KWAME NKRUMAH UNIVERSITY OF SCIENCE AND TECHNOLOY

TRANSSHIPMENT PROBLEM OF A NON-ALCOHOLIC BEVERAGE INDUSTRY: CASE STUDY OF COCA COLA BOTTLING COMPANY, GHANA.

By

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DECLARATION

I hereby declare that this submission is my own work towards the Master of Science degree and that, to the best of my knowledge it contains no material previously published by another person nor material which has been accepted for award of any other degree of the University except where due acknowledgement has been made in the text.

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DEDICATION

I dedicate this work to my father Rev. Isaac Cudjoe and to my dear friend Sheritta Lord Amankwah**.**

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May the Good Lord bless you all.

ABSTRACT

The transportation problem is a special class of the linear programming problem. It deals with the situation in which a commodity is transported from Sources to Destinations. The transshipment problem is an extension of the transportation problem where intermediate nodes which are also referred to as transshipment nodes are added to account for locations such as warehouses. My main objective is to model Coca Cola transportation as a transshipment problem and also minimize the cost in transporting them. We will formulate the Transshipment problem as a Transportation problem and use the Transportation algorithm to solve it. The QM for windows Software will be used to analyze the data. It was concluded that if Coca Cola Bottling Company adapts this method, transportation cost will be minimized.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND OF THE STUDY

One of the key problem most organizational manager's faces is how to allocate scarce resources among various activities or projects. Linear programming [LP], which is one of most widely used operations research tools and has been a decision making aid in almost all manufacturing industries as well as service organizations, is a method of allocating resources in an optimal way. Linear programming deals with mathematical programming that serve as a planning process to allocates resources which includes labor, materials, machines and capital in the best possible (optimal) way so that costs are minimized or profits are maximized (Reeb and Leavengood, 2002).

According to Hillier and Lieberman (1995), in linear programming, resources are known as decision variables and the criterion for selecting the best values of the decision variables to maximize profits or minimize costs is known as the objective function. They however hope that, limitations on resource availability form what is known as a constraint set whiles the word linear indicates that the criterion for selecting the best values of the decision variables can be described by a linear function of variables.

A linear programming problem can also be expressed in terms of straight lines, analogous geometrical figures and planes. In addition to the linear requirements, non-negativity restrictions state that variables cannot assume negative values. That is, it is not possible to have negative resources. It would be mathematically impossible to solve linear programming problem using more resources than are available without that condition. (Reeb and Leavengood, 2002)

Physical distribution of resources is one important application of linear programming from one place to another to meet a specific set of supplies. Transportation problem [TP] mathematically is very easy to express in terms of an LP model, simplex method can be used to solve such model. In view of this, the study seeks to focus on the transshipment problem of non-alcoholic beverage industry by applying linear programming to minimize transshipment cost.

The contribution of transportation in industries in the global world cannot be overemphasized. Transportation is said to contribute to customer satisfaction in most industry, more especially the brewery industry by providing additional customer value when products arrives on time, in the quantities required and undamaged. This serves as the basis of enhancing market share, customer satisfaction and profitability. According to Grant and Damel, (2006*)*, the transportation sector of most industrialized economics is so pervasive that often there is a failure to comprehend the magnitude of its impact on our way of life.

Furthermore, Agyeman (2011), emphasized that transportation is one of the largest logistics costs which accounts for important portion of the selling price of most products. Hence, the efficient management of transportation becomes more vital to a firm as both inbound and outbound transportation costs increases.

Nevertheless, Gibbons and Machin (2006) explained that transportation management and operation has significant bearing on a firm's operations. Road, rail and air transport networks for instance bring migrant workers into the cities, convey commuters to and from work, and move the finished products of production to their place of consumption.

Transportation problem refers to a class of linear programming problems that involves selection of most economical shipping routes for transfer of a uniform commodity from a number of sources to a number of destinations. However, unbalanced transportation problem deals with the total availability which is not equal to total demand, hence some of the source or destination constraints are satisfied as inequalities.

Transportation problem concerns the amount to be sent from each origin, the amount to be received at each destination, and the cost per unit shipped from any origin to any given destination is specified. The transshipment problem is an extension of the transportation problem in which the commodity can be transported to a particular destination through one or more intermediate or transshipment nodes where each of these nodes in turn supply to other destinations. Therefore, for transshipment, each point acts as shipper only or as a receiver only. Hence, shipments may go through any sequence of points rather than being restricted to direct connections from one origin to one of the destinations.

The unit cost considered from a point considered as a shipper to the same point considered as a receiver is set equal to zero. It is also assumed that a large amount of material to be shipped is available at each point and act as stockpile, which can be drawn or replenished. The main aim of transshipment problem is to ascertain the number of units to be shipped over each node so that all the demand requirements are met with the minimum transportation cost. However, transshipment problem can be converted easily into an equivalent transportation problem. This makes it possible to apply the algorithm for solving transportation problem.

Transportation problem have been studied extensively by many scholars in the past years. According to Brigden (1974), transportation problem (TP) deals with mixed constraints. Brigden (1974) solved this problem by considering a related standard transportation problem having two additional supply points and two additional destinations. Also, Klingman and Russel (1975) applied a specialized method for solving transportation problem with several additional linear constraints. Furthermore, Adlakha (2006) also designed a heuristic for solving transportation problem with mixed constraints.

Firms in Ghana are currently facing a number of difficulties in managing transportation due to recent fuel hikes in the country and the current economic crises. This has affected the operational cost, efficiency and reduces the profit margin of firms who are not able to manage its transport well. However, many firms are still striving to strengthen its internal process in order to minimize cost and gain competitive advantage. Transshipment and transportation are adopted for planning bulk distribution in most of the industries. Normally, in the absence of the transshipment, the transportation cost goes higher. In the transshipment problem all the sources and destinations can function in any direction. Therefore, transshipment is regarded as very important instrument to reduce the transportation cost.

1.1.1 History of Transportation and Mode of Transport

The history of transportation dates back to the pre-historic ages when man learned to live in groups and traveled extensively in search of food and shelter. The pre-historic means of transportation mostly consisted of walking and swimming (when required). Progressively man learned to use animals to transport himself as well as his belongings. The use of animals as means of transportation was revolutionized by the invention of the wheel. The introduction of the wheel can be given the credit for changing the whole concept of traveling and transportation. With time, man wanted speed as well as capacity in his transportation. This need led to the invention of various machines like steam engines and aircraft, (www.buzzle.com).

Transportation evolution can be broadly divided into four main categories based on their medium, namely road, rail, water, air and space transport.

Road Transport

Road transport is the oldest method of transportation. Roads were first constructed by Romans in order to enable the armies to travel faster. Man knew how to walk, both on twos and fours as early as the pre-historic times. Initially, sledges were pulled by animals and with the advent of machines, animals were relieved of this burden. Animals are still widely used in many places. The invention of wheels made road transport to revolutionize in a big way. Roads made the use of wheels very comfortable which was otherwise inconvenient and uncomfortable on rugged surfaces.

Furthermore, Man kept inventing means that would speed the transport, giving birth to machines like buggies, bicycles, and the introduction of cars and various machinery using engines. However, transportation was very slow and costly till the Industrial revolution. After the revolution, the invention of engines in this time period (steam and fuel) subsequently led to inventions of vehicles of different capacities and speeds. We can now boast of bikes, cars, trucks, buses, and many more machines that help us travel and transport faster and more efficiently. Road transport is very important to the Ghanaian economy as it is estimated that road transport accounts for 94% of freight ton-miles and about 97% of passenger miles in the country. According to the Ministry of Roads and Transport, road transport may be categorized into four main categories, namely urban, rural-urban, rural and express services. The demand for urban passenger transport is mainly by residents traveling to work, school, and other economic, social and leisure activities. In Ghana most urban transportation is by road and provided by private transport including mini-buses, taxis and state or private-supported bus services.

60% of passenger movements are accounted for by buses, while taxis account for only 14.5% with the remaining accounted for by private cars. One important trend in road transport (especially inter-city) is that there has been a shift from mini-buses towards medium and large cars with capacities of 30 to 70 seats. There has been a rising preference for good buses as the sector continues to offer more options to passenger in tons of quality of vehicles used.

Ghana's road transport infrastructure is made up of 50,620km of road network connecting the whole country. The Ghana Highways Authority controls about 14,047 km, Department of Urban Roads controls about 4,063 km and the Department of Feeder Roads controls about 32,594 km. About 15.7% of the total road network is paved.

Traffic densities are low on the whole, except in the large cities of Kumasi and Accra, where peak hour densities are relatively high.

Rail Transport

Rail as a means of transportation started around 500 years ago. Primarily, it consisted of man or horse power and sometimes also of rails made of wood. The current railway system finds its origin in England in early 1800's. Historically it was identified as wagon way and its traces have been found to exist as early as 600 BC in Greece. The Greek rail system also used wagons that were pulled by either men or animals on the grooves made in limestone. Grooves making on flat surfaces made the wagons follow a particular path without being manually directed to do so. In their early days, they were only 6 -9 km long. There are discoveries that indicate their use in the Roman Empire also.(www.gipcghana.com).

As the centuries passed, man invented various machines that assisted the growth of railways indirectly. Like the invention of steam engine gave rail transportation a new meaning and remained an astounding invention of the century, which was upgraded only in late 1900s by the fuel engine. Railways of late form the backbone of any given economy. This system of transportation is a very fast and economic way of travelling through huge distances.

Ghana's rail network (of 950km) links the three cities of Kumasi in the heart of the country, Accra-Tema in the east and Takoradi in the west. The main agricultural and mining regions are connected to the ports of Tema and Takoradi by the network. It has mainly served the purpose of transportation minerals, cocoa and timber. Considerable passenger traffic is also carried on the network. The Government has firm plans to develop the rail network more extensively to handle up to 60% of solid and liquid bulk cargo haulage between the ports and the interior and or the landlocked neighbouring countries to the north of Ghana and elsewhere. The government has set out plans to privatize the State-owned Ghana Railways Corporation (GRC) through concession and to provide much greater capacity for rail haulage of containers and petroleum products.

Water Transport

Historically, water transportation became very important because of man's tendency to settle down around water bodies. Cities were established along the banks of rivers and shores of oceans in order to make sure that the civilization never ran out of both food and water. Transportation by water actually came into picture when man discovered his business skills and also when man's greed to conquer and rule grew. Some people traveled to learn whiles others traveled to fight. In both ways human beings had to find means of transportation on water. It was simpler initially; boats were constructed by tying bamboo shafts together. Nevertheless, this was a very basic invention and came handy when traveling for short distances.

Man learned to create bigger boats that used manpower or winds for propagation, with the passing time and as a result of rising needs. Man is known to have traveled far and wide using boats and ships. The invention of engines revolutionized this mode as greatly as any other mode of transport. Now, huge ships travel the lengths of ocean in the form of either the naval forces of a country or luxurious cruise ships, often use by the rich and famous. They are unsinkable cities that are propelled by huge engines, on the water surface. This is however, a slow means of transportation these days, as compared to many others that have come into picture.

The Volta Lake in Ghana was created in the early 1960's by building a dam at Akosombo and flooding the long valley of the River Volta. The Volta lake is the largest man-made lake in the world stretching 415km form Akosombo 101km north of Accra, to Buipe in northern Ghana, about 200km from Ghana's border with Burkina Faso. As a waterway, it plays a key role in the Ghana Corridor programme by providing a useful and low cost alternative to road and rail transport between the north and the south. Ghana is in a better position, by virtue of her seaports and inland lake transport system, to service the maritime needs of land-locked countries to the north of Ghana.

Volta Lake Transport Company (VLTC) in Ghana uses a fleet of pusher tugs and assorted barges to provide regular north-south services for general cargo and liquid bulks, and tramping service for local traders. Volta Lake Transport Company (VLTC) carries 88,000 tons of cargo annually. Northbound, one of the most important cargoes is diesel oil, which is piped to Akosombo from the Tema Oil Refinery and taken on to final destination (Buipe) by barge. Other cargos include sulphate, alumina, fertilizer, cement, stores and oil products, all of which are conveyed to Akosombo by truck.

Southbound, the barges carry a range of agricultural produce including, cotton lint, cassava chips cottonseed, and sheanuts. All these items are trucked south (from Akosombo) to Accra and Tema, from where sheanut and cottonseeds are exported. Volta Lake Transport Company also operates a 300-passenger capacity vessel between

Akosombo and Yeji in Northern Ghana (293km). This vessel is designed to carry cargo as well as passengers

Air Transport and Space Travel

One of the most revolutionizing inventions of the history of mankind is airplanes. The Wright brothers in 1903 invented the first airplane. Ever since their invention, it has been modified and glorified into the fastest known method of transportation and travel. People now travel thousands of miles in just few hours. Few centuries ago this would have sounded like a wild dream, but man's curiosity and willingness have made it a reality. The inventions and discoveries of fuels, that are efficient both in terms of money and usage, have given man easy accessibility to this mode of transport.

Air transport is not only being used to connect two places on Earth these days, but it is also being used in connecting two random places in the Universe. As a result of the inventions and discoveries, man can now travel to the moon and learn about the moon. Hence, the most speculated phenomenon since ages.

Ghana is at the hub of an extensive international (and national) airline network that connects the country to Africa and the rest of the world. Most international carriers fly regularly to Kotoka International Airport (KIA) in Accra, the main entry point to Ghana by air. This is the result of Ghana’s open skies policy, which frees an air space regulator from the constraints on capacity, frequency, route, structure and other air operational restrictions. The policy, in effect allows the Ghana Civil Aviation Authority (GCAA) to operate with minimal restrictions from aviation authorities, except in cases of safety and standards and/or dominant position to distort market conditions. Ghana is working to spot herself as the gateway to West Africa. Kotoka International Airport remains the leading and preferred airport in the sub-region, having attained Category One status by the US Federal Aviation Administration (FAA) audit as part of their International Aviation Safety Audit (IASA) programme. Currently, Ghana is one of the four countries in sub-Saharan Africa in this category. The others are Egypt, South Africa and Morocco. It handles the highest volume of cargo in the sub-region and has all the requisite safety facilities, recommended practices and security standards.

A rehabilitation program embarked upon since 1996 has brought about an expansion and refurbishment and upbringing of facilities at the international terminal building, as well as the domestic terminal. This has result to increased traveler and cargo capacity. The airport's runway has been extended to cater for all types of aircraft allowing direct flights from Ghana at maximum take-off weight without the need for technical stops en-route.

1.1.2 Profile of the Coca Cola Bottling Company Limited

The Coca-Cola Bottling Company of Ghana Limited (TCCBCGL) was set up as a joint venture between the following partners in March 7, 1995: Coca-Cola Export Corporation (25%), Africa Growth Fund (20%) Government of Ghana (55%) with the Management contract granted to The Coca- Cola Company. The current ownership structure upon further re-structuring and acquisition, changed initially in 2000 to: Equatorial Coca-Cola Bottling Company (68%), Government of Ghana (32%).The Equatorial Coca-Cola Bottling Company of Barcelona, Spain bought over the Ghana Government shares in 2003 and assumed 100% ownership. The mission Statement of

TCCBCGL is to deliver high quality products and services that meet the needs of our customers and consumers. To this end, the company manufactures and market products which comply with its specifications and the requirements of the consumers and endeavors to exceed. Administratively, TCCBCGL is headed by a General Manager/CEO who is assisted by eight Heads of Departments namely, Finance, Technical, Human Resource, Commercial Manager, Internal Control, Supply Chain, and Administrative Plant Manager and External Facilities Plant Manager in Accra. The company employs around 760 workers, more than 8,000 Mini-Table operators and 77 independent Mini-Depot Operators, each of which employs at least 4 persons. Equally, the Company outsources other non-core operators to outside bodies.

Coca-Cola Bottling Company manufactures a variety of (8) brands of products which includes Coca-Cola, Fanta, Minute Maid, Sprite, Krest Burn, Schweppes, and Bon-Aqua. Seventeen (17) flavors are currently bottled under the above mentioned brands, namely: Coca-Cola ,Fanta Orange, Fanta Lemon, Fanta Fruit Cocktail, Sprite, Krest Bitter Lemon, Krest Ginger-Ale, Krest Soda, Water Krest, Tonic Water, Bon-Aqua drinking water, Schweppes Tonic Water, Fanta Pineapple, Schweppes Bitter Lemon, Schweppes Soda Water, Fanta blackurrant, Coke light, Burn Energy drinks, Schwepps Malt, and Minute maid.

The TCCBCGL operates two plants, one in Accra and other in Kumasi, made up of 5 production lines: four in Accra plant and one in Kumasi plant. From a sixty percent (60%) market share in 1995, the company in 2005 controls eighty six percent (86%) and as at March 2007, the company controls ninety five percent (95%) of the beverage industry in Ghana. As a market leader in its own right, the company has established extensive marketing and distribution networks since 1995 throughout the country. Presently, the company has created 31,000 new outlets; 8,000 Mini-Tables and 8,000 Electric Coolers.(www.coca-cola.com/gh)

1.2 STATEMENT OF THE PROBLEM

The Coca-Cola Bottling Company is the Ghana's largest beverage company, largest manufacturer, distributor and marketer of non-alcoholic beverage concentrates and syrups. Coca Cola currently distributes its product at regional sales centres in Ghana at prices that vary as a function of the location distance from the factory. The Prices of commodities are determined by a number of factors: the prices of raw materials labour and largely transport. When price of raw material increase, so does the price of the commodity. Transshipment cost also affects the pricing system. The problem that will be addressed in this study centres on the transshipment. The thesis seeks to address the problem of determine the optimal transshipment schedule that will minimizes the total cost of transporting Coca Cola products from the production site (Spintex) Accra to the companies depot and to the destinations centres.

1.3 OBJECTIVE OF THE STUDY

a. To model Coca-Cola Bottling Company's transport as transshipment problem.

b. To minimize transportation cost in the transshipment of non-alcoholic beverages at Coca cola Bottling Company.

1.4 METHODOLOGY

Our proposed methodology to our problem would be solved by using the transshipment model with intermediate destinations between the sources and the destinations. The transshipment problem will be converted to a transportation problem and the transportation algorithm will be used to solve it. A Data from Coca-Cola Bottling Company which is a secondary data for one year period (2012) would be analyzed. . Transporting Coca Cola products to their various depots and to the retail outlet destinations is considered as a transshipment problem. Example, goods are often transported from manufacturing plants to distribution centers or warehouse then finally to the stores. Given *m* pure supply nodes with demand a_i, n pure demand nodes with demand b_j and *l* transshipment nodes. Suppose the unit transportation cost from supply node *i* to transshipment node k is C_{ik} and the unit transportation cost for transshipment node *k* to demand node *j* is C_{kj}.The transshipment problem will be converted to a transportation problem and modeled as a linear programming model of transportation type, and represent the Linear Programming or the transportation problem as tableau and solve it, Management Science for windows software will be used to analyze the data.

1.5 JUSTIFICATION OF THE STUDY

Taking cognizance of the fact that transportation cost largely affect production companies, the recommendations and suggestion from this research when implemented will help Coca cola Bottling Company and all matching companies or organizations achieve cost saving on transshipment so as to be profitable and remain competitive in today's market place. Coca cola and for that matter other production firms will be able to expand their operations when they are able to maximize profit. This will create more employment opportunities for people, which will in effect have a positive impact on the economy of the country. Living conditions of people will also be improved. When organizations are able to make cost savings on transportation costs, such savings could also be utilized in enhancing corporate social responsibilities which will benefit the communities in which these organizations operate in particular and the country as a whole. A company seeking to achieve more efficient transportation and greater profitability must make significant changes in the way it perform every phase of the transshipment process. The study will help the Coca cola Company to implement new transportation and distribution strategies to improve carrier capacity utilization in a time of constrained supply to minimize cost. The research work will also serve as a reference material to the academia for future research work.

1.6 ORGANIZATION OF THE THESIS

The study consist of five chapters with the Introduction as the Chapter one, Chapter two is on the related literature review .The methodology used in the study is discussed in the Chapter three while Chapter four is the presentation of the data collected, analysis of the data and results. Chapter five deals with the conclusion and recommendations.

CHAPTER 2

LITERATURE REVIEW

This chapter reviews other studies on the problem understudy and this would serve as a yardstick for comparing the findings of the study to those undertaken elsewhere.

One of the most important problems in supply chain management is the distribution network design problem system which involves locating production plants and distribution warehouses, and determining the best strategy for distributing the product from the plants to the warehouses and from the warehouses to the customers. Vahidreza et al., (2009) studied a model which allows for multiple levels of capacities available to the warehouses and plants. The authors developed a mixed integer programming model for the problem and solved it by a heuristic procedure which contains 2 sub-procedures. The authors used harmony-search meta-heuristic as the main procedure and linear programming to solve a transshipment problem as a subroutine at any iteration of the main procedure.

Lin et al., (2003) addressed a limited form of the two-stage lightering practice for large tankers, first stage at an offshore location farther from the refinery and the second stage at the lightering location closer to the refinery using an event-based approach. The authors assumed single-compartment vessels, did not restrict the number of simultaneous services for a single tanker, did not allow pickups from more than two tankers within one voyage of a lightering vessel, ignored differences in crude densities, and did not allow the freedom to select lightering crudes. In their paper, the authors developed a new continuous-time MILP formulation that addresses all of the above drawbacks. Thus, the authors allowed multi-compartment lightering vessels, restricted the number of simultaneous transfers to two, allowed more than two pickups in one voyage for any lightering vessel, considered the impact of varying crude densities, selected optimally the right lightering crudes, and most importantly used a realistic cost-based scheduling objective. The authors MILP model generated optimal lightering schedule with lightering volumes, sequence, times, and assignments, which minimized the operating costs of lightering vessels, the demurrage costs of tankers as well as the delivery times of crude oil from the lightering location to refinery ports.

Dong and Rudi (2004) examined a different aspect by looking at the benefits of lateral transshipments for a manufacturer that supplies a number of retailers. The authors compared the case where the manufacturer is the price leader to the case of exogenous prices. For exogenous prices, it was found that retailers benefited more when demand across the network was uncorrelated. For the endogenous price case, modeled as a Stackelberg game, the manufacturer exploits his leadership to increase his benefits, leaving retailers worse off if they use transshipments. These results were restricted to demand that follows a normal distribution.

In a more retail case study based approach, Bendoly (2004) studied a model with internet and store based customers. The authors utilized lateral transshipment ideas to show how partial pooling of goods can improve a system's performance. The examined model considered a modern retail environment where stores are operated alongside internet channels and is an example of the practical uses of lateral transshipments.

Krishnan and Rao (1965) studied a reactive mode of transshipment. The authors considered a model that was similar to the periodic lateral transshipment model which has negligible transshipment times, but aim to minimize cost through transshipments once all demand is known. The author's model provided an optimal solution for a multi-location, multiperiod model. However, this solution can only be determined for networks with either two nonidentical locations or any number of identical locations. For more than two non-identical locations, a LP based heuristic solution procedure is proposed and shown to perform well for a number of scenarios.

In a system with two echelons there are several ways in which stockouts can be satisfied through emergency stock movements. Lateral transshipments are one possibility but there could be situations where it is beneficial also to perform emergency shipments from the central warehouse. Wee and Dada (2005) considered this problem with five different combinations of transshipments, emergency shipments and no movements at all and devises a method for deciding which setup is optimal under a given model description. The author's research allows the structure of the emergency stock movements to be established.

Cross docking is a logistic technique which seeks to reduce costs related to inventory holding, order picking, transportation as well as the delivery time. Most of the existing studies in the area are interested in the dock assignment problem and the design of the cross dock transportation networks. Little attention has been given to the transshipment operations inside a cross docking platform. Larbi et al., (2003) studied the transshipment scheduling problem in a simple cross dock with a single strip door and single stack door.

The authors proposed a graph based model for the problem. The shortest path in the graph gives the schedule which minimizes the total cost of transshipment operations.

Deniz et al., (2009) studied transshipment problem of a company in the apparel industry with multiple Sub-contractors and customers, and a transshipment depot in between. Unlike a typical transshipment problem that considers only the total cost of transportation, the authors model also considered the supplier lead times and the customer due dates in the system and can be used for both supplier selection and timely distribution planning. The authors proposed their model can also be adapted easily by other companies in the industry.

Tagaras (1989) used the fill rate and the probability of no-stock out to reflect the level of service. For an identical demand structure, balancing the fill rate is equivalent to starting with identical beginning inventory at each location. In the authors study, while analyzing the effect of risk pooling in a setting with one central warehouse and three stocking locations, the author compared random allocation with a ‘risk balancing’ transshipment policy. In risk balancing, transshipment quantities are determined so as to equalize the probability of a stockout in the following period, and for an identical demand structure, risk of stockout will be balanced if each location starts with the same inventory.

Kut (2006) studied a distribution system consisting of multiple retail locations with transshipment operations among the retailers. Due to the difficulty in computing the optimal solution imposed by the transshipment operations and in estimating shortage cost from a practical perspective, the authors proposed a robust optimization framework for analyzing the impact of transshipment operations on such a distribution system. The authors demonstrated that their proposed robust optimization framework is analytically tractable and is computationally efficient for analyzing even large-scale distribution systems. From a numerical study using this robust optimization framework, the authors addressed a number of managerial issues regarding the impact of transshipment on reducing the costs of the distribution system under different system configurations and retailer characteristics. The authors considered two system configurations, line and circle, and studied how inventory holding cost, transshipment cost, and demand size and variability affect the effectiveness of transshipment operations for the cases of both homogeneous and non-homogeneous retailers. The results obtained from the robust optimization framework helped to evaluate the potential benefits when investing in transshipment operations.

In situations where a seller has surplus stock and another seller is stocked out, it may be desirable to transfer surplus stock from the former to the latter. Krishnan and Rao (1965) studied the transshipment problems with multiple retail locations with identical cost structure, and examined how the possibility of such transshipments between two independent locations affects the optimal inventory orders at each location. If each location aims to maximize its own profits—the authors called this local decision making—their inventory choices will not, in general, maximize joint profits. The authors found transshipment prices which induce the locations to choose inventory levels consistent with joint-profit maximization. The authors showed that the optimal stocking quantities satisfy the equal fractile property.

Deniz et al., (2006) considered coordination among stocking locations through replenishment strategies that explicitly take into account lateral transshipments, i.e., transfer of a product among locations at the same echelon level. The basic contribution of our research is the incorporation of supply capacity into the traditional emergency transshipment model. The authors formulated the capacitated production case as a network flow problem embedded in a stochastic optimization problem. The authors developed a solution procedure based on infinitesimal perturbation analysis (IPA) to solve the stochastic optimization problem numerically. The authors analyzed the impact on system behavior and on stocking locations' performance when the supplier may fail to fulfill all the replenishment orders and the unmet demand is lost. The authors found that depending on the production capacity, system behavior can vary drastically. Moreover, in a production-inventory system, the authors found evidence that either capacity flexibility (i.e., extra production) or transshipment flexibility is required to maintain a certain level of service.

Tagaras (1989) presented a model which deals with the analysis of two-location periodic review inventory systems with non-negligible replenishment lead times. Emergency transshipments were used in these systems as a recourse action to reduce the occurrence of shortages. A class of partial pooling policies is proposed for the control of transshipments. The cost performance of this class of policies was shown to be inferior to that of complete pooling. An approximate model and a heuristic algorithm were introduced to compute near-optimal stocking policy solutions. Comparisons with simulation results verified the satisfactory performance of the approximate model and algorithm. Numerical sensitivity analysis provided additional insight into the nature of optimal transshipment behavior. The author's model also allowed for a service constraint on the minimum acceptable fill rates.

Supply chain designs are constrained by the cost-service trade-off. Cost minimization typically leads to physically efficient or lean supply chains at the expense of customer responsiveness or agility. Recently, the concept of leagility has been introduced. Research on leagility, defined as the capability of concurrently deploying the lean and agile paradigms, hinges heavily on the identification of the decoupling point, which, in turn, is enabled by postponement. Postponement strategies, however, present a cross-functional challenge for implementation. As a tactical solution to achieve leagility

without postponement, Yale et al.,(2002) studied transshipments problem, which represented a common practice in multi-location inventory systems involving monitored movement of stock between locations at the same echelon level of the supply chain.

Through a series of models, the authors established how transshipments can be used to enhance both agility and leanness.

Taragas and Cohen (1992) studied two-location transshipment model which allow for positive replenishment lead-times. With positive replenishment lead-times, it might be beneficial to hold back stock for future demands, and so it is not necessarily optimal to always transship from the other location (complete pooling) when shortages occur. However, their numerical results showed that complete pooling generally dominates partial pooling.

Herer and Rashit (1999) studied the two-location transshipment problem to include fixed and joint replenishment costs and the multiperiod case and defined a set of assumptions that lead to "complete pooling." Complete pooling means that if one location has excess stock while another location is short, the number of units transshipped will be the minimum of the excess and the shortage. The authors showed that no transshipments will occur if both locations are short or if both have excess stock, and derived several properties regarding the structure of the corresponding optimal replenishment and transshipment policies.

Dong and Rudi (2004) studied how transshipments affect manufacturers and retailers, considering both exogenous and endogenous wholesale prices. For a distribution system where a single manufacturer sells to multiple identical-cost retailers, the authors considered both the manufacturer being a price taker and the manufacturer being a price setter in a single-period setup under multivariate normal demand distribution. In the case of the manufacturer being a price taker, the authors provided several analytical results regarding the effects of key parameters on order quantities and profits. In the case of the manufacturer being a price setter, the authors characterized the Stackelberg game that arises, and provided several insights into how the game dynamics are affected by transshipments. Specifically, the authors found that risk pooling makes retailers' order quantities less sensitive to the wholesale price set by the manufacturer; hence, in general, the manufacturer benefits from retailers' transshipments by charging a higher wholesale price, while retailers are often worse off. The author's model captures the effect of demand correlation and the effect of the number of retailers throughout, and it illustrates the findings by a numerical example. The authors also provided an interactive Web page for numerical experiments.

Mangal and Chandna (2007) examined the antecedents of retailer - retailer partnership and to explore its impact on the supply chain performance. The authors considered coordination among stocking locations through replenishment strategies that take explicitly into consideration transshipments, transfer of a product among locations at the same echelon level. A continuous review inventory system was adopted, in which lateral transshipments are allowed. In general, if a demand occurs at a location and there is no stock on hand, the demand is assumed to be backordered or lost. Lateral transshipments serve as an emergency supply in case of stock out and the rule for lateral transshipments is to always transship when there is a shortage at one location and stock on hand at the other. The aim is to explore the role of lateral transshipment to control inventory and associated cost within supply chain and, from this, to develop an exploratory framework that assists understanding in the area. A simple and intuitive model was presented that enables us to characterize optimal inventory and transshipment policies for ‘n’ locations. The study was based on a case study of a bi-wheeler company in India by using its data and to strengthen its supply chain. The results obtained enabled the managers to overcome the uncertainties of demand and lead-time resulting into customer satisfaction and cost reduction.

Roberto et al., (2009) considered the stochastic capacitated transshipment problem for freight transportation where an optimal location of the transshipment facilities, which minimizes total cost, must be found. The total cost is given by the sum of the total fixed cost plus the expected minimum total flow cost, when the total throughput costs of the facilities are random variables with unknown probability distribution. By applying the asymptotic approximation method derived from the extreme value theory, a deterministic nonlinear model, which belongs to a wide class of Entropy maximizing models, is then obtained. The computational results showed a very good performance of this deterministic model when compared with stochastic one.

Banu and Sunderesh (2008) studied a single-item two-echelon inventory system where the items can be stored in each of N stocking locations is optimized using simulation. The aim of this study was to minimize the total inventory, backorder, and transshipments costs, based on the replenishment and transshipment quantities. In this study, transshipments which are the transfer of products among locations at the same echelon level and transportation capacities which are the transshipment quantities between stocking locations, were also considered. Here, the transportation capacities among the stocking locations are bounded due to transportation media or the locations' transshipment policy. Assuming stochastic demand, the system is modeled based on different cases of transshipment capacities and costs. To find out the optimum levels of the transshipment quantities among stocking locations and the replenishment quantities, the simulation model of the problem was developed using ARENA 10.0 and then optimized using the Opt Quest tool in this software.

Mabel et al., (2006) developed an analytical framework for studying a two-echelon distribution system consisting of one central warehouse and multiple retail locations with transshipment operations among the retailers. The authors' framework can be used to model very general distribution systems and analyze the impact of transshipment under different system configurations. The authors demonstrated that their proposed analytical framework was analytically tractable and computationally efficient for analyzing even large-scale distribution systems. From a numerical study using the authors framework, they addressed a number of managerial issues regarding the impact of transshipment on reducing the costs of the distribution system under different system configurations and retailer characteristics. The managerial insights obtained from the authors analysis was able to evaluate the potential benefits by investing in transshipment operations.

Transshipments, monitored movements of material at the same echelon of a supply chain, represent an effective pooling mechanism. With a single exception, research on transshipments overlooks replenishment lead times. The only approach for two-location inventory systems with non-negligible lead times could not be generalized to a multi-location setting, and the proposed heuristic method cannot guarantee to provide optimal solutions. Gong and Yucesan (2006) studied a model that uses simulation optimization by combining an LP/network flow formulation with infinitesimal perturbation analysis to examine the multi-location transshipment problem with positive replenishment lead times, and demonstrates the computation of the optimal base stock quantities through sample path optimization. From a methodological perspective, the authors deployed an elegant duality-based gradient computation method to improve computational efficiency. In test problems, the author’s algorithm was also able to achieve better objective values than an existing algorithm.

Glover et al., (2005) developed a primal simplex procedure to solve transshipment problems with an arbitrary additional constraint. The procedure incorporates efficient methods for pricing-out the basis, determining certain key vector representations, and implementing the change of basis. These methods exploit the near triangularity of the basis in a manner that takes advantage of computational schemes and list structures used to solve the pure transshipment problem. The authors implemented these results in a computer code, I/O PNETS-I. Computational results (necessarily limited) confirm that this code is significantly faster than APEX-III on some large problems. The authors also developed a fast method for determining near optimal integer solutions. Computational results showed that the near optimum integer solution value was usually within 0.5% of the value of the optimum continuous solution value.

A transshipment problem with demands that exceed network capacity can be solved by sending flow in several waves. How can this be done using the minimum number of iterations? This is the question tackled in the quickest transshipment problem. Hoppe and Tardos (1997) described the only known polynomial time algorithm that finds an integral solution to this problem. The author’s algorithm repeatedly minimizes sub-modular functions using the ellipsoid method, and is therefore not at all practical. Fleischer presented an algorithm that finds a fully integral quickest transshipment with a polynomial number of maximum flow computations. When there is only one sink, the quickest transshipment problem is significantly easier. For this case, the authors showed how the algorithm can be sped up to return an integral solution using O(k) maximum flow computations, where k is the number of sources.

Herer and Tzur (2001) investigated the strategy of transshipments in a dynamic deterministic demand environment over a finite planning horizon. The authors considered a system of two locations which replenish their stock from a single supplier, and where transshipments between the locations are possible. The authors model included fixed (possibly joint) and variable replenishment costs, fixed and variable transshipment costs, as well as holding costs for each location and transshipment costs between locations. The problem was to determine how much to replenish and how much to transship each period; thus the study can be viewed as a synthesis of transshipment problems in a static stochastic setting and multilocation dynamic deterministic lot sizing problems. The authors provided interesting structural properties of optimal policies which enhance the understanding of the important issues which motivate transshipments and allowed the development of an efficient polynomial time algorithm for obtaining the optimal strategy. By exploring the reasons for using transshipments, the model enabled practitioners to envision the sources of savings from using this strategy and therefore motivated them to incorporate it into their replenishment strategies. With this model the authors were able to minimize the total replenishment, holding and transshipment costs over a finite horizon.

The transportation problem has offered two mathematical facets: (1) as a specialized type of linear programming problem, (2) as a method of representation of some combinatorial problems. Orden (1956) developed a third aspect of the mathematical properties of the transportation problem. It was shown that the same mathematical framework can be extended beyond pair-wise connections, to the determination of optimum linked paths over a series of points. This extension although viewed here as a linear programming problem, takes advantage of the combinatorial aspect of the transportation problem, and applications may arise which, like the assignment problem, appear to be combinatorial problems, but which can be solved by linear programming

A dynamic network consists of a graph with capacities and transit times on its edges. The quickest transshipment problem is defined by a dynamic network with several sources and sinks; each source has a specified supply and each sink has a specified demand. The problem is to send exactly the right amount of flow out of each source and into each sink in the minimum overall time. Variations of the quickest transshipment problem have been studied extensively; the special case of the problem with a single sink is commonly used to model building evacuation. Similar dynamic network flow problems have numerous other applications; in some of these, the capacities are small integers and it is important to find integral flows. There are no polynomial-time algorithms known for most of these problems. Hoppe and Tardos (1997) presented the first polynomial-time algorithm for the quickest transshipment problem. The author's algorithm provides an integral optimum flow. Previously, the quickest transshipment problem could only be solved efficiently in the special case of a single source and single sink.

Rosa et al., (2001) studied the Arc Routing and Scheduling Problem with Transshipment (ARPT), a particular Arc Routing Problem whose applications arise in garbage collection. In the ARPT, the demand is collected by specially equipped vehicles, taken to a transfer station, shredded or compacted and, finally, transported to a dump site by means of high-capacity trucks. A lower bound, based on a relaxation of an integer linear formulation of the problem, was developed for the ARPT. A tailored Tabu Search heuristic was also devised. Computational results on a set of benchmark instances were reported which proved to be efficient as compared with existing methods.

Ozdemir et al., (2003) studied a supply chain model, which consists of N retailers and one supplier. The retailers may be coordinated through replenishment strategies and lateral transshipments, that is, movement of a product among the locations at the same echelon level. Transshipment quantities may be limited, however, due to the physical constraints of the transportation media or due to the reluctance of retailers to completely pool their stock with other retailers. The authors introduced a stochastic approximation algorithm to compute the order-up-to quantities using a sample-path-based optimization procedure. Given an order-up-to S policy, the authors determined an optimal transshipment policy, using an LP/network flow framework. Such a numerical approach allows the authors to study systems with arbitrary complexity.

The decentralized transshipment problem is a two-stage decision making problem where the companies first choose their individual production levels in anticipation of random demands and after demand realizations they pool residuals via transshipment. The coordination will be achieved if at optimality all the decision variables, i.e. production levels and transshipment patterns, in the decentralized system are the same as those of centralized system. Hezarkhani and Kubiak (2009) studied a model with the coordination via transshipment prices. The authors proposed a procedure for deriving the transshipment prices based on the coordinating allocation rule introduced by Anupindi et al. (2006). With the transshipment prices being set, the companies are free to match their residuals based on their individual preferences. The authors drew upon the concept of pair-wise stability to capture the dynamics of corresponding matching process. As the main result of the study, the authors showed that with the derived transshipment prices, the optimum transshipment patterns are always pair-wise stable, i.e. there are no pairs of companies that can be jointly better off by unilaterally deviating from the optimum transshipment patterns.

Topkis (1984) developed a complement and substitution principles applicable to sittings in transshipment dual stage problems such as those encountered in factories and warehouses. Direct examination of the basic property of this transportation problem suggested that two locations of a similar nature would be reasonable substitutes. Such elements may not apply to location pairs where there were one or more warehouses. Where no warehouse was present, complement and substitution principles are functional. Model illustrations of factory warehouses and demand centre locations were highlighted in the author's results.

Huang and Greys (2008) studied a newsvendor game with transshipments, in which n retailers face a stochastic demand for an identical product. Before the demand was realized, each retailer independently orders her initial inventory. After the demand was realized, the retailers selected an optimal transshipment pattern and ship residual inventories to meet residual demands. Unsold inventories were salvaged at the end of the period. The authors compared two methods for distribution of residual profit—transshipment prices (TPs) and dual allocations (DAs)—that were previously analyzed in literature. TPs are selected before the demand is known, and DAs, which were obtained by calculating the dual prices for the transshipment problem, were calculated after observing the true demand. The authors first studied the conditions for the existence of the Nash equilibrium under DA and then compared the performance of the two methods and showed that neither allocation method dominates the other. The author's analysis suggested that DAs may yield higher efficiency among "more asymmetric" retailers, whereas TPs worked better with retailers that were "more alike," but the difference in profits does not seem significant. The authors also linked expected dual prices to TPs and used those results to develop heuristics for TPs with more than two symmetric retailers. For general instances with more than two asymmetric retailers, the authors proposed a TP agreement that uses a neutral central depot to coordinate the transshipments (TPND). Although DAs in general outperform TPND in our numerical simulations, its ease of implementation makes TPND an attractive alternative to DAs when the efficiency losses are not significant (e.g., high critical fractiles or lower demand variances).

Asmuth et al., (1979) studied a multi-commodity transshipment problem where the prices at each location are an affine function of the supplies and demands at that location and the shipping costs are an affine function of the quantities shipped. A system of prices, supplies, demands, and shipments is defined to be equilibrium, if there is a balance in the shipments, supplies, and demands of goods at each location, if local prices do not exceed the cost of importing, and if shipments are price efficient. The authors used Lemke's algorithm to compute the equilibrium.

Perincherry and Kikuchi (1990) presented a transshipment problem in which the projected demand and supply at different locations on different days are known in fuzzy quantities. The formulation of the model follows that of fuzzy linear programming in that the solution is a shipment schedule which satisfies the objective at a `reasonable cost'. Priorities for satisfying requirements at demand points and supply points on selected days were incorporated by multiplying corresponding weights to h, the level of satisfaction. The authors provided several examples for their formulation.

Dahan (2009) studied a model which considered two retailers between which transshipments can take place at the end of the period. The retailers differ in cost and demand distributions, operate in a single period, and cooperate to minimize joint costs. The authors work differs from previous analyses as it considered the possibility that customers are not always willing to wait for transshipments. Instead, only some customers are willing to wait and return to the retailer for transshipments. The objective of the study was to find the replenishment levels and transshipment quantities that minimize the total expected system cost. The authors considered two cases - a partially deterministic case, and a fully stochastic case. In the partially deterministic case, the number of returning customers was a known fraction of those that could not be satisfied off-the-shelf. The fully stochastic case treated the number of returning customers as a random variable whose probability density function is known and whose expected value was a fraction of the customers that could not be satisfied off-the-shelf. In the partially deterministic case, the authors showed that the transshipment decision has a form similar to complete pooling. They proved that the objective function was convex in the replenishment levels, and suggested numerical methods for finding the optimal replenishment levels

Herer et al., (2006) considered coordination among stocking locations through replenishment strategies that take explicitly into consideration transshipments, that is, transfer of a product among locations at the same echelon level. The authors incorporated transportation capacity such that transshipment quantities between stocking locations are bounded due to transportation media or the location's transshipment policy. The authors modeled different cases of transshipment capacity as a capacitated network flow problem embedded in a stochastic optimization problem. Under the assumption of instantaneous transshipments, the authors developed a solution procedure based on infinitesimal perturbation analysis to solve the stochastic optimization problem, where the objective was to find the policy that minimizes the expected total cost of inventory, shortage, and transshipments. Such a numerical approach provides the flexibility to solve complex problems. Investigating two problem settings, the authors showed the impact of transshipment capacity between stocking locations on system behavior. The authors found out that transportation capacity constraints do not only increase total cost, but also modify the inventory distribution throughout the network.

Lateral transshipments in multi-echelon stochastic inventory systems imply that locations at the same echelon of a supply chain share inventories in some way, in order to deal with local uncertainties in demands. While the structure of a transshipment policy will depend on many important factors, a commonly observed phenomenon at the retail level, called "customer switching", may be of some significance. Under such a phenomenon, a customer, who cannot obtain a desired product at a specific location, may visit one or more other retail locations in search of the item. Liao (2010) studied the inventory replenishment and transshipment decisions in the presence of such stochastic "customer switching" behavior, for two firms which were either under centralized control, or operate independently. The first model adopted in this study considered two retailers that sell the same product to retail customers. After demand was realized, transshipments occur if only one location has insufficient inventory. Under this circumstance, a random fraction of the unfulfilled demand from the stocked out firm (which was referred to as the "shortage firm") may switch to the other firm with surplus inventory (which was referred to as the "surplus firm"). We examine the impact of such customer switching behavior on the firms' inventory decisions, and found out that the firm with surplus inventory is willing to (1) transship the entire quantity requested ("complete pooling policy"), (2) transship a portion of the amount requested ("inventory keeping policy"), or (3) transship nothing ("no-shipping policy") to the shortage firm. The authors demonstrated that a unique pair of optimal order quantities exists if the two firms are centeredly coordinated.

Zhaowei et al., (2009) studied a new type of transshipment problem, the flows through the cross dock are constrained by fixed transportation schedules and any cargos delayed at the last moment of the time horizon of the problem will incur relative high inventory penalty cost. The problem is known to be NP-complete in the strong sense. The authors therefore focused on developing efficient heuristics. Based on the problem structure, the authors proposed a Genetic Algorithm to solve the problem efficiently. Computational experiments under different scenarios showed that GA outperforms CPLEX solver.

Herer and Tzur (1998) investigated the strategy of transshipments in a dynamic deterministic demand environment over a finite planning horizon. The authors considered a system of two locations which replenished their stock from a single supplier, and where transshipments between the locations are possible. The authors model included fixed and variable replenishment costs, fixed and variable transshipment costs, as well as holding costs for each location and transshipment costs between locations. The problem was to determine how much to replenish and how much to transship in each period. The authors provided interesting structural properties of optimal policies which enhanced the understanding of the important issues which motivate transshipments and allowed the development of an efficient algorithm for obtaining the optimal strategy. By exploring the reasons for using transshipments, the model enabled practitioners to envision the sources of savings from using this strategy and therefore motivated them to incorporate it into their replenishment strategies.

Belgasmi et al., (2008) studied a multi-location inventory system where inventory choices at each location are centrally coordinated. Lateral transshipments are allowed as recourse actions within the same echelon in the inventory system to reduce costs and improve service level. However, this transshipment process usually causes undesirable lead times. The authors proposed a multi-objective model of the multi-location transshipment problem which addressed optimizing three conflicting objectives: (1) minimizing the aggregate expected cost, (2) maximizing the expected fill rate, and (3) minimizing the expected transshipment lead times. The authors applied an evolutionary multi-objective optimization approach using the strength Pareto evolutionary algorithm (SPEA2), to approximate the optimal Pareto front. Simulation with a wide choice of model parameters showed the different trades-off between the conflicting objectives.

Glover et al., (1974) presented a primal simplex procedure to solve transshipment problems with an arbitrary additional constraint. The procedure incorporates efficient methods for pricing-out the basis, determining representations, and implementing the change of basis. These methods exploit the near triangularity of the basis in order to take full advantage of the computational schemes and list structures used in solving the pure transshipment problem. Also reported was the development of a computer code, I/O PNETS-I for solving large scale singularly constrained transshipment problems. This code has demonstrated its efficiency over a wide range of problems and has succeeded in solving a singularly constrained transshipment problem with 3000 nodes and 12,000 variables in less than 5 minutes on a CDC 6600. Additionally, a fast method for determining near optimal integer solutions is also developed. Computational results showed that the near optimum integer solution value is usually within a half of one percent of the value of the optimum continuous solution value.

Cheng and Karimi (2002) addressed a special case of the general chemical transshipment problem, namely the tanker lightering problem. When tankers are fully loaded with crude oil, they may not be able to enter the shallow channels or refinery ports due to the draft limitation. Under such circumstances, it is necessary to transfer some part of the crude oil from the tanker to lightering vessels in order to make the tanker “lighter”. After such transshipment operation, the tanker can travel to the refinery port, which it previously cannot. And, the lightering vessels also travel to the refinery port to deliver the lightered crude oil. With tanker lightering operation, large tankers can also deliver crude oil to shallow-draught refinery ports. Furthermore, it helps to reduce the demurrage costs of tankers as well as inventory holding costs (Chajakis, 2000) at the refinery. During congested time, tankers could spend days awaiting lightering service. Since the demurrage costs of tankers are extremely high, effective scheduling of lightering operation is crucial for minimizing the system cost by reducing the waiting times of tankers and increasing the utilization of lightering vessels.

Chajakis (1997) considered a scheduling problem faced by a shipping company that provides lightering services to multiple refineries clustered in a region. The company operates a fleet of multi-compartment lightering vessels with a mix of different configurations such as numbers of compartments, sizes, speeds, heating equipment, and so on. When a tanker arrives at the lightering location, one lightering vessel pumps off crude oil from one side of the tanker. Therefore, at most two lightering services can take place simultaneously for a tanker, one at each side of the tanker. And, these multi-compartment lightering vessels can pick up multiple types of crude from the same/different tankers during a voyage. After enough crude oil has been offloaded, the tanker leaves the lightering system and travels to its designated refinery port. However, lightering vessels travel to the refinery ports, deliver the crude oils, and then return to the lightering location to continue their service. In other words, the lightering vessels make multiple voyages among the refinery ports and lightering location in order to service multiple tankers. Furthermore, the authors considered a two-stage lightering practice for large tankers, first stage at an offshore location farther from the refinery and the second stage at the lightering location closer to the refinery.

Transshipments, monitored movements of material at the same echelon of a supply chain, represent an effective pooling mechanism. Earlier papers dealing with transshipments either do not incorporate replenishment lead times into their analysis, or only provide a heuristic algorithm where optimality cannot be guaranteed beyond settings with two locations. Gong and Yucesan (2010) presented a method that uses infinitesimal perturbation analysis by combining with a stochastic approximation method to examine the multi-location transshipment problem with positive replenishment lead times. It demonstrates the computation of optimal base stock quantities through sample path optimization. From a methodological perspective, this study deploys a duality-based gradient computation method to improve computational efficiency. From an application perspective, it solves transshipment problems with non-negligible replenishment lead times.

Gilbert et al., (1997) examined a multiperiod capacity transshipment model with upgrading. There are multiple product types, corresponding to multiple classes of demand, and the firm purchases capacity of each product before the first period. Within each period, after demand arrives, products are allocated to customers. Customers who arrive to find that their product has been depleted can be upgraded by at most one level. The authors showed that the optimal allocation policy is a simple two-step algorithm: First, they used any available capacity to satisfy same-class demand, and then upgrade customers until capacity reaches a protection limit, so that in the second step the higher-level capacity is rationed. The authors showed that these results hold both when all capacity is salvaged at the end of the last demand period as well as when capacity can be replenished (in the latter case, an order up to policy is optimal for replenishment). Although finding the optimal protection limits was computationally intensive, the authors described bounds for the optimal protection limits that take little effort to compute and can be used to effectively solve large problems.

Hsu and Bassok (1999) considered a single period problem with one input resulting in a random yield of multiple, downward substitutable products. They showed how the network structure of the problem can be used to devise an efficient algorithm.

McGillivray and Silver (1978) considered a case where products had identical costs and there is a fixed probability that a customer demand for a stocked-out product can be substituted by another available product.

Bertrand and Bookbinder (1998) extend the complete pooling policy problem, whereby the amount transshipped from location i to location j is the minimum between the excess at location i and the shortage at location j: to a system whose stock keeping locations have non-identical costs. In particular, the authors considered a warehouse following a periodic order up to S policy based on the system stock. Once the warehouse receives a shipment, it is entirely allocated to the retailers, who experience independent and (not necessarily identically distributed) normal demand. Prior to a new replenishment (order by the warehouse), system stock is redistributed—in a preventive transshipment mode—among the retailers to minimize the expected holding, backorder and transshipment costs. In the case with identical retailers, the authors analytically showed that redistribution reduces the variance of the net inventory prior to a new order. For the case with non-identical retailers, a one-parameter-at-a-time simulation experiment showed that higher values of the length of the replenishment cycle, the number of retailers, holding costs, lead times (LTs) from the warehouse to the retailers, coupled with low values for transhipment costs, supplier LTs, and shortage penalties, favor a redistribution policy.

Mues et al., (2005) stated that the transshipment Problems and Vehicle Routing Problems with Time Windows (VRPTW) are common network flow problems and well studied. Combinations of both are known as intermodal transportation problems. This concept describes some real world transportation problems more precisely and can lead to better solutions, but they are examined rarely as mathematical optimization problems.

According to White, (1972) the movement of vehicles and goods in a transportation system can be represented as flows through a time-dependent transshipment network. An inductive out-of-kilter type of algorithm was presented which utilizes the basic underlying properties of the dynamic transshipment network to optimize the flow of a homogeneous commodity through the network, given a linear cost function.

Banerjee et al. (2003) examined the effects of transshipments in different operating conditions one of which is based on the concept of inventory balancing via transshipments. Under their redistribution policy, the beginning inventory at each location is equalized. Bertrand and Bookbinder (1998) also use the balancing of the beginning inventory as a redistribution policy for identical retailers.

CHAPTER 3

METHODOLOGY

3.1 INTRODUCTION

In this chapter we shall discuss the transportation and the transshipment problems and their solution procedures.

3.1.1 The Transportation Problem

Transportation problems are primarily concerned with the optimal (best possible) way in which a product produced at different factories or plants (called supply origins) can be transported to a number of warehouses (called demand destinations). The transportation problem basically seeks to find the best way to fulfill the demand of say n demand points using the capacities of say m supply points. The objective in a transportation problem is to fully satisfy the destination requirements within the operating production capacity constraints at the minimum possible cost.

Whenever there is a physical movement of goods from the point of manufacture to the final consumers through a variety of channels of distribution (wholesalers, retailers, distributors etc.), there is a need to minimize the cost of transportation so as to increase the profit on sales. Transportation problems arise in all such cases as providing assistance to top managers in ascertaining how many units of a particular product should be transported from each supply origin to each demand destinations so that the total prevailing demand for the company's product is satisfied, while at the same time the total transportation costs are minimized. Bressler and King (1978) noted that widely separated regions may not engage in trading because the costs of transfer exceed the price differences that exist in absence of trade, therefore, great distances and expensive transportation restrict trade whiles technological developments that reduces transfer cost can increase trade. Identification of surplus and deficit regions, quantities shipped, and which region should ship available surplus to which deficit region often becomes a complicated task. Consideration of transportation cost causes the pattern of distribution of the commodity to become an essential factor in determining the total transportation cost.

Sasieni et al., (1959) noted that problems of allocation arise whenever there are a number of activities to perform, but limitations on either the amount of resources or the way they can be spent prevent us from performing each separate activity in the most effective way conceivable. In such situations we wish to allot the available resources to the activities in a way that will optimize the total effectiveness.

The standard scenario where a transportation problem arises is that of sending units of a product across a network of highways that connect a given set of cities. Each city is considered either as a "source," (supply point) or a "sink," (demand point). Each source has a given supply, each sink has a given demand, and each highway that connects a source-sink pair has a given transportation cost per unit of shipment. This can be visualized in the form of a network, as depicted in Figure 3.1

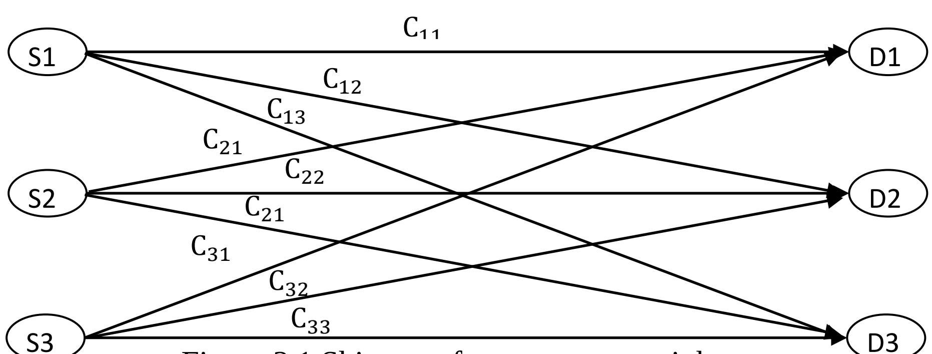

Figure 3.1 Shipment from sources to sinks

Given such a network, the problem of interest is to determine an optimal transportation scheme that minimizes the total cost of shipments, subject to supply and demand constraints. Problems with this structure arise in many real-life situations. The transportation problem is a linear programming problem, which can be solved by the regular simplex method but due to its special structure a technique called the transportation technique is used to solve the transportation problem. It got its name from its application to problems involving transporting products from several sources to several destinations, although the formation can be used to represent more general assignment and scheduling problems as well as transportation and distribution problems. The two common objectives of such problems are either to:

- minimize the total transportation cost of shipping a single commodity from *m* sources to *n* destinations, or
- maximize the profit of shipping from *m* sources to *n* destinations.

3.1.2 Characteristics of a Transportation Problem

a) Objective function is to reduce the transportation cost to the minimum.

b) Maximum quantity available at the sources is limited. This is a constraint.

c) Maximum quantity required at the destination is specified. This cannot be exceeded, this is another constraint.

d) Transportation cost is specified for each item.

e) Sum of the products available from all sources is equal to sum of the products distributed at various destinations

Maximum quantity available at the source, maximum quantity required at the destination and the cost of transportation, all refer to a single product.

3.1.3 THE TRANSPORTATION ALGORITHM

Also from Amponsah (2009), the transportation problem deals with a special class of linear programming problems in which the objective is to transport a homogeneous product manufactured at several plants (origins) to a number of different destinations at a minimum total cost. The total supply available at the origin and the total quantity demanded by the destinations are given in the statement of the problem. The cost of shipping a unit of goods from a known origin to a known destination is also given. The objective is to determine the optimal allocation that results in minimum total shipping cost. The model deals with how to get the minimum-cost plan to transport a commodity from a number of sources to number of destinations.

3.2 MATHEMATICAL FORMULATION

Supposed a company has m warehouses and n retail outlets. A single product is to be shipped from the warehouses to the outlets. Each warehouse has a given level of supply, and each outlet has a given level of demand. We are also given the transportation cost between every pair of warehouse and outlet, and these costs are assumed to be linear. More explicitly, the assumptions are:

- The total Supply a_i of the products from the ith warehouse where $i = 1, 2, 3.., m$
- The total Demand b_j, of the products at the jth outlet, where j $= 1, 2, 3 \dots, n.$.

- The cost of sending one unit of the product from warehouse i to outlet j is equal to C_{ij}, where $i = 1, 2, 3 \dots, m$ and $j = 1, 2, 3 \dots m$, The total cost of a shipment is linear in size of shipment.

Let the cost of transporting one unit of goods form i^{th}origin to j^{th} destination be C_{ij}, i = 1,2, …m, j=1,2,….n. If $X_{ij} \geq 0$ is the amount of goods to be transported from i^{th} origin to j^{th} destination, then the problem is to determine X_{ij}

so as to

Minimize Z = $\sum_{i=1}^{m} \sum_{j=1}^{n} X_{ij} C_{ij}$

Subject to the constraints

$\sum_{j=1}^{n} X_{ij} = a_i$, (i = 1, 2, ..., m)

$\sum_{i=1}^{m} X_{ij} = b_j$, (j = 1, 2, ..., n)

and

$x_{ij} \geq 0$, for all i and j, where a_i and b_j are demand and supply availabilities.

3.2.1 Feasible Solution

A set of non-negative allocations, X_{ij}

which satisfies the row and column restrictions is known as a feasible solution. A feasible solution to an m-origin and n-destination problem is said to be a basic feasible solution if the number of positive allocations are (m+n–1).

3.2.2 Non – Degenerate Basic Feasible Solution

A basic feasible solution of an (m × n) transportation problem is said to be non-Degenerate if it has following two properties:

a) Initial basic feasible solution must contain exactly (m+n–1) number of individual allocations.

b) These allocations must be in independent positions. Independent positions of a set of allocations means that it is always impossible to form any closed loop through these allocations.

Definition (Loop)

Given a transportation table, an ordered set of four or more cells is said to form a loop if:

a) Any two adjacent cells in the ordered set lie in the same row or in the same column.

b) Any three or more adjacent cells in the ordered set do not lie in the same row or in the same column.

3.2.3 Degenerate Basic Feasible Solution

A basic feasible solution that contains less than (m + n – 1) non – negative allocations is said to be a degenerate basic feasible solutions.

3.2.4 Degeneracy in Transportation Problem

Transportation with m-origins and n-destinations can have (m+n-1) positive basis variables or allocations, otherwise the basic solution degenerates. So whenever the number of basic cells or occupied cells is less than (m + n-1), the transportation problem is degenerate.

3.2.5 How to resolve degeneracy in transportation problem

To resolve the degeneracy, the positive variables are augmented by as many zero-valued variables as is necessary to complete (m +n –1) basic variables.

3.3 BALANCED TRANSPORTATION PROBLEM

If total supply equals total demand, the problem is said to be a balanced transportation problem: that is

$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$

3.3.1 Unbalanced Transportation Problem

If the transportation problem is known as an unbalanced transportation problem then, there are two cases.

Case (1)

Here, $\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$

In solving this, we first balance it by introducing a dummy destination in the transportation table. The cost of transporting to this destination is all set equal to zero. The requirement at this destination is assumed to be equal to

$$\sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

Case (2)

Here, $\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$

To solve this, we first balance it by introducing a dummy origin in the transportation table; the costs associated with are set equal to zero. The availability is

$\sum_{j=1}^{n} b_j = \sum_{i=1}^{m} a_i$

3.4 METHODS OF FINDING INITIAL BASIC FEASIBLE SOLUTION FOR A BALANCED TRANSPORTATION PROBLEM

The three basic methods are:

The Northwest Corner Method

The Least Cost Method

The Vogel's Approximation Method

3.4.1 Northwest-Corner Method

The steps below are used in the Northwest- Corner method:

Step (1) The first assignment is made in the cell occupying the upper left-hand (North West) corner of the transportation table. The maximum feasible amount is allocated there, i.e.; $x_{11} = \min(a_1, b_1)$.

Step (2) If $b_1 > a_1$, the capacity of origin O_1 is exhausted but the requirement at D_1 is not satisfied. So move downs to the second row, and make the second allocation: $x_{21} = \min(a_2, b_1 - x_{11})$ in the cell (2,1). If $a_1 > b_1$, allocate $x_{12} = \min(a_1 - x_{11}, b_2)$ in the cell (1,2) .

Continue this until all the requirements and supplies are satisfied.

3.4.2 Least-Cost Method

The least cost method uses shipping costs in order to come up with a basic feasible solution that has a lower cost. To begin the minimum cost method, first we find the decision variable with the smallest shipping cost x_{ij}. Then assign x_{ij} its largest possible value, which is the minimum of s_i. and d_j. After that, as in the Northwest Corner Method we should cross out row i and column j and reduce the supply or demand of the non crossed-out row or column by the value of x_{ij}, then we will choose the cell with the minimum cost of shipping from the cells that do not lie in a crossed-out row or column and we will repeat the procedure.

3.4.3 Vogel's Approximation Method (VAM)

Step 1 For each row of the transportation table, identify the smallest and the next to-smallest costs. Determine the difference between them for each row. Display them alongside the transportation table by enclosing them in parenthesis against the respective rows. Similarly compute the differences for each column.

Step 2 Identify the row or column with the largest difference among all the rows and columns. If a tie occurs, use any arbitrary tie breaking choice. Let the greatest difference correspond to ith row and the minimum cost be C_{ij}. Allocate a maximum feasible amount $x_{ij} = \min(a_i, b_j)$ in the (i, j)th cell, and cross off the ith row or jth column.

Step 3. Re compute the column and row differences for the reduced transportation table and go to step 2. Repeat the procedure until all the rim requirements are satisfied.

3.5 OPTIMAL SOLUTION

A feasible solution (not necessarily basic) is said to be optimal if it minimizes the total transportation cost.

3.5.1 Theorem for Testing Optimality.

If we have a B.F.S. consisting of m + n – 1 independent positive allocations and a set of arbitrary number u and v (i =1,2,...m; j =1,2,...n) such that $c_{rs} = u_r + v_s$ for all occupied cells (r,s) then the evaluation d_{ij} corresponding to each empty cell (i, j) is given by $d_{ij} = c_{ij} - (u_i + v_j)$

3.5.2 Solution to Optimality

As mentioned above, the solution method for transportation problems is a streamlined version of the Simplex algorithm. As such, the solution method also has two phases. In the first phase, the aim is to construct an initial basic feasible solution; and in the second phase, to iterate to an optimal solution. For optimality, we need a method, like the simplex method, to check and obtain the optimal solution. The two methods used are:

a. Stepping-stone method

b. Modified distributed method (MODI)

3.5.3 Stepping Stone

a) Consider an initial tableau
b) Introduce a non-basic variable into basic variable
c) Add the minimum value of all the negative cells into cells that has " positive sign", and subtracts the same value to the "negative" cells

d) Repeat this process to all possible non-basic cells in the tableau until one has the minimum cost. If it does not give the optimal solution or yield a good results, Introduce the MODI method for optimality

3.5.4 Modified distributed method (MODI)

It is a modified version of the stepping stone method

MODI determines if a tableau is the optimal, tells which non-basic variable should be firstly considered as an entry variable, and makes use of stepping-stone to get its answer of next iteration

Procedure (MODI)

Step 1: let $u_i,\ v_j,\ c_{ij}$ variables represent rows, columns, and cost in the transportation tableau, respectively.

Step 2: (a) form a set of equations that uses to represent all basic variables $u_i + v_j = c_{ij}$

(b) Solve variables by assign one variable = 0

Step3: (a) form a set of equations use to represent non-basic variable (or empty cell) as Such $c_{ij} - u_i - v_j = k_{ij}$

(b) Solve variables by using step 2b information.

Step 4: Select the cell that has the most negative value in 3b.

Step 5: Use stepping-stone method to allocate resource to cell in Step 4.

Step 6: Repeat the above steps until all cells in 3a has no negative Value.

3.6 THE TRANSSHIPMENT PROBLEM

In a transportation problem shipment of commodity takes place among sources and destinations. But instead of direct shipments to destinations, the commodity can be transported to a particular destination through one or more intermediate or trans-shipment points. Each of these points in turn supply to other points. Thus, when the shipments pass from destination to destination and from source to source, we have a trans-shipment problem. Since transshipment problem is a particular case of transportation problem hence to solve transshipment problem, we first convert transshipment problem into equivalent transportation problem and then solve it to obtain optimal solution using MODI method of transportation problem. In a transportation problem, shipments are allowed only between source-sink pairs. In many applications, this assumption is too strong. For example, it is often the case that shipments may be allowed between sources and between sinks. Moreover, some points may exist through which units of a product can be transshipped from a source to a sink. Models with these additional features are called transshipment problems. Interestingly, it turns out that any given transshipment problem can be converted easily into an equivalent transportation problem. The availability of such a conversion procedure significantly broadens the applicability of our algorithm for solving transportation problems.

A transportation problem allows only shipments that go directly from supply points to demand points. In many situations, shipments are allowed between supply points or between demand points. Sometimes there may also be points (called transshipment points) through which goods can be transshipped on their journey from a supply point to a demand point. Fortunately, the optimal solution to a transshipment problem can be found by solving a transportation problem.

Some Definitions:

Supply point: It can send goods to another point but cannot receive goods from any other point.

Demand point: It can receive goods from other points but cannot send goods to any other point.

Transshipment point: It can both receive goods from other points send goods to other points.

Orden (1956) introduced the concept of Transshipment Problem. He extended the concept of original transportation problem so as to include the possibility of transshipment. In other words, we can say that any shipping or receiving point is also permitted to act as an intermediate point. In fact, the transshipment technique is used to find the shortest route from one point to another point representing the network diagram. Rhody (1963) considered Transshipment model as reduced matrix model. King and Logan (1964) argued that the problem of determining simultaneously the flows of primary products through processor to the market as final product has been formulated alternatively as a transshipment model. However Judge et al. (1965) formulated the Transshipment model into general linear programming model. Garg and Prakash (1985) studied time minimizing Transshipment model. However Herer and Tzura (2001) discussed dynamic Transshipment Problem. In the standard form, the Transshipment problem is basically a linear minimum cost network. For such types of optimization problems, a number of effective solutions are available in the literature since many years. Recently Khurana and Arora (2011) discussed transshipment problem with mixed constraints.

3.6.1 Mathematical Statement of the Problem

If we let the sources and destinations in a transshipment problem as T, then x_ij would represent the amount of goods shipped from the i^{th} terminal (T_i) to the j^{th} terminal (Tj) and c_ij would represent the unit cost of such shipment. Naturally, x_ij would equal to zero because no units would be shipped from a terminal to itself. Now, assume that at m terminals (T_1, T_2, ... , T_m), the total out shipment exceeds the total in shipment by amounts equal to a_1, a_2, ... , a_m respectively and at the remaining n terminals (T_{m+1}, T_m+_2, ... , T_m+_n), the total in shipment exceeds the total out shipment by amounts b_m+_1, b_m+_2, ... , b_m+_n respectively. If the total in shipment at terminals T_1, T_2, ... , T_m be t_1, t_2, ... , t_m respectively and the total out shipment at the terminals $T_{m+1}, T_{m+2}, \dots, T_{m+n},$ be $t_{m+1},$, $t_{m+2}, \dots, t_{m+n},$ respectively, we can write the transshipment problem as:

$$minimize\ \ z = \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} c_{ij}x_{ij}$$

Subject to

$$x_{i1} + x_{i2} + \cdots x_{i,i-1} + x_{i,i+1} + \cdots x_{i,m+n} - t_i = a_i(i = 1,2, \dots, m) \quad (1)$$

$$x_{1i} + x_{2i} + \cdots x_{i-1,i} + x_{i+1,i} + \cdots x_{m+n,i} = t_i(i = 1,2, \dots, m) \quad (2)$$

$$x_{1j} + x_{2j} + \cdots x_{j-1,j} + x_{j+1,j} + \cdots x_{m+n,j} - t_j = b_j (j = m+1, m+2, ..m+n) \quad (3)$$

$$x_{j1} + x_{j2} + \cdots x_{j,j-1} + x_{j,j+1} + \cdots x_{j,m+n} = t_j (j = m+1, m+2, ..m+n) \quad (4)$$

$$\sum_{i=1}^{m} a_i = \sum_{j=m+1}^{m+n} b_j \quad (5)$$

As can be easily observed, these constraints are similar to the constraints of a transportation problem with m+n sources and m+n destinations, with the differences that here there are no x_{ii} and x_{jj} terms, and that $b_j = 0$ for j=1,2,...,m and $a_i = 0$for i=m+1,m+2,...,m+n. The terms t_i and tj in these constraints may be seen as the algebraic equivalents of x_{ii} and x_{jj}. Now, we can view this problem as an enlarged problem and solve it by using the transportation method. The transshipment problem can be depicted in the form shown in Table 3.2

Table 3.2 Transshipment Problem

TERMINAL $T_I \downarrow T_j \rightarrow$	1	2	…	M	m+1	…	m+n	Supply a_i
1	$-t_i$	x_{12}		x_{1m}	x_1,m+1		x_1,m+n	a_1
2	x_{21}	$-t_i$		x_{2m}	$x_{2,}m+1$		x_2,m+n	a_2
□			□			□		
M	x_{m1}	x_{m2}		$-t_m$	$x_m, m+1$		$x_m, m+1$	a_m
m+1	$x_{m+1,1}$	$x_{m+1,2}$		$x_{m+1,m}$	$-t_{m+1}$		x_{m+1}, m	0
□			□			□		
m+n	$x_{m+n,1}$	$x_{m+n,2}$		$x_{m+n,m}$	x_{m+n}, m		$-t_{m+n}$	0
Demand b_j	0	0	…	0	b_{m+1}	…	b_{m+n}	$\sum a_i = \sum b_j$

The first m rows represent the m constraints given in (1) while the remaining n rows show the constraints given in (4). The constraints in (2) and (3) are represented by the first m columns and the remaining n columns respectively. All the t values are placed on the diagonal from left top to right bottom. Each of them bears negative sign which must be considered carefully when it is involved in the readjustment (during the solution process).

3.6.2 Solution of the Transshipment Problem

The following are steps for solving a Transshipment problem so far available in the literature.

Step1 If necessary, add a dummy demand point (with a supply of 0 and a demand equal to the problem's excess supply) to balance the problem. Shipments to the dummy and from a point to itself will be zero. Let s= total available supply.

Step2 Construct a transportation table as follows: A row in the table will be needed for each supply point and transshipment point, while a column will be needed for each demand point and transshipment point.

In transshipment problem we consider the following concept. Let each supply point will have a supply equal to its original supply, and each demand point will have a demand to its original demand. Let s= total available supply. Then each transshipment point will have a supply which is equal to point's original supply + s and a demand which is equal to point's original demand + s. This ensures that any transshipment point that is, a net supplier will have a net outflow equal to point's original supply and a net demander will have a net inflow equal to point's original demand. Although we don't know how much will be shipped through each transshipment point. However, we can be sure that the total amount will not be exceeded.

3.6.3 Illustrated Example

In this section first we solve numerical example related with transshipment problem using the method available so far..

Example 3.6.1.1 Consider a firm having two factories to ship its products from the factories to three-retail stores. The number of units available at factories X and Y are 200 and 300 respectively, while those demanded at retail stores A, B and C are 100,150 and 250 respectively. Instead of shipping directly from factories to retail stores, it is asked to investigate the possibility of transshipment. The transportation cost (GH) per unit is given in the table 3.3

Table 3.3 Illustrated Example

		Factory		Retail store		
		X	**Y**	**A**	**B**	**C**
Factory	**X**	0	8	7	8	9
	Y	6	0	5	4	3
Retail Store	**A**	7	2	0	5	1
	B	1	5	1	0	4
	C	8	9	7	8	0

Find the optimal shipping schedule.

Solution We solve this problem using VAM to find the initial solution and then use this initial solution to obtain optimal solution using MODI method in the following way.

Table 3.4 showing Initial solution using VAM

	X	Y	A	B	C	SUPPLY
X	0 0	8	100 7	100 8	9	200
Y	6	0 0	5	50 4	250 3	300
A	7	2	0 0	5	1	0
B	1	5	1	0 0	4	0
C	8	9	7	8	0 0	0
DEMAND	0	0	100	150	250	500

Table 3.5 showing optimal solution using MODI

	X	Y	A	B	C	SUPPLY	
X	0 0	8	100 7	100 8	9	200	$u_1 = 4$
Y	6	0 0	5	50 4	250 3	300	$u_2 = 0$
A	7	2	0 0	5	1	0	$u_3 = 3$
B	1	5	1	0 0	4	0	$u_4 = 4$
C	8	9	7	8	0 0	0	$u_5 = 3$
DEMAND	0	0	100	150	250	500	
v_j	$v_{1=4}$	$v_{2=0}$	$v_{3=3}$	$v_{4=4}$	$v_{5=3}$		

$d_{12} = c_{12} - (u_1 + v_2) = 8 - 4 = 4$

$d_{12} = c_{15} - (u_1 + v_5) = 9 - 7 = 2$

$d_{21} = c_{21} - (u_2 + v_1) = 6 + 4 = 10$

$d_{23} = c_{23} - (u_2 + v_3) = 5 - 3 = 2$

$d_{31} = c_{31} - (u_3 + v_1) = 7 + 7 = 14$

$d_{32} = c_{32} - (u_3 + v_2) = 2 + 3 = 5$

$d_{34} = c_{34} - (u_3 + v_4) = 5 - 1 = 4$

$d_{35} = c_{35} - (u_3 + v_5) = 1 - 0 = 1$

$d_{41} = c_{41} - (u_4 + v_1) = 1 + 8 = 9$

$d_{42} = c_{42} - (u_4 + v_2) = 5 + 4 = 9$

$d_{43} = c_{43} - (u_4 + v_3) = 1 + 1 = 2$

$d_{45} = c_{45} - (u_4 + v_5) = 4 + 1 = 5$

$d_{51} = c_{51} - (u_5 + v_1) = 8 + 7 = 15$

$d_{52} = c_{52} - (u_5 + v_2) = 9 + 3 = 12$

$d_{53} = c_{53} - (u_5 + v_3) = 7 - 0 = 7$

$d_{54} = c_{54} - (u_5 + v_4) = 8 - 1 = 7$

Since opportunity cost corresponding to each unoccupied cell is positive, therefore, the solution given in Table 3.5 is optimal.

Total cost = (100*7) + (100*8) + (50*4) + (250*3) = 700+800+200+750 = **2450.** This solution is optimal solution

CHAPTER 4

MODEL FORMULATION AND ANALYSIS

4.1 Introduction

The Coca Cola Bottling Company of Ghana Limited (TCCBCGL) is number one producers of non-alcoholic beverage concentrates and syrups in the world. Transshipment cost represents about 25% of the total production cost. The company has outsourced its transportation to external logistics services Providers. Coca Cola Bottling Company limited has registered about 10 transporters who operate with 120 trucks. Each of the plants at the various sites namely Spintex Road (Accra) and Ahinsan (Kumasi) has its own constraint with respect to plant and warehouse capacity. Thus, there is a limit capacity at each plant. The total plant capacity for Spintex Road and Ahinsan per hour is 2000 and 1200 crates dependants on the plant efficiency. The company works maximum of eight hours a day.

This thesis is intended to minimize the total transshipment cost from the Accra Plant production site at Spintex Road (Accra) to the following company Depots and finally to the destination centres.

4.2 Data Description

The data was obtained from Coca Cola Bottling Company. The company has two production points or source in Ghana, one in Accra and the other in Kumasi. The data was obtained from Accra; hence the Accra Plant source would be used as the main source.

Their products are shipped by road from this plant (source) to their Company Depot Offices before they are transported to final destination hereby referred to as Manual Distribution centers.

The data is a quantitative data which is made up of the distances from sources to the destinations. The table 4.1 is a display of names of towns acting as sources and destinations. Columns l and 2 are the various sources and the names of the towns in which these sources are located respectively. Column 3 is a list of codes representing the towns serving as sources. Columns 4 and 5 show the destinations and the towns representing these destinations respectively. Column 6 indicates the codes of the destinations.

Table 4.1: Names of sources and destinations

SOURCES	TOWNS	CODE	DEST..	TOWNS	CODE
1	Spintex	SPI	1	Kaneshie	KAN
2	Koforidua	KOF	2	Tema	TEM
3	Cape coast	CAP	3	Ada	ADA
4	Takoradi	TAR	4	Winneba	WIN
5	Kata lapaz	KAT	5	Korle-Bu	KOR
			6	Prestea	PRE
			7	Tarkwa	TAR
			8	Enchi	ENC
			9	Mankessim	MAN
			10	Ho	HO

NETWORK REPRESENTATION OF DATA SHOWING THE DISTANCES BETWEEN LOCATIONS (KM)

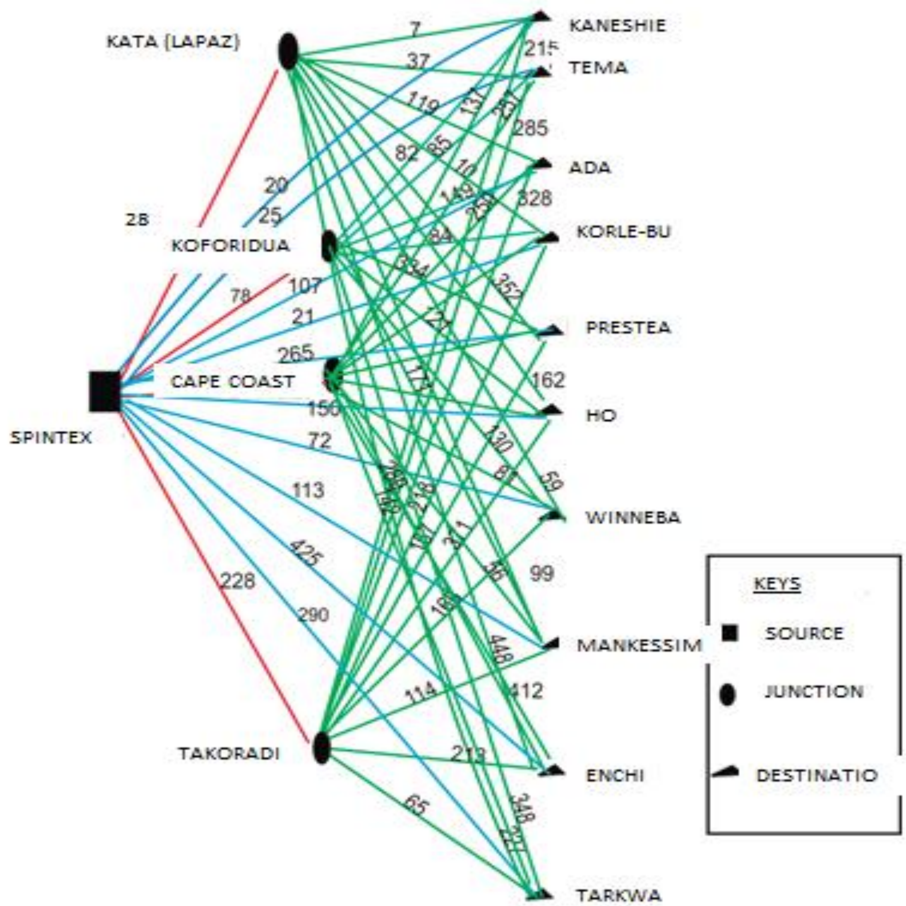

Figure 4.1: This network shows the main source, junctions and the destination

The following distance data in table 4.2 which indicates the distances from sources to destination were also obtained from the Coca- Cola Company.

The codes of the various sources are listed in columns one and two. Columns three to column fourteen are the distances from the various sources to destinations

Table 4.2: Distances (in kilometers) from Sources to Destinations

		KOF	CAP	TAK	KAT	KAN	TEM	ADA	WIN	KOR	PRE	TAR	ENC	MAN	HO
		D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14
SPIN	S1	78	153	228	28	20	25	107	72	21	265	290	425	113	150
KOF	S2		211	286	75	82	85	149	130	84	338	348	448	171	121
CAP	S3	211		79	142	137	167	250	88	140	218	142	289	36	293
TAK	S4	286	79		220	215	245	328	166	218	167	65	213	114	371
KAT	S5	75	142	220		7	37	119	59	10	352	227	412	99	162

Source one (S1) is seen as a pure source. Sources S2, S3, S4 and S5, are the junctions, that is, they are serving as both sources and destinations. Destinations D8, D9, D10, D11, D12, D13 and D14, are pure destination nodes.

An average fuel cost of 2.10cedis is incurred in transporting products per kilometer. The ratio of this amount to the truck load of 1400crates was found to be 1.5×10^{-3} . This amount was used to multiply all the distances in table 4.2 to obtain the unit cost in transporting products from sources to the various destinations. This is summarized in table 4.3.

Table 4.3: Unit cost of transporting a crate of Coca-Cola Product from sources to destination

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	SUPPLY
S1	0.117	0.2295	0.342	0.042	0.03	0.0375	0.1605	0.108	0.0315	0.3975	0.435	0.6375	0.1695	0.225	9142
S2	0	0.3165	0.429	0.1125	0.123	0.1275	0.2235	0.195	0.126	0.507	0.522	0.672	0.2565	0.1815	5379
S3	0.3165	0	0.1185	0.213	0.2055	0.2505	0.375	0.132	0.21	0.327	0.213	0.4335	0.054	0.4395	4743
S4	0.425	0.1185	0	0.33	0.3225	0.3675	0.492	0.249	0.327	0.2505	0.0975	0.3195	0.171	0.5563	7178
S5	0.1125	0.213	0.33	0	0.0105	0.0555	0.1785	0.0885	0.015	0.528	0.3465	0.618	0.1485	0.243	1493
DEMAND	2332	2552	2711	2160	1849	2088	1823	1699	1070	1714	1584	2312	1067	2946	

Column one and row one are the list of the various sources and destinations respectively. Columns two to column fourteen display the unit cost of transporting products from sources to the destinations. The last column indicates the supply quantities and the last row shows the demand quantities (in thousands) from January 2012 to December 2012.

Total supply = 27935000

Total demand = 27907000

A dummy was created with a total demand equal to the difference between the demand and supply, thus 28000 to make the transportation problem a balanced one.

4.3 FORMULATION OF PROBLEM

Let Y_i = Plant site at Spintex(Accra)

.X_{ij} = the units in crates from node i to node j

i=1,2,3,4,...15

j=1,2,3,4,...15

Using the shipping cost data of Table 4.2 above, the annual transshipment cost in thousand of cedis is written as

Minimize

$0.117X_{11}+0.2295X_{12}+0.342X_{13}+0.042X_{14}+0.03X_{15}+0.0375X_{16}+0.1605X_{17}+0.108X_{18}$

+0.0315X_{19}+0.3975$X_{1,10}$+0.435$X_{1,11}$+0.6375$X_{1,12}$+0.1695$X_{1,13}$+0.225$X_{1,14}$+0$X_{1,15}$+0$X_{2,1}$+

0.3165$X_{2,2}$+0.429 X_{23}+0.1125 X_{24} + 0.123X_{25} + 0.1275X_{26}+ 0.2235X_{27} + 0.195X_{28} + 0.126X_{29}

0.507$X_{2,10}$ + 0.522$X_{2,11}$ + 0.672$X_{2,12}$+ 0.2565$X_{2,13}$+ 0.1815$X_{2,14}$+ 0$X_{2,,15}$+ 0.3165$X_{,31}$ + 0X_{32}
+

0.1185X_{33} + 0.213X_{34}+ 0.2055X_{35} + 0.2505X_{36} + 0.375X_{37} + 0.132X_{38} + 0.21X_{39} + 0.327$X_{3,10}$
+

0.213$X_{3,11}$ + 0.4335$X_{3,12}$ + 0.054$X_{3,13}$ + 0.4395$X_{3,14}$ + 0$X_{3,15}$ + 0.425X_{41} + 0.1185X_{42} + 0
0.33X_{43} +

0.3225X_{44} + 0.3675X_{45} + 0.492X_{46} + 0.249X_{47} + 0.327X_{48} + 0.2505X_{49}+ 0.0975$X_{4,10}$ +
0.3195$X_{4,11}$

+ 0.171$X_{4,12}$ + 0.5563$X_{4,13}$ + 0$X_{4,14}$ + 0.1125X_{51} + 0.213X_{52} + 0.33X_{53} + 0 0.0105X_{54}+
0.0555X_{55}

0.1785X_{56} + 0.0885X_{57} + 0.015X_{58} + 0.528$X_{5,10}$ + 0.3465$X_{5,11}$ + 0.618$X_{5,12}$+ 0.1485X_{513} +
0.243$X_{5,14}$

Consider Capacity Constraint

$$X_{11} + X_{12} + X_{13} + X_{14} + X_{15} + X_{16} + X_{17} + X_{18} + X_{19} + X_{1,10} + X_{1,11} + \cdots X_{1,14} \leq 9142$$

$$X_{21} + X_{22} + X_{23} + X_{24} + X_{25} + X_{26} + X_{27} + X_{28} + X_{29} + X_{2,10} + X_{2,11} + \cdots X_{2,14} \leq 5379$$

$$X_{31} + X_{32} + X_{33} + X_{34} + X_{35} + X_{36} + X_{37} + X_{38} + X_{39} + X_{3,10} + X_{3,11} + \cdots X_{3,14} \leq 4743$$

$$X_{41} + X_{42} + X_{43} + X_{44} + X_{45} + X_{46} + X_{47} + X_{48} + X_{49} + X_{4,10} + X_{4,11} + \cdots X_{4,14} \leq 7179$$

$$X_{51} + X_{52} + X_{53} + X_{54} + X_{55} + X_{56} + X_{57} + X_{58} + X_{59} + X_{5,10} + X_{5,11} + \cdots X_{5,14} \leq 1493$$

Demand Constraint

.$X_{11} + X_{21} + X_{31} + X_{41} + X_{51} \geq 2332$

$X_{12} + X_{22} + X_{32} + X_{42} + X_{52} \geq 2552$

$X_{13} + X_{23} + X_{33} + X_{43} + X_{53} \geq 2711$

$X_{14} + X_{24} + X_{34} + X_{44} + X_{41} \geq 2160$

$X_{15} + X_{25} + X_{35} + X_{45} + X_{55} \geq 1849$

$X_{16} + X_{26} + X_{36} + X_{46} + X_{56} \geq 2088$

$X_{17} + X_{27} + X_{37} + X_{47} + X_{57} \geq 1823$

$X_{18} + X_{28} + X_{38} + X_{48} + X_{58} \geq 1699$

$X_{19} + X_{29} + X_{39} + X_{49} + X_{59} \geq 1070$

$X_{1,10} + X_{2,10} + X_{3,10} + X_{4,10} + X_{5,10} \geq 1744$

$X_{1,11} + X_{2,11} + X_{3,11} + X_{4,11} + X_{5,11} \geq 1584$

.$X_{1,12} + X_{2,12} + X_{3,12} + X_{4,12} + X_{5,12} \geq 2312$

$X_{1,13} + X_{2,13} + X_{3,13} + X_{4,13} + X_{5,13} \geq 1064$

$X_{1,14} + X_{2,14} + X_{3,14} + X_{4,14} + X_{5,14} \geq 2946$

$X_{1,15} = 28$

$X_{ij} \geq 0$ for all i and j

4.4 COMPUTATIONAL PROCEDURE

The QM 32 is windows-based software designed for use with many of the techniques represented in Operations management theory book. The management scientist 6.0 software packaged was employed to solve this transshipment problem. The management scientist software is mathematical tool solver for optimization and mathematical programming in operations research. The Q M module used is based on simplified version of the simplex technique called The Transportation Simplex Method. The transportation simplex method is a special version of Simplex Method used to solve Transportation and transshipment Problems. It was run on Intel(R) Core(TM) Duo CPU Toshiba machine with 4.0GB of RAM. Based on the data gathered (in Table) that were used in running the Q M program, produced the same output for the ten trials.

Table 4.4: Results Analysis

	Shipment List				Shipment cost
	From	**To**	**Shipment**	**Cost per unit**	
1	Source 1	Destination 4	667000	0.042	28014
2	Source 1	Destination 5	1849000	0.03	55470
3	Source 1	Destination 6	2088000	0.0375	73300
4	Source 1	Destination 7	1750000	0.1605	280875
5	Source 1	Destination 8	1699000	0.108	183492
6	Source 1	Destination9	1070000	0.0315	3370.5
7	Source 1	Destination10	19000	0.3975	7552.5
8	Source 2	Destination 1	2332000	0	0
9	Source 2	Destination 7	73000	0.2235	16315.5
10	Source 2	Destination 14	2946000	0.1815	534699
11	Source 2	Dummy	28000	0	0
12	Source 3	Destination 2	2552000	0	0
13	Source 3	Destination 10	1124000	0.327	367548
14	Source 3	Destination 13	1067000	0.054	57618
15	Source 4	Destination 3	2711000	0	0
16	Source 4	Destination10	571000	0.2505	143035.5
17	Source 4	Destination11	1584000	0.0975	154440
18	Source 4	Destination 12	2312000	0.3195	738684
19	Source 5	Destination 4	1493000	0	0

4.5 Interpretation of Results

The above transshipment problem was solved with linear programming module and transportation module of the Management Scientist, and the optimal solution obtained was the same for each results. The computer solution shows that the minimum total transshipment cost is GH¢27,505.98 The values for the decision variables show the optimal amounts to ship over each route.

From Table 4.4 above the following quantities of crates of minerals 667000, 1849000, 2088000, 1750000, 1699000, 1070000, 19000 were transported from source 1 to destinations 4,5,6,7,8,9 and 10 at a unit cost of 0.042, 0.03, 0.0375, 0.1605, 0.108,0.0315 and 0.3975 respectively. A total of 2332000, 73000, 2946000 and 28000 units were shipped from source 2 to destinations 1,7,14 and Dummy at a unit cost of 0, 0.2235, 0.1815 and 0 respectively. 2552000,1124000 and 1067000 crates were transported to destination 2,10 and 13 at a unit cost of 0, 0.327 and 0.054 from source 3.

The total shipment cost was GH¢27,505.98

Table 4.5 shows final distribution of products (in thousands) between the sources and the various destinations. The last column displays the total supply form each source to the destinations and the last row indicates the demand at each destination points.

Table 4.5: Final distribution of products

	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	DUMMY	SUPPLY
S1	0	0	0	667	1849	2088	1750	1699	1070	19	0	0	0	0	0	9142
S2	2332	0	0	0	0	0	73	0	0	0	0	0	0	2946	28	5379
S3	0	2552	0	0	0	0	0	0	0	1124	0	0	1067	0	0	4743
S4	0	0	2711	0	0	0	0	0	0	571	1584	2312	0	0	0	7178
S5	0	0	0	1493	0	0	0	0	0	0	0	0	0	0	0	1493
DEMAND	2332	2552	2711	2160	1849	2088	1823	1699	1070	1714	1584	2312	1067	2946	28	

CHAPTER 5

CONCLUSION AND RECOMMENDATIONS

5.1 CONCLUSION

The transportation cost is an important element of the total cost structure for any business. The transshipment problem was formulated as a Linear Programming and solved with the standard LP solvers such as the Management scientist module to obtain the optimal solution. The computational results provided the minimal total transshipment cost and the values for the decision variables for optimality. Upon solving the LP problems by the computer package, the optimum solutions provided the valuable information for Coca Cola Company to make optimal decisions.

Through the use of this mathematical model (Transshipment Model) the business (Coca Cola Company) can easily and efficiently plan out its transportation, so that it cannot only minimize the cost of transporting goods and services but also create time utility by reaching the goods and services at the right place at right time. This in turn will enable them to meet the corporative objective such as education fund, entertainment and other support they offered to people of Ghana.

The optimal cost was GH¢27,505.98 as compared to their annual distribution expenditure of GH¢37,480.70. The value for the decision variable produced the optimal amounts to be ship to each distributor of Coca Cola Company.

5.2 RECOMMENDATIONS

Based on the findings of this work and the large difference that exists between the optimal solution and the annual expenditure on distribution of products by the Coca-Cola bottling Company from the source through the transshipment points to the various destinations, the researcher recommends to the management of Coca Cola Company to seek to the application of mathematical theories into their operations as a necessary tool when it comes to decision making.

If the Coca Cola Company managers are to employ the proposed transshipment model it will assist them to efficiently plan out its transshipment scheduled at a minimum cost. There are number of algorithms that can assist in construction of TP and LP problems. In future the researcher recommends the solution of large-scale transshipment problems through aggregation. This proposed method is applicable to any transportation and transshipment problems.

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