chromosome Representation of
a 15 City MTSP with 4 Salespersons.

In the example in Figure 3.1, cities 2, 8, 12 and 9 would be visited by salesperson 2 in that order. Cities 5, 14, 10 and 15 would be visited by salesperson 1 and likewise for the remaining cities assigned to salespersons 3 and 4. If m represents the number of salespersons, using this modeling scheme, there are $n!(m^n)$ possible solutions to the problem, many of which are redundant.

The second method is to model the mTSP as a TSP with one chromosome and lengthening the chromosome by one position for each salesperson above one that is going to be used to cover the routes. In this model the extra positions in the chromosome are filled with dummy cities that represent the change from one salesperson to the next.

2	5	14	6	-2	1	11	8	13	-3	4	10	1	-1	12	15	9	7

Figure 3.2
Example of a One Chromosome Representation
of a 15 City MTSP with 4 Salespersons.

In the example in Figure 3.2 the negative numbers are used as the dummy cities to indicate the change from one salesperson to the next. So the first salesperson would visit cities 2, 5, 14 and 6 in that order. The second salesperson would visit cities 1, 11, 8 and 13 in that order, and so on for salespersons 3 and 4. This makes the chromosome (n + m -1) in length where m is the number of salespersons (Tang et al., 2000).Using this modeling scheme, there are (n + m –1)! possible solutions to this problem, many of which are redundant.

The new modeling technique developed for the mTSP uses a two-part chromosome that also reduces redundancy of the same solution being represented in different ways. For the mTSP, the first part of the chromosome is a permutation of the n cities. The second part of the chromosome is of length m and represents the number of cities assigned to each of the m salesperson. The sum of the values assigned to all m salespersons must sum to the number of cities to be visited (n).

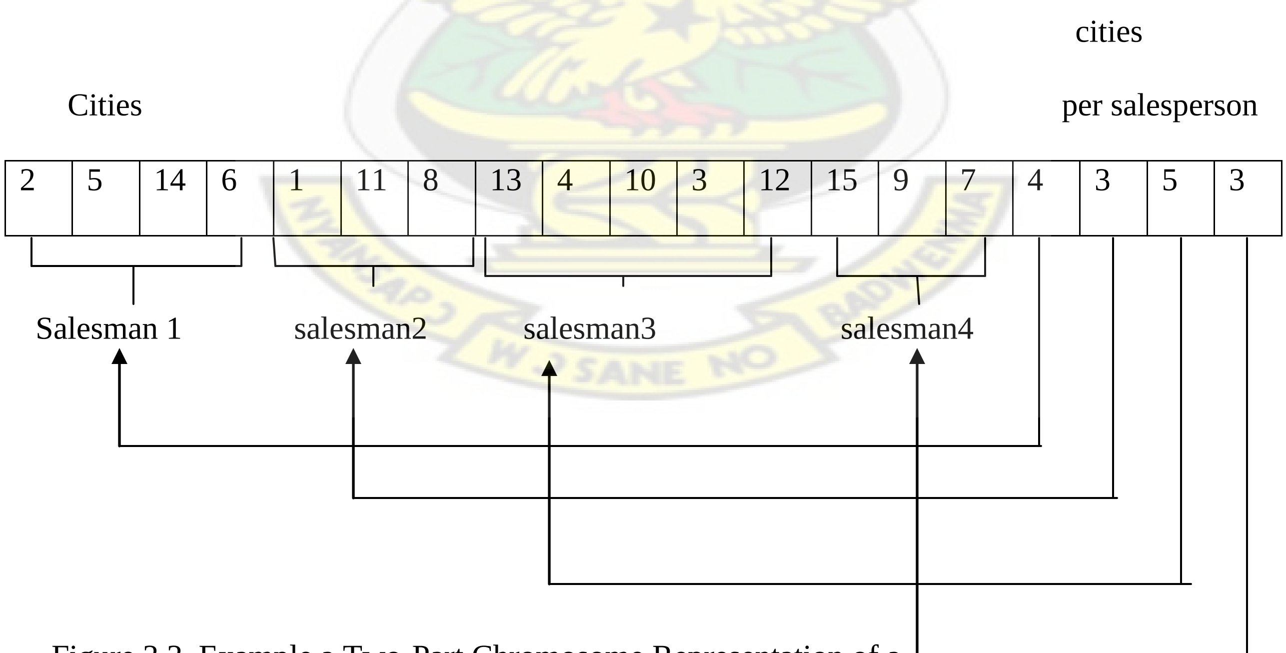

Figure 3.3. Example a Two-Part Chromosome Representation of a 15 City MTSP with 4 Salespersons**.**

In Figure 3.3, salesperson 1 would visit cities 2, 5, 14 and 6 in that order, salesperson 2 would visit cities 1, 11 and 8 in that order, and so on for salespersons 3 and 4.

Using the two-part chromosome for the mTSP, there are n! possible permutations for the first part of the chromosome.

The second part of the chromosome represents a positive vector $(x_1, x_2, \ldots, x_m)$ that must sum to n.There are $\binom{n-1}{m-1}$ distinct positive integer-valued m-vectors that satisfy this requirement(Ross,1984).Therefore,the solution space for the two-part chromosomes is of size $n!\binom{n-1}{m-1}$.

CHAPTER FOUR

4.0 DATA

The data was collected from the Engineering department, drawing section of the Electricity Company of Ghana (Accra East Region), in the softcopy form in AutoCAD. With the help of an architect, Mr.Adjin-Tettey, the data was transformed by taking the distances from one city(with substation/transformer) to the other.

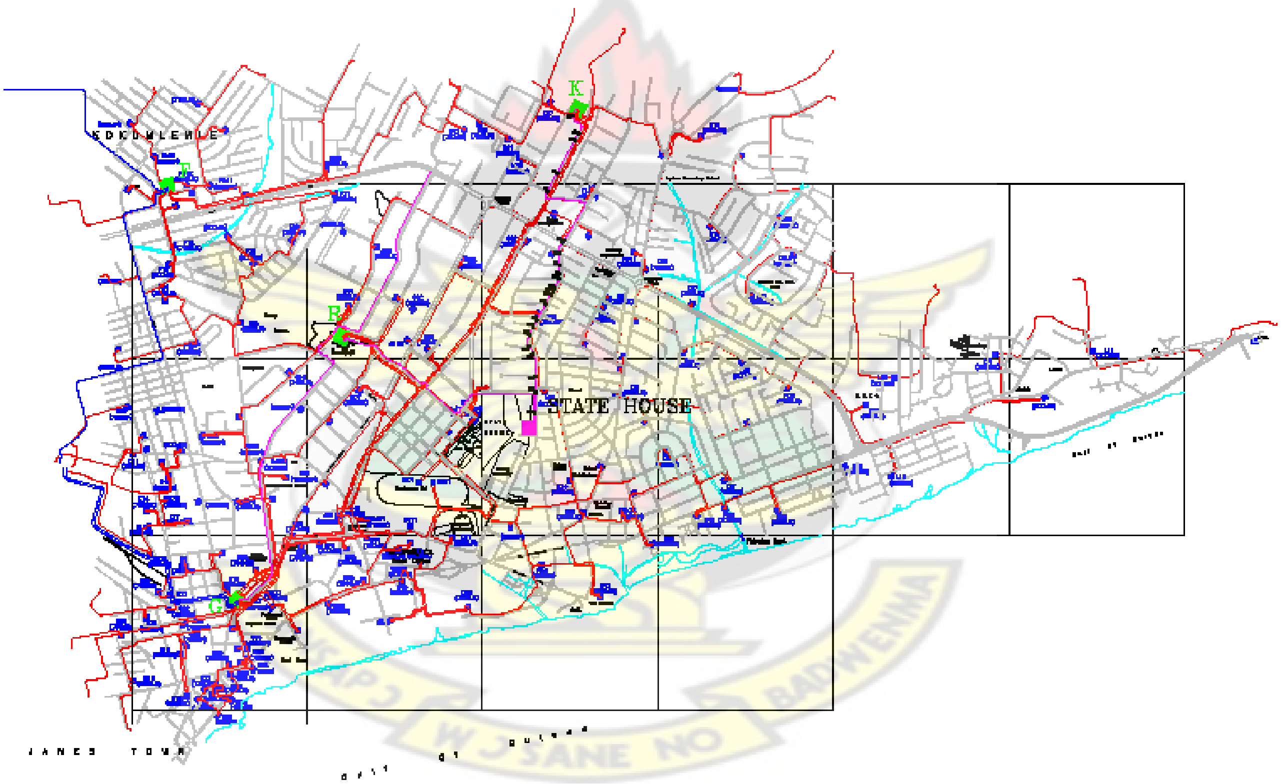

Figure 4.1 Area Map of Makola District (ECG, Accra East)

TABLE 4.1 THE CITY TO CITY DISTANCES (TRANSFORMED DATA)

	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
1	0	0.47	0.30	0.62	0.96	1.45	1.92	1.86	2.77	3.24	4.54	5.30	5.63	6.48	7.02
2	0.47	0	0.35	0.48	0.82	1.35	1.91	1.88	4.86	3.14	4.63	5.69	5.32	5.27	5.36
3	0.30	0.35	0	0.85	0.60	1.13	1.59	1.47	4.38	2.84	4.21	4.84	5.29	4.49	4.85
4	0.62	0.48	0.35	0	0.38	0.92	1.44	1.54	3.23	2.69	4.04	4.76	4.89	4.53	4.71
5	0.96	0.82	0.60	0.38	0	0.51	1.11	1.41	3.58	2.58	3.73	4.40	4.73	3.92	4.28
6	1.45	1.35	1.13	0.92	0.51	0	1.33	1.91	3.60	3.11	4.12	4.46	4.10	3.54	3.65
7	1.92	1.91	1,59	1.44	1.11	1.33	0	1.31	3.43	1.95	3.00	3.63	3.88	5.96	6.08
8	1.86	1.88	1.47	1.54	1.41	1.91	1.31	0	2.02	2.18	3.07	5.04	4.01	6.30	6.20
9	2.77	4.86	4.38	3.23	3.58	3.60	3.43	2.02	0	2.46	5.11	6.03	6.09	7.96	8.29
10	3.24	3.14	2.84	2.69	2.58	3.11	1.95	2.18	2.46	0	1.42	2.48	2.46	4.10	4.43
11	4.54	4.63	4.21	4.04	3.73	4.12	3.00	3.07	5.11	1.42	0	1.25	2.46	4.29	4.47
12	5.30	5.69	4.84	4.76	4.40	4.46	3.63	5.04	6.03	2.48	1.25	0	1.59	3.27	3.69
13	5.63	5.32	5.29	4.89	4.73	4.10	3.88	4.01	6.09	2.46	2.47	1.59	0	2.72	3.85
14	6.48	5.27	4.49	4.53	3.92	3.54	5.96	6.30	7.96	4.10	4.29	3.27	2.72	0	0.40
15	7.02	5.36	4.85	4.71	4.28	3.65	6.08	6.20	8.29	4.43	4.47	3.69	3.85	0.40	0

The numbers 1- 15 represents the various cities the substations are located. The data is presented in a matrix form. The figures 0, 0.47, 0.30, 0.62, etc represent the shortest distance path in kilometers from one city to the other. So the distance from city 1 to city 2 is 0.47km.The distance from city 1 to city 3 is 0.30km, as it applies to distances to other cities from Table 4.1.

TABLE 4.2 REPRESENTATION OF CITIES WITH SUBSTATIONS/TRANSFORMERS

Number	City
1	Makola(ECG Office)
2	Old Parliament House(High Street)
3	Public Works Department
4	Ministry of Foreign Affairs
5	America Embassy Annex
6	Sports Stadium (Osu)
7	State House
8	Ridge Hospital (Ridge)
9	Kokomlemle
10	Police Headquarters
11	Aquinas Senior High School(Cantonments)
12	Labone Senior High School(Labone)
13	G.N.T.C
14	Lababi Market(Labadi)
15	Lababi Township(Labadi)

The first column represents the numbers and the second column is the names of the cities or locations associated with the numbers in Table 4.2

4.1 Model Instance

$$Min \quad \sum_{i=1}^{n}\sum_{j=1}^{n} C_{ij}X_{ij}$$

s.t

$$\sum_{j=2}^{n} x_{1j} = m$$

$$\sum_{j=2}^{n} x_{j1} = m$$

$$\sum_{i=1}^{n} X_{ij} = 1, \qquad j = 1, \dots, n$$

$$\sum_{j=1}^{n} X_{ij} = 1, \qquad i = 1, \dots, n$$

$$\{(i, j) | i, j = 2, \dots, n; X_{ij} = 1\}$$

$$X_{ij} = \{0,1\}\ for\ all\ i, j = 1, \dots, n$$

C_{ij} corresponds to the rows and columns which is the distance matrix of Table 4.1 in the data. m is the number of technical teams, m=2, 5.

4.2 Computation Procedure

The mTSP algorithm was implemented in MATLAB using the shortest distance path in Table 4.1. The algorithm was run on Duo Core PC at 1.83 GHz (1.00 GB RAM). The algorithm was run ten different times varying the number of technical men with number of iterations being 125 for each run.

4.3 Results

TABLE 4.3 RESULTS AFTER TEN DIFFERENT RUNS

Technical teams	Best Routes	Route Breaks	Distance	Iteration
2	2 4 3 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	68
	3 4 2 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	68
	2 4 3 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	60
	2 4 3 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	64
	2 4 3 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	122
	3 4 2 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	57
	3 4 2 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	38
	2 4 3 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	48
	2 4 3 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	67
	2 4 3 5 6 15 14 13 12 11 10 9 8 7	3	21.8100	73
3	5 7 8 2 4 3 9 10 11 12 13 14 15 6	3 6	24.5500	349
	3 4 2 8 7 5 9 10 11 12 13 14 15 6	3 6	24.5500	55
	2 4 3 8 7 5 9 10 11 12 13 14 15 6	3 6	24.5500	108
	2 4 3 5 7 8 9 10 11 12 13 14 15 6	3 6	24.5500	24
	5 7 8 2 4 3 9 10 11 12 13 14 15 6	3 6	24.5500	65
	2 4 3 6 15 14 13 12 11 10 9 8 7 5	3 11	24.5500	47
	5 7 8 9 10 11 12 13 14 15 6 3 4 2	3 11	24.5500	46
	8 7 5 3 4 2 9 10 11 12 13 14 15 6	3 6	24.5500	52
	8 7 5 9 10 11 12 13 14 15 6 3 4 2	3 11	24.5500	59
	3 4 2 9 10 11 12 13 14 15 6 5 7 8	3 11	24.5500	408
4	10 13 12 11 6 15 14 5 9 8 7 2 4 3	4 8 11	33.0800	124
	10 13 12 11 9 8 7 6 15 14 5 2 4 3	4 7 11	33.0800	2116
	10 13 12 11 6 15 14 5 9 8 7 2 4 3	4 8 11	33.0800	146
	8 10 9 15 14 13 12 11 5 6 7 3 4 2	3 8 11	33.1100	61
	10 13 12 11 9 8 7 6 15 14 5 3 4 2	4 7 11	33.0800	47
	10 13 12 11 6 15 14 5 9 8 7 3 4 2	4 8 11	33.0800	261
	11 10 9 12 13 14 15 6 8 7 5 2 4 3	3 8 11	33.1400	404
	11 10 9 12 13 14 15 6 5 7 8 3 4 2	3 8 11	33.1400	43
	9 10 8 3 4 2 15 14 13 12 11 7 6 5	3 6 11	33.1100	105
	9 10 8 5 6 7 15 14 13 12 11 3 4 2	3 6 11	33.1100	226
5	4 5 9 8 10 11 12 13 14 15 7 6 2 3	2 4 10 12	32.0700	102
	3 2 6 7 10 11 12 13 14 15 9 8 4 5	2 4 10 12	32.0700	25
	7 6 5 4 2 3 9 8 10 11 12 13 14 15	2 4 6 8	32.0700	34
	9 8 4 5 7 6 15 14 13 12 11 10 2 3	2 4 6 12	32.0700	138
	7 6 3 2 4 5 9 8 15 14 13 12 11 10	2 4 6 8	32.0700	54
	4 5 8 9 10 11 12 13 14 15 6 7 2 3	2 4 10 12	32.0700	176
	9 8 10 11 12 13 14 15 3 2 7 6 5 4	2 8 10 12	32.0700	269
	9 8 15 14 13 12 11 10 3 2 7 6 5 4	2 8 10 12	32.0700	212
	4 5 2 3 9 8 7 6 15 14 13 12 11 10	2 4 6 8	32.0700	291
	5 4 2 3 9 8 7 6 15 14 13 12 11 10	2 4 6 8	32.0700	29

From table 4.3, the best route is chosen using the minimum distance and the least iteration. The first column of Table 4.3 represents the technical teams, the second column represents the best routes for each of the technical teams, column three gives the route breaks for each of the best routes, the break down into the individual routes of the technical teams. Column four gives the total distance of each of the routes. Column five of Table 4.3 gives the iteration for each of the technical teams. The best route with minimum distance is given in Table 4.4 below.

TABLE 4.4 THE BEST ROUTES AND MINIMUM DISTANCES

No. of cities	No. of technical teams	Best Routes	Route Breaks	Minimum Distance
15	2	3 4 2 5 6 15 14 13 12 11 10 9 8 7	3	21.8100
15	3	2 4 3 5 7 8 9 10 11 12 13 14 15 6	3 6	24.5500
15	4	10 13 12 11 9 8 7 6 15 14 5 3 4 2	4 7 11	33.0800
15	5	3 2 6 7 10 11 12 13 14 15 9 8 4 5	2 4 10 12	32.0700

The third column of table 4.4 represents the best routes of the technical teams, to the cities they are to visit and the fifth column represents the total distance associated with the routes. Column four of table 4.4 shows the route breaks which gives the breakdown of the various cities the technical teams are to visit.

TABLE 4.5 DESIGNATED ROUTES FOR THE TECHNICAL TEAMS

No. of technical teams	Technical teams	Designated routes for teams
2	Team one	1 3 4 2 1
	Team two	1 5 6 15 14 13 12 11 10 9 8 7 1
3	Team one	1 2 4 3 1
	Team two	1 5 7 8 1
	Team three	1 9 10 11 12 13 14 15 6 1
4	Team one	1 10 13 12 11 1
	Team two	1 9 8 7 1
	Team three	1 6 15 14 5 1
	Team four	1 3 4 2 1
5	Team one	1 3 2 1
	Team two	1 6 7 1
	Team three	1 10 11 12 13 14 15 1
	Team four	1 9 8 1
	Team five	1 4 5 1

Column three of table 4.5 shows the designated routes for the various technical teams. With a technical team of two, the two technical teams visits the cities allocated them. That is, team one starts at city 1 and moves to through the cities and return to city 1.Team two of the two technical team also starts at city 1 moves through the various cities and return to city 1.All the technical teams visits the cities designated to them in the same order.

4.4 Discussions

From Table 4.5,team one of the two technical team starts at Makola(Accra East Office) moves to Public works Department, move to the Ministry of Foreign Affairs, move to Old Parliament House(High Street) and then return to Makola(Accra East Office).Team two also starts at Makola(Accra East Office) then to America Embassy Annex to Sports stadium(Osu) to Labadi Township to Labadi market to G.N.T.C to Labone Senior High School(Labone) to Aquinas Senior High School(Cantonments) to Police Headquarters to Kokomlemle to Ridge Hospital(Ridge) to State House and return to Makola(Accra East Office).The total distance covered by the two teams is 21.8100km.For the three technical team, the first team starts at Makola(Accra East Office) move to Old Parliament House(high Street) to Ministry of Foreign Affairs to Public works Department and return to Makola(Accra East Office).The second team also starts at Makola(Accra East Office) to America embassy Annex to State House to Ridge Hospital(Ridge) and return to Makola(Accra East Office).Team three starts at Makola(Accra East Office) move to Kokomlemle to Police Headquarters to Aquinas Senior High School(Cantonments) to Labone Senior High School(Labone) to G.N.T.C to Labadi Market to Labadi Township to Sports stadium(Osu) and return to Makola(Accra East Office).The total distance covered is 24.5500km.For the four technical team, team one starts at Makola(Accra East Office) to Police Headquarters to G.N.T.C to Labone Senior High School(Labone) to Aquinas Senior High School(Cantonments) and return to Makola(Accra East Office).Team two starts at Makola(Accra East Office), move to Kokomlemle to Ridge Hospital(Ridge) to State House and return to Makola(Accra East Office).Team three starts at Makola(Accra East Office) move to Sports Stadium to Labadi Township to Labadi Market to America Embassy Annex and return to Makola(Accra East Office).Team four starts at Makola(Accra East Office) move to Public works department to Ministry of Foreign Affairs to Old Parliament House and return to Makola(Accra East Office).The total distance covered is 33.0800km.For the five technical team, team one starts at Makola(Accra East Office) to Public Works Department to Old parliament House(High Street) and return to Makola(Accra East Office).Team two starts at Makola(Accra East Office) move to Sports Stadium to State House and return to Makola(Accra East Office).Team three starts at Makola(Accra East Office) move to Police Headquarters to Aquinas Senior High School(Cantonments) to Labone Senior High School(Labone) to G.N.T.C to Labadi Market to Labadi Township and return to Makola(Accra East Office).Team four starts at Makola(Accra East Office) move to Kokomlemle to Ridge Hospital(Ridge) and return to Makola(Accra East Office).Team five starts at Makola(Accra East Office) to Ministry of Foreign Affairs to America Embassy Annex and return to Makola(Accra East Office).The total distance covered is 32.0700km.The total distance covered by all the teams is 111.5100km.

CHAPTER FIVE

5.0 CONCLUSION

The objectives of the thesis which were, to model a route path for the technical teams and also minimize the distances travelled. With the results given above, the shortest distances path for the various technical teams has been modeled using the mTSP and the total distance minimized. The total distance covered by the two technical team was 21.8100km, for the three technical team, the total distance covered was 24.5500km. The total distance covered by the four technical team was 33.0800km.The total distance covered by the five technical team was 32.0700km. The total distance covered by all the teams is 111.5100km.

5.1 RECOMMENDATION

With the above results, it is strongly recommended that the management of the Electricity Company of Ghana (ECG), Accra East Region (Makola District) should send out a technical team of two. Considering all the distances, the least distance was covered by the two technical teams since they would be able to cover all the cities and also minimize the distance involved. Using this model, ECG can design routes for their technical teams for other areas.

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APPENDIX

```
Functionvarargout=
mtspf_ga(xy,dmat,salesmen,min_tour,pop_size,num_iter,show_prog,show_res)
% MTSPF_GA Fixed Multiple Traveling Salesmen Problem (M-TSP) Genetic Algorithm (GA)
%   Finds a (near) optimal solution to a variation of the M-TSP by setting
%   up a GA to search for the shortest route (least distance needed for
%   each salesman to travel from the start location to individual cities
%   and back to the original starting place)
%
% Summary:
%     1. Each salesman starts at the first point, and ends at the first
%        point, but travels to a unique set of cities in between
%     2. Except for the first, each city is visited by exactly one salesman
%
% Note: The Fixed Start/End location is taken to be the first XY point
%
% Input:
%     XY (float) is an Nx2 matrix of city locations, where N is the number of cities
%     DMAT (float) is an NxN matrix of city-to-city distances or costs
%     SALESMEN (scalar integer) is the number of salesmen to visit the cities
%     MIN_TOUR (scalar integer) is the minimum tour length for any of the
%        salesmen, NOT including the start/end point
%     POP_SIZE (scalar integer) is the size of the population (should be divisible by 8)
%     NUM_ITER (scalar integer) is the number of desired iterations for the algorithm to run
```

```
%     SHOW_PROG (scalar logical) shows the GA progress if true
%     SHOW_RES (scalar logical) shows the GA results if true
%
% Output:
%     OPT_RTE (integer array) is the best route found by the algorithm
%     OPT_BRK (integer array) is the list of route break points (these specify the indices
%         into the route used to obtain the individual salesman routes)
%     MIN_DIST (scalar float) is the total distance traveled by the salesmen
%
% Route/Breakpoint Details:
%     If there are 10 cities and 3 salesmen, a possible route/break
%     combination might be: rte = [5 6 9 4 2 8 10 3 7], brks = [3 7]
%     Taken together, these represent the solution [1 5 6 9 1][1 4 2 8 1][1 10 3 7 1],
%     which designates the routes for the 3 salesmen as follows:
%         . Salesman 1 travels from city 1 to 5 to 6 to 9 and back to 1
%         . Salesman 2 travels from city 1 to 4 to 2 to 8 and back to 1
%         . Salesman 3 travels from city 1 to 10 to 3 to 7 and back to 1
%
% 2D Example:
%     n = 35;
%     xy = 10*rand(n,2);
%     salesmen = 5;
%     min_tour = 3;
```

```
%     pop_size = 80;
%     num_iter = 5e3;
%     a = meshgrid(1:n);
%     dmat = reshape(sqrt(sum((xy(a,:)-xy(a',:)).^2,2)),n,n);
%     [opt_rte,opt_brk,min_dist] = mtspf_ga(xy,dmat,salesmen,min_tour, ...
%         pop_size,num_iter,1,1);
%
% 3D Example:
%     n = 35;
%     xyz = 10*rand(n,3);
%     salesmen = 5;
%     min_tour = 3;
%     pop_size = 80;
%     num_iter = 5e3;
%     a = meshgrid(1:n);
%     dmat = reshape(sqrt(sum((xyz(a,:)-xyz(a',:)).^2,2)),n,n);
%     [opt_rte,opt_brk,min_dist] = mtspf_ga(xyz,dmat,salesmen,min_tour, ...
%         pop_size,num_iter,1,1);
%
% See also: mtsp_ga, mtspo_ga, mtspof_ga, mtspofs_ga, mtspv_ga, distmat
%
% Author: Joseph Kirk
% Email: jdkirk630@gmail.com
```

```
% Release: 1.3
% Release Date: 6/2/09

% Process Inputs and Initialize Defaults
nargs = 8;
for k = nargin:nargs-1
    switch k
        case 0
            xy = 10*rand(40,2);
        case 1
            N = size(xy,1);
            a = meshgrid(1:N);
            dmat = reshape(sqrt(sum((xy(a,:)-xy(a',:)).^2,2)),N,N);
        case 2
            salesmen = 5;
        case 3
            min_tour = 2;
        case 4
            pop_size = 80;
        case 5
            num_iter = 5e3;
        case 6
            show_prog = 1;
```

```
        case 7
            show_res = 1;
        otherwise
    end
end

% Verify Inputs
[N,dims] = size(xy);
[nr,nc] = size(dmat);
if N ~= nr || N ~= nc
    error('Invalid XY or DMAT inputs!')
end
n = N - 1; % Separate Start/End City

% Sanity Checks
salesmen = max(1,min(n,round(real(salesmen(1)))));
min_tour = max(1,min(floor(n/salesmen),round(real(min_tour(1)))));
pop_size = max(8,8*ceil(pop_size(1)/8));
num_iter = max(1,round(real(num_iter(1))));
show_prog = logical(show_prog(1));
show_res = logical(show_res(1));

% Initializations for Route Break Point Selection
```

```
num_brks = salesmen-1;
dof = n - min_tour*salesmen;          % degrees of freedom
addto = ones(1,dof+1);
for k = 2:num_brks
    addto = cumsum(addto);
end
cum_prob = cumsum(addto)/sum(addto);

% Initialize the Populations
pop_rte = zeros(pop_size,n);          % population of routes
pop_brk = zeros(pop_size,num_brks);   % population of breaks
for k = 1:pop_size
    pop_rte(k,:) = randperm(n)+1;
    pop_brk(k,:) = randbreaks();
end

% Select the Colors for the Plotted Routes
clr = [1 0 0; 0 0 1; 0.67 0 1; 0 1 0; 1 0.5 0];
if salesmen > 5
    clr = hsv(salesmen);
end

% Run the GA
```

```
global_min = Inf;
total_dist = zeros(1,pop_size);
dist_history = zeros(1,num_iter);
tmp_pop_rte = zeros(8,n);
tmp_pop_brk = zeros(8,num_brks);
new_pop_rte = zeros(pop_size,n);
new_pop_brk = zeros(pop_size,num_brks);
if show_prog
    pfig = figure('Name','MTSPF_GA | Current Best Solution','Numbertitle','off');
end
for iter = 1:num_iter
    % Evaluate Members of the Population
    for p = 1:pop_size
        d = 0;
        p_rte = pop_rte(p,:);
        p_brk = pop_brk(p,:);
        rng = [[1 p_brk+1];[p_brk n]]';
        for s = 1:salesmen
            d = d + dmat(1,p_rte(rng(s,1))); % Add Start Distance
            for k = rng(s,1):rng(s,2)-1
                d = d + dmat(p_rte(k),p_rte(k+1));
            end
            d = d + dmat(p_rte(rng(s,2)),1); % Add End Distance
```

```
    end
    total_dist(p) = d;
end

% Find the Best Route in the Population
[min_dist,index] = min(total_dist);
dist_history(iter) = min_dist;
if min_dist < global_min
    global_min = min_dist;
    opt_rte = pop_rte(index,:);
    opt_brk = pop_brk(index,:);
    rng = [[1 opt_brk+1];[opt_brk n]]';
    if show_prog
        % Plot the Best Route
        figure(pfig);
        for s = 1:salesmen
            rte = [1 opt_rte(rng(s,1):rng(s,2)) 1];
            if dims == 3, plot3(xy(rte,1),xy(rte,2),xy(rte,3),'.-','Color',clr(s,:));
            else plot(xy(rte,1),xy(rte,2),'.-','Color',clr(s,:)); end
            title(sprintf('Total Distance = %1.4f, Iteration = %d',min_dist,iter));
            hold on
        end
        if dims == 3, plot3(xy(1,1),xy(1,2),xy(1,3),'ko');
```

```
        else plot(xy(1,1),xy(1,2),'ko'); end
        hold off
    end
end

% Genetic Algorithm Operators
rand_grouping = randperm(pop_size);
for p = 8:8:pop_size
    rtes = pop_rte(rand_grouping(p-7:p),:);
    brks = pop_brk(rand_grouping(p-7:p),:);
    dists = total_dist(rand_grouping(p-7:p));
    [ignore,idx] = min(dists);
    best_of_8_rte = rtes(idx,:);
    best_of_8_brk = brks(idx,:);
    rte_ins_pts = sort(ceil(n*rand(1,2)));
    I = rte_ins_pts(1);
    J = rte_ins_pts(2);
    for k = 1:8 % Generate New Solutions
        tmp_pop_rte(k,:) = best_of_8_rte;
        tmp_pop_brk(k,:) = best_of_8_brk;
        switch k
            case 2 % Flip
                tmp_pop_rte(k,I:J) = fliplr(tmp_pop_rte(k,I:J));
```

```
        case 3 % Swap
            tmp_pop_rte(k,[I J]) = tmp_pop_rte(k,[J I]);
        case 4 % Slide
            tmp_pop_rte(k,I:J) = tmp_pop_rte(k,[I+1:J I]);
        case 5 % Modify Breaks
            tmp_pop_brk(k,:) = randbreaks();
        case 6 % Flip, Modify Breaks
            tmp_pop_rte(k,I:J) = fliplr(tmp_pop_rte(k,I:J));
            tmp_pop_brk(k,:) = randbreaks();
        case 7 % Swap, Modify Breaks
            tmp_pop_rte(k,[I J]) = tmp_pop_rte(k,[J I]);
            tmp_pop_brk(k,:) = randbreaks();
        case 8 % Slide, Modify Breaks
            tmp_pop_rte(k,I:J) = tmp_pop_rte(k,[I+1:J I]);
            tmp_pop_brk(k,:) = randbreaks();
        otherwise % Do Nothing
    end
  end
  new_pop_rte(p-7:p,:) = tmp_pop_rte;
  new_pop_brk(p-7:p,:) = tmp_pop_brk;
end
pop_rte = new_pop_rte;
pop_brk = new_pop_brk;
```

```
end

if show_res
    % Plots
    figure('Name','MTSPF_GA | Results','Numbertitle','off');
    subplot(2,2,1);
    if dims == 3, plot3(xy(:,1),xy(:,2),xy(:,3),'k.');
    else plot(xy(:,1),xy(:,2),'k.'); end
    title('City Locations');
    subplot(2,2,2);
    imagesc(dmat([1 opt_rte],[1 opt_rte]));
    title('Distance Matrix');
    subplot(2,2,3);
    rng = [[1 opt_brk+1];[opt_brk n]]';
    for s = 1:salesmen
        rte = [1 opt_rte(rng(s,1):rng(s,2)) 1];
        if dims == 3, plot3(xy(rte,1),xy(rte,2),xy(rte,3),'.-','Color',clr(s,:));
        else plot(xy(rte,1),xy(rte,2),'.-','Color',clr(s,:)); end
        title(sprintf('Total Distance = %1.4f',min_dist));
        hold on;
    end
    if dims == 3, plot3(xy(1,1),xy(1,2),xy(1,3),'ko');
    else plot(xy(1,1),xy(1,2),'ko'); end
```

```
    subplot(2,2,4);
    plot(dist_history,'b','LineWidth',2);
    title('Best Solution History');
    set(gca,'XLim',[0 num_iter+1],'YLim',[0 1.1*max([1 dist_history])]);
end

% Return Outputs
if nargout
    varargout{1} = opt_rte;
    varargout{2} = opt_brk;
    varargout{3} = min_dist;
end

    % Generate Random Set of Break Points
    function breaks = randbreaks()
        if min_tour == 1 % No Constraints on Breaks
            tmp_brks = randperm(n-1);
            breaks = sort(tmp_brks(1:num_brks));
        else % Force Breaks to be at Least the Minimum Tour Length
            num_adjust = find(rand < cum_prob,1)-1;
            spaces = ceil(num_brks*rand(1,num_adjust));
            adjust = zeros(1,num_brks);
            for kk = 1:num_brks
```

```
            adjust(kk) = sum(spaces == kk);
        end
        breaks = min_tour*(1:num_brks) + cumsum(adjust);
    end
  end
end
```

(Kirk, 2009)